Vol. XXXI. Parts I and II July, 1939

BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

FOUNDED BY

W. F. R. WELDON, FRANCIS GALTON AND KARL PEARSON

EDITED BY

EGON S. PEARSON

IN CONSULTATION WITH

HARALD CRAMÉR — J. B. S. HALDANE

R. C. GEARY — G. M. MORANT

MAJOR GREENWOOD — JOHN WISHART

Reprinted by offset-litho 1952, 1963

ISSUED BY THE BIOMETRIKA OFFICE
UNIVERSITY COLLEGE, LONDON
AND PRINTED AT THE
UNIVERSITY PRESS, CAMBRIDGE

PRINTED IN GREAT BRITAIN

[*Issued* 25 *July* 1939]

VOL. XXXI JULY, 1939 PARTS I AND II

RANDOM AND SYSTEMATIC ARRANGEMENTS

By HAROLD JEFFREYS, F.R.S.

I HAVE followed with interest the recent discussions of this question, though I cannot claim personal knowledge of the types of problem where the proposed arrangements are used. My personal feeling, as a seismologist dealing with natural earthquakes, is one of envy of workers in subjects where it is possible to design experiments at all, and especially where they can be designed in such a way that the normal equations for the various parameters to be estimated will contain no cross terms. Nevertheless, I think that some points concerning experimental design are in danger of being overlooked even in the fortunate subjects where design is possible. In particular, the word "randomness" appears to be used in two senses, which are not equivalent. To take an illustration from seismic survey, the experiment would consist in the discharge of an explosion at a known point and the recording of the times of transmission of elastic waves through the ground to a set of recorders at known distances. The choice of the distances is part of the design of the experiment; the problem is to find a pair of parameters a and b as accurately as possible from observations of the times t_r at distances x_r. This will be done by the method of least squares, the equations of condition being

$$a + bx_r = t_r.$$

Now the question of design will be, how should the arbitrary distances x_r be chosen? In practice the intervals are usually made as nearly equal as possible, so that there is a high degree of system in the design. But an advocate of extreme randomness in design would apparently assign all the distances by Tippett's numbers or some analogous device. A possible consequence of this would be that they would all be nearly equal, and it would be impossible to determine either a or b without external evidence concerning the value of the other. Or they might all be concentrated near two values; then a and b could be determined, but there would be no means of testing whether the time of transmission is adequately represented by a linear formula—there are cases in practice where a square or a cube term needs to be included, though the linear form is usually adequate. Thus randomness in design may in some circumstances lead to the loss of information that would be provided by a systematic design.

On the other hand it is claimed that a systematic design may lead to an underestimate of the uncertainty, and it is here that the ambiguity of the use of "randomness" enters. The validity of the method of least squares rests on the hypothesis of randomness in the sense that parameters a and b exist such that, given their values and the standard error, the probabilities of the residuals at all the distances used are independent. This may be true without the distances themselves being assigned at random. Indeed, if the ground is not level, the variations of height will affect the times of transmission for some waves; random-

UNIVERSITY OF UTAH LIBRARIES

ness of design will then permit several observations to be near the same distance, and the probabilities of the residuals, given the distances, will not be independent. If in an actual case the variation of level was harmonic, and the recorders were at uniform intervals so as always to be on the crests, there would be a systematic error in the least squares solution; but this can in this problem be calculated and allowed for. In practice, however, variations of level are usually irregular, and independence of the errors is best achieved by using uniform intervals so as to minimize the increase of uncertainty due to possible correlations between neighbouring distances. This does not exclude the possibility that a closer approximation might be attempted by including a term proportional to the height in the law assumed; but that would still not eliminate possible effects of local variation in the velocities immediately below the recording instruments, and the systematic design would still approximate to the condition of independence of the errors better than the random one would.

My excuse for mentioning this problem in a mainly biological journal is that it is one that I am fairly familiar with, and that it seems to illustrate two points that have not been adequately analysed in the biological literature. The first is the absolute necessity of distinguishing between the probabilities of the same proposition assessed on different data. Before designing the above experiment we may have only the vague knowledge of a and b suggested by similar experiments elsewhere. If we are to make a systematic design we do know something about what the relations between the intervals will be; if we are to make a random one we do not. But in either case, as soon as the experiment is designed, the distances are definitely known in both cases, and are not random any longer. The knowledge of the possible types of design that might have been adopted is no longer relevant in either case, since we know the particular design that *has* been adopted; it is now part of the data of the problem. The estimates will in either case be made using the actual distances as data, not some aggregate of the possible distances that might have occurred with that method of design. The hypothesis of the randomness of the residuals, which is needed for the validity of the method of least squares, has nothing to do, intrinsically, with the intended randomness of the original design; in this case the latter randomness would in practice often vitiate the validity of the former, while in any case increasing the uncertainty of the estimates.

The second point is that any method of estimation presupposes a law of error or chance containing adjustable parameters, such that if these were known the probabilities of various experimental results are assigned—in other words, that the likelihood, given these parameters, has a definite value. Now I maintain that this is always what logicians call a *considered* proposition, not an *asserted* one. There is no proof in any actual case that no parameters other than those so far thought of will ever be required; any theory of significance tests contemplates the possibility that others may be needed. But if a new parameter, whose value

is not yet known, may be relevant, then the law of error, given those already considered, is not unique and the likelihood has not a definite value. In the seismic survey, we do not assert that the relation between distance and time of travel *is* linear; we merely consider it and use it until we find some definite evidence against it in a particular case. But we need the information that will find out if it is wrong; and this must be done by making *sure* that the presence of higher powers can be tested. This is done by the systematic design. The random design would leave it to chance, and abandon the attempt to test the law if it should give an arrangement with all the distances concentrated near two values. It would be pleasant to be able to say that we already know what parameters can be relevant; but we have to face the fact that we do not. What we can do is to provide a procedure for testing them as they arise; but this necessarily implies that we must proceed from the hypothesis involving fewer adjustable parameters to the one involving more. The hypothesis that a limited number of parameters is adequate must be *considered* in any case, but that is not the same as saying that no others will ever be needed. Estimates, however, are always subject to the hypothesis that no parameters other than those explicitly considered are relevant. There appears to be no escape from this dilemma; we can only admit it, state the hypothesis explicitly, and say that the results are the best that we can give in the actual state of our knowledge at the time. Confidence in it will depend mainly on the failure to find evidence for other parameters that might conceivably have been relevant to the observations already available.

Let us now consider how this may apply to problems of sampling a population and the design of agricultural experiments. In the former case the population consists of several different types, of numbers $\nu_1, \nu_2, \ldots, \nu_p$, total ν. The numbers of these types in our sample are $n_1, n_2, \ldots, n_p$, total n. It is not in our power to change the ν_r, and practical considerations may fix n. But $n_1, n_2, \ldots$ are at our disposal. The question is, how should they be chosen? Neyman (1934) has shown that if the standard errors of the respective types are $\sigma_1, \sigma_2, \ldots$, and the means found for the types are x_r, then provided that the ν_r are known the most accurate estimate of the mean for the entire population would be got by taking the n_r in proportion to $\nu_r\sigma_r$, and the estimate as $\Sigma\nu_r x_r/\nu$. If all the σ_r are equal, the n_r should be taken in proportion to the ν_r, and the best estimate of the mean for the population will be the mean over the sample, $\Sigma n_r x_r/n$, irrespective of the differences between different types. The standard error also is independent of these differences. It reduces simply to a matter of estimating the mean for each type from a sample of that type, and its uncertainty is determined only by the variation within types; and the mean for the whole population is the mean of the type means, weighted in proportion to the ν_r, which are in proportion to the n_r. Now this is a highly systematic method of sampling. The random method would be to choose n individuals from the entire population by selecting from a directory by some set of "random numbers". The numbers of the types in the sample

would then depart from proportionality, on account of the usual sampling errors, and if the mean of the sample is taken as the estimate of the mean of the population it will have an additional error on account of the contributions from the products of the type differences and the sampling errors. It is true that the standard error of the result so obtained is correctly estimated from the mean square variation within the sample as a whole; but it will be larger than that of the mean of a proportional sample. If instead of taking the mean of the sample as our estimate of the mean of the population, thus weighting the type means according to n_r, we weight them according to ν_r, we again recover an estimate that is independent of the variation of the type means and therefore has a higher accuracy, approaching that for the proportional sample.

To apply this method it is necessary that the ν_r should be known. If they are unknown the only way of estimating them would be by taking a random sample from the whole population, and their most probable values would be in the ratios of the n_r. Then the mean of the sample would be the best estimate of the population mean, and its standard error would be correctly found from the variation in the whole sample. The point is that information about the ν_r is relevant to the estimate of the population mean and its standard error, given the sample; and this will be our actual position, since our problem is in practice to make inferences about the population given the sample. If we do not know the population numbers apart from what the sample can tell us about them, there is no more to be said; but if we do know them and omit to use them we are discarding valuable information and introducing avoidable error into the estimate of the mean. The conclusions drawn from a given sample are not the same if the ν_r are also part of the data as they are if the ν_r are unknown.

On the other hand it is claimed that the method of random sampling is unbiased. Now the word "bias" also seems to be capable of several interpretations. If the standard deviations within types are different, but their numbers are known, it has been seen that the best estimate will be obtained by deliberately introducing a departure from proportionality into the sampling and allowing for it afterwards by weighting according to ν_r. It might be said, not unfairly, that any procedure that gives an estimate different from the best one using the whole of the available data is biased. It is true that random sampling at the outset is designed to give every possible sample of given size an equal chance of being the actual sample; but when the sample has actually been taken, that is the sample and the intentions of the designer are no longer relevant. If we know that it is not in fact proportional we are entitled to allow for the departure and shall gain accuracy by doing so. The fact that the use of the sample mean as the estimate might give an error with either sign in a future experiment does not imply that in a particular case it is equally likely to have given one with either sign. It is sometimes said that by taking a random sample we avoid dangers that might arise through a systematic sample not being in proportion. Thus Yule & Kendall

(1937, p. 340) mention the problem of sampling in a planned town, where, if we choose houses at equal intervals equal to the length of the blocks, we should get a sample containing either no corner houses or nothing else. But here the difficulty can be avoided either by choosing an interval that is prime to the number of houses in the blocks, or by deliberately taking samples of corner and other houses separately. It is true that such a sample does not give all combinations of houses an equal chance of being chosen, but that appears to be a merit. It excludes the possibility that might still arise in random sampling, that we should miss all the corner houses; and it does not necessarily invalidate the hypothesis of the randomness of the errors. In this case it is not necessary that the houses already chosen should be irrelevant to the identity of the next; it is sufficient that they should be irrelevant to the amount of the departure of the next from the mean of its type, which is a much less stringent condition, and the considerations mentioned in relation to the problem of seismic survey will apply. My point is that if we omit to make use of relevant information that is in our possession, we are making avoidable errors; and in this problem the possible difference between corner and other houses is a known piece of information, and the best procedure is to design the work so as to determine it as accurately as possible and not leave it to chance whether it can be determined at all. It is possible in any case that the hypothesis of independence of the errors may be wrong; no absolute provision against that can be made, and it may lead to unexpected discrepancies. But that is dealt with by recognizing that the hypothesis is a considered proposition, and that the argument is inductive and not deductive; it is not dealt with by including an estimable error that has nothing to do with it.

I have been puzzled by the recent criticisms of Fisher by "Student" (1937). Reading "Student's" paper and Fisher's *Design of Experiments* I find myself in almost complete agreement with both; and I should therefore have expected them to agree with each other. On the matter of the half-drill strip I have no opinion whatever, and leave that to specialists. But it seems to me that "Student" is wrong in regarding Fisher as an advocate of extreme randomness, and possibly Fisher has not sufficiently emphasized the amount of system in his methods. Consider an arrangement intended to compare five varieties or treatments in three 5×5 squares. A die-hard randomist might proceed by numbering the plots in order, 1 to 75, and throw a die till numbers other than sixes have occurred 75 times. If the nth throw, neglecting sixes, gives a five, he will assign the nth plot to the fifth variety, and so on. He might carry out an analogous procedure with some series of random numbers. The result might be that some variety might not appear at all in one square; there would usually be many cases where the same variety occurred two or three times in the same row or column. Different varieties would not occur equally often in the design, and the comparisons between them would have different weights. It is excessively improbable that the result would be three Latin squares. When Fisher makes each variety occur

equally often he introduces one systematic restriction; when he applies this to each square separately he introduces another; when he makes it occur just once in every row and every column he introduces others. The complete random procedure has never, I think, been adopted in such a case. It may be too absurd to have even been considered; but statement of it shows how far Fisher has gone in the direction advocated by "Student". The latter does in fact point out that the Latin square is both random and balanced, but the two properties refer to different features of the design. The systematic features enable the maximum accuracy to be obtained for each variety, consistently with uniformity. But I think that some further attention should be given to the function of the randomizing of rows and columns that is carried out after the main features of the Latin square design are fixed. There would be no point in this if the probabilities of the plot errors, given the varietal, row and column values, were all independent. The hypothesis required by the method of least squares would be satisfied, and the analysis of variance is equivalent to the method of least squares applied to a system where the normal equations contain only diagonal terms. On the other hand a systematic ground effect that upsets the independence of the errors would require the explicit introduction of a new parameter to express it; this would not necessarily enter orthogonally to the others, and the analysis of variance analysis would have to be replaced by a detailed solution by least squares. Now the absence of such terms cannot be guaranteed. To illustrate how they can enter, let us suppose axes of position taken parallel to the sides, with the centre of the square as origin. Then if (x, y) are the coordinates of the centre of a plot, suppose first that the fertility may be expressed in the form

$$F = a_0 + a_1 x + b_1 y + a_2 x^2 + b_2 y^2 + \ldots + b_4 y^4.$$

The elimination of rows and columns, in a 5×5 square, allows for and eliminates all powers of x and y separately up to x^4 and y^4; evidently any set of row and column totals could be fitted by choosing the nine coefficients suitably. But neither the design of the square nor the analysis of variance technique would deal with a term like xy, which might be present. Whatever its coefficient may be, it will contribute nothing to the row and column totals; but Σxy for all plots of a particular variety will not in general vanish—indeed it is possible for xy to have the same sign for four of them, vanishing for the other. Thus the presence of such a term will make a contribution to the estimates of the varietal differences unless provision is made against it. Now this is not merely a theoretical danger. If terms in x^2 and y^2 are relevant it will be only for one particular pair of directions of the sides that the coefficient of xy in the fertility will vanish, and it must be presumed that if there is reason to attend to x^2 and y^2 we should also attend to xy. The most accurate way of doing so would be to introduce it explicitly into the equations of condition and estimate it by the method of least squares. In estimating the uncertainty allowance would have to be made for the fact that an additional parameter has been estimated for every square. But since it will not

enter orthogonally to the varietal differences the labour of calculation would be greatly increased. Again, if x^3 and y^3 are relevant, there is reason to expect x^2y and xy^2 to be, and so on. The analysis would become prohibitively difficult.

Now this is where the use of randomization comes in. In a single 5×5 Latin square it is impossible for Σxy to vanish for every variety. (I do not know whether it would be possible for larger squares, but even if it is, higher product terms would require attention.) But it is possible to arrange the design so that, without evaluating the contributions from xy and analogous terms explicitly, they will contribute independently to the totals. This is done by repeating the square, using separate acts of randomization for each square. It is important to notice that the object would not be achieved if the first square was simply copied. If this was done, the values of Σxy for each treatment in one square, and for one treatment in each of the others, would determine the values for all the other treatments in the other two squares, and the hypothesis of independence would be wrong. The separate randomizations ensure that, with three replications, the contributions of the xy and similar terms to the varietal totals are each the sum of three independent components. Omission to evaluate them explicitly may sacrifice information that could possibly be recovered, and thereby lead to loss of accuracy in estimating the main effects; but so far as that is a criticism it does not apply here to the randomization but to the analysis of variance technique, which will be valid in these circumstances only if some artificial device is introduced to convert the systematic disturbance into one that can legitimately be treated as random. The real justification of using this technique rather than a complete least squares solution, taking into account xy and possibly higher product terms, is practical convenience. The justification of the separate randomization in relation to it is that it prevents differences from being interpreted as due to varietal or treatment differences when they are really due to the accumulation of neglected ground effects. To advocate a least squares solution for the neglected terms may well be analogous to saying that an investigator should carry out all computations to seven figures when he knows quite well that the second will be uncertain in any case.

It seems to me that Fisher sums up the situation very well in his advice to balance or eliminate the larger systematic effects as accurately as possible and randomize the rest. It provides against two dangers: first, that the main object of the work may be lost by an imperfect estimate of the larger effects, especially through great departures of the normal equations from orthogonality; and secondly, that a neglected term, small in any one observation, may mount up when many are combined. But what is worth balancing, what is worth randomizing, at the cost of a small increase of uncertainty, but not worth definitely eliminating, and what is random anyhow, must be a matter of the particular problem. The only rule is to attend to actual conditions and the types of systematic variation likely to arise in them. I should see no point, for instance, in an

elaborate sampling of people by a fixed list of random numbers when a list in alphabetical order is available; if uniform intervals in such a list do not give a random sample in the sense of the independence of the errors I do not know what will. The only systematic variation that could bias the mean would be a periodicity in terms of the position in the table, the wave-length being an exact submultiple of the interval chosen; and this seems too remote a possibility to need much consideration. But a similar difficulty might arise in relation to one experimental layout mentioned by "Student", namely

$$A\ B\ B\ A\ A\ B\ B\ A.$$

Here, as "Student" indeed remarks, the estimated difference between A and B would be biased if there was a harmonic variation of fertility whose period is an odd multiple of the width of a quartet. He dismisses this as a not particularly likely occurrence. But land is often ridged in just this way, and an unwary designer might easily lay out his experiment in such a way that the difference sought could not be separated from the periodic ground effect. In such land I should say that the harmonic effect is likely to be the dominant ground effect and that the best treatment would be to introduce it explicitly into the equations of condition, and lay out the experiment so that the possible phases are uniformly distributed both for A and B.

In the 5×5 Latin square, there is no *logical* reason why we should stop before an expression of possible ground effects by an expression of twenty-five terms with adjustable coefficients; then there would be no means left for separating treatment effects from ground ones. Equally there seems to be none for not treating the whole variation in terms of block mean, treatments, and random error. The place to draw the line between these two extremes cannot be decided by pure theory; it is indicated by previous experience, which has indicated that some types of ground effect are habitually significant, others occasionally significant, and others legitimately treated as random. What Fisher's randomization does is really to combine members of the second class in such a way that they may be included in the third. But extreme randomization would also try to randomize the first class, and the disadvantages of this seem far to outweigh any possible advantage.

However, I must now return to a subject where the normal equations are never orthogonal; where the normal law of error never holds; and where it is impossible to separate the ground effects in different regions from possible systematic differences between the observers inhabiting those regions.

REFERENCES

NEYMAN, J. (1934). *J.R. Statist. Soc.* **97**, 577–80.

"STUDENT" (1937). *Biometrika*, **29**, 363–79.

YULE, G. U. & KENDALL, M. G. (1937). *Introduction to the Theory of Statistics*. London: Griffin.

A NOTE ON NORMAL CORRELATION*

By E. J. G. PITMAN
University of Tasmania

In a recent paper Finney (1938) discussed the distribution of the ratio of estimates of the two variances in a sample from a normal bi-variate population, and showed how a test for significance could be applied when the population correlation coefficient is known. He also showed how the test may be adapted when only a sample estimate of this correlation is available, but this adaptation is not completely satisfactory. This note shows how, by using a different distribution, an exact test for significance can be obtained, and fiducial limits for the ratio of the population variances determined, when the population correlation coefficient is unknown.

Suppose that x, y are normally correlated variables with probability function

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-a)^2}{\sigma_1^2}-2\rho\frac{(x-a)(y-b)}{\sigma_1\sigma_2}+\frac{(y-b)^2}{\sigma_2^2}\right]\right\},$$

and that $x_s, y_s \quad (s = 1, 2, \ldots, n)$

are n pairs of observed values of x, y. Write

$$\bar{x} = \Sigma x_s/n, \qquad \bar{y} = \Sigma y_s/n,$$

$$S_1 = \Sigma(x_s-\bar{x})^2, \qquad S_2 = \Sigma(y_s-\bar{y})^2,$$

$$r = \frac{\Sigma(x_s-\bar{x})(y_s-\bar{y})}{\sqrt{\{\Sigma(x_s-\bar{x})^2 . \Sigma(y_s-\bar{y})^2\}}}.$$

Since
$$-\frac{1}{2(1-\rho^2)}\left[\frac{(x-a)^2}{\sigma_1^2}-2\rho\frac{(x-a)(y-b)}{\sigma_1\sigma_2}+\frac{(y-b)^2}{\sigma_2^2}\right]$$
$$= -\frac{1}{2}\left[\frac{1}{2(1+\rho)}\left\{\frac{x-a}{\sigma_1}+\frac{y-b}{\sigma_2}\right\}^2+\frac{1}{2(1-\rho)}\left\{\frac{x-a}{\sigma_1}-\frac{y-b}{\sigma_2}\right\}^2\right],$$

it is evident that $u = \frac{x}{\sigma_1}+\frac{y}{\sigma_2}, \quad v = \frac{x}{\sigma_1}-\frac{y}{\sigma_2}$

are *independent* normal variables with variances $2(1+\rho)$, $2(1-\rho)$ respectively.

Thus, if $u_s = \frac{x_s}{\sigma_1}+\frac{y_s}{\sigma_2}, \quad v_s = \frac{x_s}{\sigma_1}-\frac{y_s}{\sigma_2}, \qquad (s = 1, 2, \ldots, n),$

the n pairs of numbers u_s, v_s constitute a sample from a normal bi-variate

* [This paper and that which follows by W. A. Morgan were received for publication at about the same date. The subject of both was suggested by D. J. Finney's paper, but the two treatments and applications of the improved solution of his problem, when ρ is unknown, follow quite different lines. Ed.]

population with zero correlation. Hence the distribution of their correlation coefficient,

$$R = \frac{\Sigma(u_s-\bar{u})\,(v_s-\bar{v})}{\sqrt{\{\Sigma(u_s-\bar{u})^2 \cdot \Sigma(v_s-\bar{v})^2\}}},$$

is known.

$$R = \frac{\Sigma\left(\frac{x_s-\bar{x}}{\sigma_1}+\frac{y_s-\bar{y}}{\sigma_2}\right)\left(\frac{x_s-\bar{x}}{\sigma_1}-\frac{y_s-\bar{y}}{\sigma_2}\right)}{\sqrt{\left\{\Sigma\left(\frac{x_s-\bar{x}}{\sigma_1}+\frac{y_s-\bar{y}}{\sigma_2}\right)^2 \cdot \Sigma\left(\frac{x_s-\bar{x}}{\sigma_1}-\frac{y_s-\bar{y}}{\sigma_2}\right)^2\right\}}}$$

$$= \frac{\frac{S_1}{\sigma_1^2}-\frac{S_2}{\sigma_2^2}}{\sqrt{\left\{\left(\frac{S_1}{\sigma_1^2}+\frac{S_2}{\sigma_2^2}+\frac{2r\sqrt{(S_1S_2)}}{\sigma_1\sigma_2}\right)\left(\frac{S_1}{\sigma_1^2}+\frac{S_2}{\sigma_2^2}-\frac{2r\sqrt{(S_1S_2)}}{\sigma_1\sigma_2}\right)\right\}}}$$

$$= \frac{\frac{S_1}{\sigma_1^2}-\frac{S_2}{\sigma_2^2}}{\sqrt{\left\{\left(\frac{S_1}{\sigma_1^2}+\frac{S_2}{\sigma_2^2}\right)^2-4r^2\frac{S_1S_2}{\sigma_1^2\sigma_2^2}\right\}}}.$$

Putting $\omega = \sigma_1^2/\sigma_2^2, \quad w = S_1S_2,$

we have $$R = \frac{w-\omega}{\sqrt{\{(w+\omega)^2-4r^2w\omega\}}}.$$

The values of w and r are given by the sample, hence, from the known distribution of R, fiducial limits for ω can be determined. If we merely wish to test whether the values of S_1 and S_2 are significantly different, i.e. if we wish to test the null hypothesis $\omega = 1$, we simply insert this value of ω and test for significance the corresponding value of the correlation coefficient R. In determining fiducial limits for ω the arithmetic is a little easier if we make use of "Student's" z-distribution, or Fisher's t-distribution. We know that R^2 has a $B(\frac{1}{2}, \frac{1}{2}n-1)$ distribution, and that

$$\frac{R}{\sqrt{(1-R^2)}} = \frac{w-\omega}{\sqrt{\{4(1-r^2)\,w\omega\}}}$$

is distributed like "Student's" z for a sample of $n-1$, while

$$t = \frac{R\sqrt{(n-2)}}{\sqrt{(1-R^2)}} = \frac{(w-\omega)\sqrt{(n-2)}}{\sqrt{\{4(1-r^2)\,w\omega\}}}$$

is distributed like Fisher's t with $n-2$ degrees of freedom.

If α is any given number less than 1, we can determine k such that

$$P\{|t| \leqq k\} = \alpha.$$

The inequality $|t| \leqq k$ is equivalent to

$$(w-\omega)^2 \leqq \frac{4(1-r^2)\,k^2w\omega}{n-2},$$

i.e. $$\omega^2 - 2Kw\omega + w^2 \leqq 0,$$

where $$K = 1 + \frac{2(1-r^2)\,k^2}{n-2};$$

hence $$P\{w(K-\sqrt{(K^2-1)}) \leqq \omega \leqq w(K+\sqrt{(K^2-1)})\} = \alpha.$$

In the example given by Finney (1938, p. 192),

$$n = 173, \quad w = \frac{S_1}{S_2} = \frac{(5{\cdot}299)^2}{(4{\cdot}766)^2}, \quad r = 0{\cdot}878.$$

Substituting these values in the expression for t, and putting $\omega = 1$, we obtain $t = 2{\cdot}902$. As n is large the distribution of t is approximately normal with zero mean and unit standard deviation, and therefore the value of t is significant at the 1% level. It may be noted that in this particular case the exact test gives less significance than Finney's.

For comparison with the case of uncorrelated normal variables we may note that if $\rho = 0$,

$$\frac{w}{w+\omega}$$

has a $B\{\frac{1}{2}(n-1), \frac{1}{2}(n-1)\}$ distribution, and therefore

$$\frac{(w-\omega)^2}{(w+\omega)^2}$$

has a $B\{\frac{1}{2}, \frac{1}{2}(n-1)\}$ distribution. As shown above, for any value of ρ,

$$R^2 = \frac{(w-\omega)^2}{(w+\omega)^2 - 4r^2w\omega}$$

has a $B\{\frac{1}{2}, \frac{1}{2}(n-2)\}$ distribution.

The same method gives very easily* the distribution required for the estimation of ρ when the standard deviations of x and y are the same, $\sigma_1 = \sigma_2 = \sigma$.

$$u = x+y, \quad v = x-y$$

are then independent normal variables with variances $2\sigma^2(1+\rho)$, $2\sigma^2(1-\rho)$;

hence $$\frac{\Sigma(u_s-\bar{u})^2}{2\sigma^2(1+\rho)}, \quad \frac{\Sigma(v_s-\bar{v})^2}{2\sigma^2(1-\rho)}$$

are independent chance variables each distributed like χ^2 with $n-1$ degrees of freedom, and so the distribution of their quotient

$$w = \frac{1-\rho}{1+\rho} \cdot \frac{\Sigma(u_s-\bar{u})^2}{\Sigma(v_s-\bar{v})^2}$$

is known; $w/(1+w)$ has a $B\{\frac{1}{2}(n-1), \frac{1}{2}(n-1)\}$ distribution. The expression for w is

$$w = \frac{1-\rho}{1+\rho} \cdot \frac{1+r}{1-r},$$

where $$r = \frac{2\Sigma(x_s-\bar{x})\,(y_s-\bar{y})}{\Sigma(x_s-\bar{x})^2 + \Sigma(y_s-\bar{y})^2}.$$

* Cf. De Lury (1938), where the same problem is dealt with by a different method.

If x and y have the same mean value a as well as the same standard deviation σ, the mean value of v is 0, and therefore

$$\frac{\Sigma v_s^2}{2\sigma^2(1-\rho)}$$

is distributed like χ^2 with n degrees of freedom. Thus the distribution of the quotient,

$$w = \frac{1-\rho}{1+\rho} \cdot \frac{\Sigma(u_s-\bar{u})^2}{\Sigma v_s^2},$$

is known; $w/(1+w)$ has a $B\{\frac{1}{2}(n-1), \frac{1}{2}n\}$ distribution. The expression for w is

$$w = \frac{1-\rho}{1+\rho} \cdot \frac{1+r}{1-r},$$

where

$$r = \frac{2\Sigma(x_s-\xi)(y_s-\xi)}{\Sigma(x_s-\xi)^2+\Sigma(y_s-\xi)^2},$$

$$\xi = \Sigma(x_s+y_s)/2n.$$

The procedure in this case corresponds exactly to the treatment of fraternal correlation by the analysis of variance.

REFERENCES

FINNEY, D. J. (1938). *Biometrika*, **30**, 190–2.
DE LURY, D. B. (1938). *Ann. Math. Statist.* **9**, 2, 149–51.

A TEST FOR THE SIGNIFICANCE OF THE DIFFERENCE BETWEEN THE TWO VARIANCES IN A SAMPLE FROM A NORMAL BIVARIATE POPULATION

By W. A. MORGAN

Department of Statistics, University College, London

1. Derivation of likelihood ratio test

In a paper published in a recent issue of this Journal D. J. Finney (1938) considered the following questions. A sample of n pairs of variables (x, y) has been drawn from the bivariate normal distribution whose probability law is

$$p(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho_{12}^2)}}\exp\left[-\frac{1}{2(1-\rho_{12}^2)}\left\{\left(\frac{x-\xi_1}{\sigma_1}\right)^2 - 2\rho_{12}\frac{(x-\xi_1)(y-\xi_2)}{\sigma_1\sigma_2} + \left(\frac{y-\xi_2}{\sigma_2}\right)^2\right\}\right], \quad \text{.....(1)}$$

then

(i) What is the probability law of the ratio, ω, where

$$\omega = s_1'/s_2' \quad \text{.....(2)}$$

and
$$(s_1')^2 = \sum_i (x_i-\bar{x})^2/(n-1), \quad (s_2')^2 = \sum_i (y_i-\bar{y})^2/(n-1)? \quad \text{.....(3)}$$

(ii) Could this ratio be used as a criterion to test the hypothesis that $\sigma_1 = \sigma_2$?

Using a more direct method he was first able to confirm a previous result of Bose (1935), giving the probability distribution of ω in the case where $\sigma_1 = \sigma_2$, and hence to show that the chance that ω exceeds a given value, say Ω, could be obtained from the *Tables of the Incomplete Beta Function* (1934), using the relation

$$P\{\omega > \Omega\} = I_x\left(\frac{n-1}{2}, \frac{n-1}{2}\right), \quad \text{.....(4)}$$

where
$$x = \frac{1}{2}\left(1 - \frac{\Omega-\Omega^{-1}}{\sqrt{\{(\Omega+\Omega^{-})^2 - 4\rho_{12}^2\}}}\right). \quad \text{.....(5)}$$

Since the probability expression (4) is dependent on the population correlation ρ_{12}, which will be in general unknown, Finney pointed out that the ratio ω was not altogether a satisfactory criterion to use in testing the hypothesis $\sigma_1 = \sigma_2$, but he put forward a possible method of getting round this difficulty.

The question arises as to whether there is not some other more suitable criterion for testing whether the variances are equal, whose sampling distribution will be independent of ρ_{12} and of any other parameters whose values are not specified by the hypothesis itself. The likelihood ratio method of approach

of Neyman and Pearson may be followed; this method has proved of service in a number of instances where the appropriate criterion was not immediately obvious. Summarized briefly, it involves the following steps:

(*a*) A specification of the set of admissible hypotheses. In the present case these will be defined by the joint probability law of the n pairs of observations,

$$p(x_1, \ldots, x_n; y_1, \ldots, y_n \mid \xi_1, \xi_2, \sigma_1, \sigma_2, \rho_{12})$$
$$= \{2\pi\sigma_1\sigma_2\sqrt{(1-\rho_{12}^2)}\}^{-n} \exp\left[-\frac{n}{2(1-\rho_{12}^2)}\left\{\left(\frac{\bar{x}-\xi_1}{\sigma_1}\right)^2 - 2\rho_{12}\frac{(\bar{x}-\xi_1)(\bar{y}-\xi_2)}{\sigma_1\sigma_2}\right.\right.$$
$$\left.\left. + \left(\frac{\bar{y}-\xi_2}{\sigma_2}\right)^2 + \frac{s_1^2}{\sigma_1^2} - \frac{2\rho_{12}r_{12}s_1s_2}{\sigma_1\sigma_2} + \frac{s_2^2}{\sigma_2^2}\right\}\right], \qquad \ldots\ldots(6)$$

where $\quad -\infty \leqslant \xi_1, \xi_2 \leqslant \infty, \quad 0 \leqslant \sigma_1, \sigma_2 \leqslant \infty, \quad -1 \leqslant \rho_{12} \leqslant 1.$

In this expression $\bar{x}$ and $\bar{y}$ are the sample means, r_{12} the sample correlation coefficient, and

$$s_1^2 = \sum_i (x_i - \bar{x})^2/n, \quad s_2^2 = \sum_i (y_i - \bar{y})^2/n. \qquad \ldots\ldots(7)$$

(*b*) A determination of those values of the five unknown parameters, as functions of the observations, which jointly maximize the expression (6). The solution is known to be obtained when

$$\xi_1 = \bar{x}, \quad \xi_2 = \bar{y}, \quad \sigma_1 = s_1, \quad \sigma_2 = s_2, \quad \rho_{12} = r_{12}. \qquad \ldots\ldots(8)$$

The maximum value of (6) is thus

$$p_1(\max) = \{2e\pi s_1 s_2 \sqrt{(1-r_{12}^2)}\}^{-n}. \qquad \ldots\ldots(9)$$

(*c*) A specification of the hypothesis tested. This hypothesis assumes that the probability law is of the form

$$p(x_1, \ldots, x_n; y_1, \ldots, y_n \mid \xi_1, \xi_2, \sigma, \rho_{12}),$$

where the function is obtained by putting $\sigma_1 = \sigma_2 = \sigma$ in (6).

(*d*) A determination of the values of the four unknown parameters ξ_1, ξ_2, σ, and ρ_{12} which maximize this expression. These values may be shown to be

$$\xi_1 = \bar{x}, \quad \xi_2 = \bar{y}, \quad \sigma = \sqrt{\{\tfrac{1}{2}(s_1^2 + s_2^2)\}}, \quad \rho_{12} = 2r_{12}s_1s_2/(s_1^2 + s_2^2). \qquad \ldots\ldots(10)$$

The maximum value of the probability function defined in (*c*) then becomes

$$p_2(\max) = \{e\pi\sqrt{[(s_1^2 + s_2^2)^2 - 4r_{12}^2 s_1^2 s_2^2]}\}^{-n}. \qquad \ldots\ldots(11)$$

(*e*) The likelihood ratio criterion is then

$$\lambda = \frac{p_2(\max)}{p_1(\max)} = \left\{\frac{4s_1^2 s_2^2(1-r_{12}^2)}{(s_1^2+s_2^2)^2 - 4r_{12}^2 s_1^2 s_2^2}\right\}^{\frac{1}{2}n} \qquad \ldots\ldots(12a)$$
$$= \left\{1 - \frac{(s_1^2 - s_2^2)^2}{(s_1^2+s_2^2)^2 - 4r_{12}^2 s_1^2 s_2^2}\right\}^{\frac{1}{2}n}. \qquad \ldots\ldots(12b)$$

(*f*) The hypothesis tested becomes less and less likely as λ moves from 1 to 0. To complete the test it is necessary to know the sampling distribution of λ, or of a single valued function of λ, when the hypothesis tested is true.

It will be seen at once that the test differs from Finney's because the criterion,

unlike his ω, is a function of r_{12} as well as s_1^2 and s_2^2. The meaning of the criterion which has been picked out by the λ-method becomes clear if we make the following transformation of the original variables:

Write $$x = X + Y, \quad y = X - Y, \qquad \text{.....(13)}$$

so that $$X = \tfrac{1}{2}(x+y), \quad Y = \tfrac{1}{2}(x-y). \qquad \text{.....(14)}$$

Then the population variances of x and y may be expressed as functions of the variances and correlation for X and Y, as follows:

$$\left.\begin{aligned} \sigma_1^2 &= \sigma_X^2 + \sigma_Y^2 + 2\rho_{XY}\sigma_X\sigma_Y, \\ \sigma_2^2 &= \sigma_X^2 + \sigma_Y^2 - 2\rho_{XY}\sigma_X\sigma_Y. \end{aligned}\right\} \qquad \text{.....(15)}$$

The necessary and sufficient condition that the hypothesis tested is true, or that $\sigma_1 = \sigma_2$, is that $$\rho_{XY} = 0. \qquad \text{.....(16)}$$

Since X and Y are normally correlated variables, the appropriate criterion to test the hypothesis, $\rho_{XY} = 0$, is the sample correlation coefficient between the transformed variables, i.e. r_{XY}. If the hypothesis is true, this coefficient has the well-known probability law

$$p(r_{XY} \mid \rho_{XY} = 0) = \text{constant} \times (1 - r_{XY}^2)^{\frac{1}{2}(n-4)}. \qquad \text{.....(17)}$$

Making use of the transformations (14), it is found that

$$r_{XY} = \frac{s_1^2 - s_2^2}{\{(s_1^2 + s_2^2)^2 - 4r_{12}^2 s_1^2 s_2^2\}^{\frac{1}{2}}}. \qquad \text{.....(18)}$$

Hence the likelihood criterion of (12b) is seen to be

$$\lambda = \{1 - r_{XY}^2\}^{\frac{1}{2}n}, \qquad \text{.....(19)}$$

and as the hypothesis tested becomes less and less likely, $\lambda \to 0$ or $r_{XY}^2 \to 1$. The test may therefore be carried out by (a) referring the r_{XY} of (18) to the probability distribution (17), or (b) alternatively referring

$$t = \frac{r_{XY}\sqrt{(n-2)}}{\sqrt{(1 - r_{XY}^2)}} \qquad \text{.....(20)}$$

to "Student's" distribution with degrees of freedom $f = n - 2$, and (c) rejecting the hypothesis when $| r_{XY} |$ or $| t |$ fall beyond the desired probability level. The test, it will be seen, is independent of the unknown correlation ρ_{12} between x and y.

2. THE POWER OF THE TEST

While the probability distribution of r_{XY} is independent of ρ_{12} if the hypothesis ($\sigma_1 = \sigma_2$) is true, the chance that in using the rejection rule of the test we shall detect real differences between σ_1 and σ_2 will depend on the value of ρ_{12}. Suppose that we fix a probability level $r(\alpha)$ such that

$$\alpha = 2\int_{r(\alpha)}^{1} c(1 - r^2)^{\frac{1}{2}(n-4)}\, dr, * \qquad \text{.....(21)}$$

where α, for example, equals 0·05.

* c is the constant of the probability law (17).

Then the chance of rejecting the hypothesis that

$$\gamma = \sigma_1/\sigma_2 = 1, \qquad \text{.....(22)}$$

when $\gamma = \gamma_1 \neq 1$, is given by the expression

$$P\{r_{XY}^2 > r^2(\alpha) \,|\, \gamma = \gamma_1\} = P\{r^2 > r^2(\alpha) \,|\, \rho = \rho_1 \neq 0\}$$
$$= 1 - \int_{-r(\alpha)}^{r(\alpha)} p(r \,|\, \rho = \rho_1)\, dr. \qquad \text{.....(23)}$$

This expression Neyman & Pearson (1936) have termed the power of the test of the hypothesis that $\gamma = \sigma_1/\sigma_2 = 1$ with regard to an alternative hypothesis $\gamma = \gamma_1$. In the expression (23), $p(r \,|\, \rho = \rho_1)$ denotes the general probability law for r in samples from a bivariate normal population, first obtained by R. A. Fisher (1915). It will be seen that a relation similar to (18) holds between ρ_{XY}, $\gamma = \sigma_1/\sigma_2$ and ρ_{12}, namely

$$\rho_{XY} = \frac{\gamma - \gamma^{-1}}{\{(\gamma - \gamma^{-1})^2 + 4(1-\rho_{12}^2)\}^{\frac{1}{2}}}. \qquad \text{.....(24)}$$

Owing to the symmetry of the distribution of r when $\rho = 0$, the power of the test will be the same for alternatives ρ_{XY} and $-\rho_{XY}$; it will therefore be the same for alternatives γ and γ^{-1}. For example, the test is as likely to reject the hypothesis $(\sigma_1 = \sigma_2)$ when $\sigma_1 = 2\sigma_2$ as when $\sigma_1 = \frac{1}{2}\sigma_2$. This is clearly what we should expect, as σ_1 and σ_2 are in no way differentiated.

In Fig. 1 I have shown the power function of the test, taking $\alpha = 0{\cdot}10$* and sample size $n = 25$, for alternatives $\gamma > 1$, in the three cases $\rho_{12} = 0$, $\rho_{12} = 0{\cdot}5$, and $\rho_{12} = 0{\cdot}8$. It will be noticed that the test is more powerful when ρ_{12} is large. This of course follows from (24), since for a given value of γ, ρ_{XY} will be further from zero the nearer $|\rho_{12}|$ is to unity.

The computations were made with the help of F. N. David's recently published *Tables* (1938). The work was simplified by taking ρ_{XY} at convenient values $0{\cdot}1, 0{\cdot}2, \ldots, 0{\cdot}9$, and finding the corresponding values of γ by means of (24).

The table on p. 18 shows, in the columns headed Test (a), the values of the power function computed in this way for this case of $n = 25$ and also for $n = 12$ and $n = 100$.

3. Comparison with Finney's test in the case where ρ_{12} is known

In the case where ρ_{12} is not known, Finney has suggested that his test criterion ω might be used by making a double appeal to significance levels on the lines proposed in another case by Hirschfeld (1937). It does not, however, appear easy to determine numerically the power of the resulting test. In the case where ρ_{12} is known, it may be shown that the likelihood ratio criterion now becomes

$$\lambda = \left\{1 + \frac{(s_1 - s_2)^2}{2s_1 s_2(1 - r_{12}\rho_{12})}\right\}^{-n} = \left\{1 + \frac{(\omega - 1)^2}{2\omega(1 - r_{12}\rho_{12})}\right\}^{-n}. \qquad \text{.....(25)}$$

* This means that the hypothesis is to be rejected when r_{XY} falls beyond the 5% level in either tail of the distribution (17), or for the case $n = 25$, when $|r_{XY}| > 0{\cdot}3365$.

This is not solely a function of Finney's criterion ω, since it depends again on the sample correlation r_{12}. The form of this expression is interesting, as it shows that if ρ_{12} is known and differs from zero, in view of the correlation which exists between s_1, s_2, and r_{12},* the sample values of all three are relevant in examining the hypothesis that $\sigma_1 = \sigma_2$. I have not succeeded in determining the sampling distribution of the λ of (25), or of any single-valued function of λ.

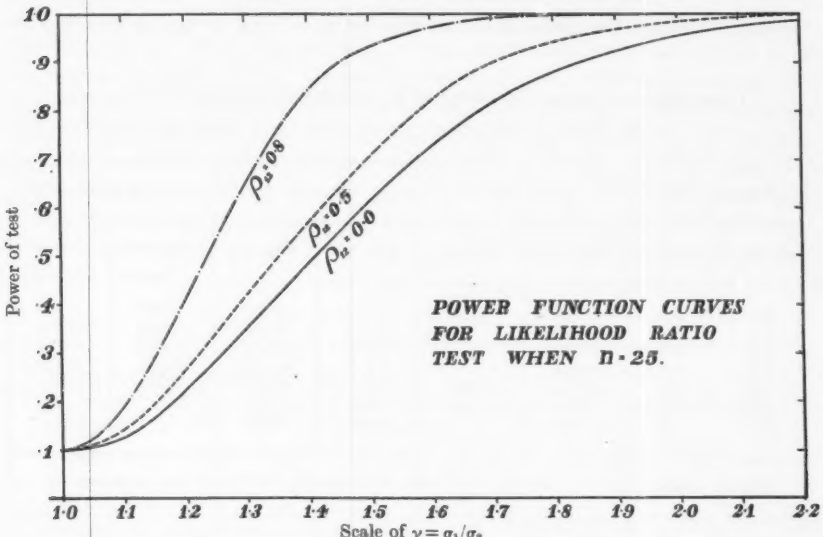


Fig. 1.

In the case of ρ_{12} known, it is, however, possible to compare Finney's test (involving ρ_{12}) with the likelihood test appropriate in the case where ρ_{12} is unknown, but still of course applicable when it is known. The power function for Finney's test may be computed as follows.

When using this test the hypothesis that $\gamma = 1$ is rejected if $\omega > \Omega$ or if $\omega < \Omega^{-1}$, where Ω is a constant chosen by using relations (4) and (5) so that

$$\int_{\Omega}^{\infty} p(\omega \mid \gamma = 1)\, d\omega = \tfrac{1}{2}\alpha. \qquad \text{.....(26)}$$

The power of the test with regard to an alternative hypothesis, $\gamma \neq 1$, is therefore given by

$$F(\gamma) = 1 - P\{\Omega^{-1} < \omega < \Omega\} = \int_0^{\Omega^{-1}} p(\omega \mid \gamma)\, d\omega + \int_{\Omega}^{\infty} p(\omega \mid \gamma)\, d\omega, \qquad \text{.....(27)}$$

where

$$p(\omega \mid \gamma) = \text{constant} \times (1-\rho_{12}^2)^{\frac{1}{2}(n-1)} \frac{\gamma^2+\omega^2}{\gamma\omega^2} \left\{ \left(\frac{\gamma^2+\omega^2}{\gamma\omega} \right)^2 - 4\rho_{12}^2 \right\}^{-\frac{1}{2}n}. \qquad \text{.....(28)}$$

* See K. Pearson (1913).

The above integrals may be transformed to give

$$F(\gamma) = 1 - I_{x_1}\left(\frac{n-1}{2}, \frac{n-1}{2}\right) + I_{x_2}\left(\frac{n-1}{2}, \frac{n-1}{2}\right), \qquad \text{.....(29)}$$

where $x_1 = \frac{1}{2}[1 + (\gamma\Omega - \gamma^{-1}\Omega^{-1})\sqrt{\{(\gamma\Omega - \gamma^{-1}\Omega^{-1})^2 + 4(1-\rho_{12}^2)\}}]$

and $x_2 = \frac{1}{2}[1 - (\gamma^{-1}\Omega - \gamma\Omega^{-1})\sqrt{\{(\gamma^{-1}\Omega - \gamma\Omega^{-1})^2 + 4(1-\rho_{12}^2)\}}]$ $\Bigg\}$, ...(30)

so that values of the power function may be calculated using the *Tables of the*

Comparison of power functions of (a) Likelihood test (based on r_{XF}), (b) Finney's test (based on ω), when ρ_{12} is known

Case $\rho_{12} = 0{\cdot}0$		$n = 12$		$n = 25$		$n = 100$	
$\gamma = \sigma_1/\sigma_2$	ρ_{XY}	Test (*a*)	Test (*b*)	Test (*a*)	Test (*b*)	Test (*a*)	Test (*b*)
1·00	0	0·100	0·100	0·100	0·100	0·100	0·100
1·11	0·1	0·116	—	0·138	0·139	0·262	0·262
1·22	0·2	0·167	—	0·254	0·254	0·642	0·642
1·36	0·3	0·254	—	0·437	0·438	0·922	0·922
1·53	0·4	0·379	0·382	0·652	0·653	0·979	—
2·00	0·6	0·710	0·715	0·953	0·954	—	—
3·00	0·8	0·965	—	1·000	1·000	—	—

Case $\rho_{12} = 0{\cdot}5$		$n = 12$		$n = 25$		$n = 100$	
$\gamma = \sigma_1/\sigma_2$	ρ_{XY}	Test (*a*)	Test (*b*)	Test (*a*)	Test (*b*)	Test (*a*)	Test (*b*)
1·00	0	0·100	0·100	0·100	0·100	0·100	0·100
1·09	0·1	0·116	0·116	0·138	0·138	0·262	0·261
1·19	0·2	0·167	0·166	0·254	0·253	0·642	0·643
1·31	0·3	0·254	—	0·437	0·437	0·922	0·924
1·45	0·4	0·379	0·379	0·652	0·656	0·979	—
1·84	0·6	0·710	0·722	0·953	0·958	—	—

Case $\rho_{12} = 0{\cdot}8$		$n = 12$		$n = 25$		$n = 100$	
$\gamma = \sigma_1/\sigma_2$	ρ_{XY}	Test (*a*)	Test (*b*)	Test (*a*)	Test (*b*)	Test (*a*)	Test (*b*)
1·00	0	0·100	0·100	0·100	0·100	0·100	0·100
1·06	0·1	0·116	0·115	0·138	0·137	0·262	0·260
1·13	0·2	0·167	—	0·254	0·250	0·642	0·643
1·21	0·3	0·254	—	0·437	0·436	0·922	0·926
1·30	0·4	0·379	0·375	0·652	0·660	0·979	—
1·55	0·6	0·710	0·735	0·953	0·964	—	—
2·08	0·8	0·965	0·981	1·000	1·000	—	—

Incomplete Beta Function. These values are compared in the table on p. 18 above with those for the likelihood ratio test. It will be seen that:

(1) When $\rho_{12} = 0$, i.e. when the two variables are known to be independent, the test based on ω or the ratio of sample variances is the better. This is a result already known, but the table shows how small is the difference between the tests.

(2) When ρ_{12} is 0·5 or 0·8, however, the likelihood ratio test is somewhat more sensitive for the small departures in γ from unity, and less sensitive for large departures than the ω test. Practically, this means that when ρ_{12} is known, the ω test is slightly the better, since we are most interested in situations where the chance of detection of a difference becomes large, say greater than 0·5 at any rate. It is possible that a test based on the criterion given in equation (25), if it could be obtained, would be more powerful than either of the other two tests when ρ_{12} is known. In practical cases, however, it will nearly always happen that ρ_{12} is unknown, and in such cases the r_{XY} of (18) appears the appropriate test criterion to use.

REFERENCES

BOSE, S. (1935). *Sankhyā*, **1**, 65.
DAVID, F. N. (1938). *Tables of the Correlation Coefficient. Biometrika* publication.
FINNEY, D. J. (1938). *Biometrika*, **30**, 190.
FISHER, R. A. (1915). *Biometrika*, **10**, 507.
HIRSCHFELD, H. O. (1937). *Biometrika*, **29**, 65.
PEARSON, K. (1913). *Biometrika*, **9**, 1.
NEYMAN, J. & PEARSON, E. S. (1936). *Statist. Res. Mem.* **1**, 1.
Tables of the Incomplete Beta Function (1934). *Biometrika* publication.

THE DISTRIBUTION OF RANGE IN SAMPLES FROM A NORMAL POPULATION, EXPRESSED IN TERMS OF AN INDEPENDENT ESTIMATE OF STANDARD DEVIATION

By D. NEWMAN

Department of Statistics, University College, London

1. Introduction

Starting from the contribution of Tippett (1925), a considerable amount of computational work has been carried out in recent years with the object of making possible the use of range, i.e. the distance between the highest and lowest observation, when dealing with samples from a normal population. Thus Tippett's tables of the mean range expressed in terms of the population standard deviation, σ, for sample sizes $n = 2$ to 1000 have been republished in *Tables for Statisticians and Biometricians*, Part II, Table XXII (K. Pearson, 1931). Later E. S. Pearson (1932) gave a table containing the standard deviation of range, and also the approximate upper and lower 10, 5, 1 and 0·5 % probability levels for sample sizes $n = 2$ to 100, again in terms of σ as unit. In doing this he used empirical Pearson-type curves with correct moments, and checked his results against some experimental sampling distributions.

If a number of small samples are available, it has been shown that a rapid estimate of σ may be obtained from the mean value of the range, which is only slightly less accurate than the estimate obtained from the sums of squares. Again, owing to the high correlation between range and standard deviation in a sample of size 10 or less, it was pointed out by Pearson & Haines (1935) that range may be usefully substituted for standard deviation in control charts used to study changes in the variation of quality in industry. In all these cases, however, the basic sampling distribution used has been that of the ratio of range to σ.

Not very long ago "Student" (the late Mr W. S. Gosset) suggested to Prof. E. S. Pearson that it might be useful to know more about the sampling distribution of the ratio

$$q = w/s,$$

where w is the range in a sample of n observations from a normal population with standard deviation σ, and s^2 is an *independent* and unbiassed estimate of σ^2 based on f degrees of freedom, obtained from a sum of squares in the usual manner. The type of problem which "Student" had in mind was one in which, as a result of an experiment, a number of 'treatment' means, say $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$, are available, and also an independent estimate, s^2, of their sampling variance. Then a rapid method of judging whether any treatment differences exist would consist in comparing the difference between the highest and lowest treatment means, say

$w = \bar{x}_n - \bar{x}_1$, with s. Should this difference be clearly significant, having regard to the values of n and f, the more divergent of the extremes, say $\bar{x}_1$, could be set aside, and the difference $\bar{x}_n - \bar{x}_2$ compared with s, using $n-1$ and f. This procedure would in fact be similar to that suggested by "Student" himself in his paper on "Errors in routine analysis" (1927, pp. 161–2), except that the ratio used would now be w/s rather than w/σ. Of course the probability levels for the former will tend to those of the latter ratio as $f \to \infty$.

In the following sections I shall first make use of the results obtained by Prof. Pearson in computing probability levels for w/σ, to determine appropriate levels for w/s, and then illustrate "Student's" suggestion on three practical examples. It should be noted that in a recent paper H. O. Hartley (1938) has suggested a systematic method of obtaining probability levels for "studentized" functions. It is hoped that before long fuller and more accurate tables may be available to supplement the present tables which rest to some extent on an empirical basis.

2. The expectation of $q = w/s$

Before describing the method of quadrature by which the probability levels were obtained, it may be useful to give a table from which the expectation of q for various values of n and f can be calculated. Since $s^2 = \sum_{i=1}^{f+1}(x_i - \bar{x})^2/f$, we have for the probability distribution of s,

$$p(s) = \frac{f^{\frac{1}{2}f}}{2^{\frac{1}{2}(f-2)}\Gamma(\frac{1}{2}f)\,\sigma^f}\, s^{f-1} e^{-fs^2/2\sigma^2} \qquad (1)$$

If we write $p(w)$ for the probability distribution of range, a function whose precise value is only known for the cases $n = 2, 3$, and denote an expectation by the symbol E, we have

$$\begin{aligned} E(q) &= \int_0^\infty p(q)\, q\, dq \\ &= \int_0^\infty \int_0^\infty w s^{-1} p(w)\, p(s)\, dw\, ds, \text{ since } w \text{ and } s \text{ are independent,} \\ &= \int_0^\infty w p(w)\, dw \times \int_0^\infty s^{-1} p(s)\, ds \\ &= E(w) \times \int_0^\infty s^{-1} p(s)\, ds. \end{aligned}$$

Since the values of $E(w/\sigma)$ for changing n have been tabled by Tippett, it is only necessary to consider the integral

$$\begin{aligned} \int_0^\infty \sigma s^{-1} p(s)\, ds &= \frac{f^{\frac{1}{2}f}}{2^{\frac{1}{2}(f-2)}\Gamma(\frac{1}{2}f)\,\sigma^{f-1}} \int_0^\infty s^{f-2} e^{-fs^2/2\sigma^2}\, ds \\ &= \sqrt{(\tfrac{1}{2}f)}\,\Gamma\{\tfrac{1}{2}(f-1)\}/\Gamma(\tfrac{1}{2}f). \end{aligned}$$

Hence it follows that

$$E(q) = E(w/\sigma) \times \sqrt{(\tfrac{1}{2}f)}\,\Gamma\{\tfrac{1}{2}(f-1)\}/\Gamma(\tfrac{1}{2}f). \tag{2}$$

A brief table of the second function is given below; the values of $E(w/\sigma)$ may be obtained from Tippett (1925), pp. 386–7, or from *Tables for Statisticians and Biometricians*, Part II, Table XXII.

TABLE I

Factors by which to multiply $E(w/\sigma)$ to obtain $E(q)$

Degrees of freedom	3	5	7	10	20	30	∞
Factor	1·382	1·189	1·126	1·084	1·040	1·026	1·000

3. Computation of table of 5 and 1% significance levels for $q = w/s$

The problem is to determine, for different values of n and f, values q_α such that

$$\alpha = \int_{q_\alpha}^{\infty} p(q)\,dq$$
$$= \int_0^{\infty} \{p(w) \int_0^{w/q_\alpha} p(s)\,ds\}\,dw, \tag{3}$$

where $\alpha = 0{\cdot}05$ and $0{\cdot}01$. Since the distribution of q will clearly be independent of the population standard deviation σ, we may take σ as unity in the probability functions used in (3). Except in the cases $n = 2, 3$, which will be referred to again below, the procedure adopted was as follows:

(*a*) The ordinates of the empirical curves, say $y(w)$, obtained by E. S. Pearson (1932) for the cases $n = 4, 6, 10$, and 20 were used in place of the unknown $p(w)$. These ordinates had been calculated at equal intervals of $0{\cdot}1$ for w, the population standard deviation being the unit.

(*b*) Taking a trial value of q_α, the integrals $J(w, q_\alpha) = \int_0^{w/q_\alpha} p(s)\,ds$ were calculated, with the help of the *Tables of the Incomplete Gamma Function* (K. Pearson, 1922), for each value of w used in (*a*). Thus $J(w, q_\alpha) = I(u, p)$ in the notation of the tables, where

$$u = (\sqrt{\tfrac{1}{2}}f)\left(\frac{w}{q_\alpha}\right)^2, \quad p = \tfrac{1}{2}f - 1. \tag{4}$$

(*c*) It was then necessary to apply quadrature to the products $y(w) \times J(w, q_\alpha)$, calculated at intervals $0{\cdot}5$ for w, through as much of the range $w = 0$ to $w = \infty$ as was necessary to obtain the required degree of accuracy.

(*d*) The resulting expression would of course not correspond to α exactly, as q_α had been guessed. Other trial values were taken for q_α, and the final value corresponding to $\alpha = 0{\cdot}05$ or $0{\cdot}01$ obtained by backward interpolation. Starting with the case $n = 2$, where the exact value of q_α could be obtained for all f, and knowing for $n > 2$ the limits to which q_α tended as $f \to \infty$, this process of trial and error was not found too laborious.

Special case when $n = 2$

In this case, taking $\sigma = 1$, the distribution of w assumes the simple form

$$p(w) = \frac{1}{\sqrt{\pi}} e^{-\frac{1}{4}w^2} \quad \text{for } 0 \leqslant w < \infty. \tag{5}$$

Hence the joint probability distribution of w and s is

$$p(w, s) = \frac{f^{\frac{1}{2}f}}{2^{\frac{1}{2}(f-2)}\pi^{\frac{1}{2}}\Gamma(\frac{1}{2}f)} s^{f-1} e^{-\frac{1}{2}w^2 - \frac{1}{2}fs^2} \tag{6}$$

Transforming to variables $q = w/s$ and s, since

$$\left| \frac{\partial(w, s)}{\partial(q, s)} \right| = s, \tag{7}$$

we obtain $$p(q, s) = \text{constant} \times s^f e^{-\frac{1}{2}s^2(q^2+2f)}. \tag{8}$$

Now integrating for s between the limits 0 and ∞, we obtain for the probability distribution of q

$$p(q) = \sqrt{\left(\frac{2}{f\pi}\right)} \frac{\Gamma\{\frac{1}{2}(f+1)\}}{\Gamma(\frac{1}{2}f)} \left(1 + \frac{q^2}{2f}\right)^{-\frac{1}{2}(f+1)} \quad \text{for } q \geqslant 0. \tag{9}$$

This corresponds to the positive half of a "Student" distribution having f degrees of freedom. Values of q_α satisfying the relation $\alpha = \int_{q_\alpha}^{\infty} p(q)\,dq$ may therefore be obtained from R. A. Fisher's (1938) tables of the percentage points for t. Thus

$$q_\alpha = t_\alpha \sqrt{2}, \tag{10}$$

where t_α will be respectively the 5 and 1 % levels for t.*

Special case when $n = 3$

For this case McKay & Pearson (1933) have given an expression for the true distribution of w. The quadrature method employed when $n > 3$ was again used, but the true values of $p(w)$ were taken, and not the ordinates of the empirical curve upon which E. S. Pearson (1932) based his original tables of percentage limits for w.

The following table shows the framework of values for $q_{0\cdot05}$ and $q_{0\cdot01}$ obtained as has been described. Values for $f = \infty$ were of course already known, and values for $n = 2$ and 3 are exact.

From Table II, the more complete working Tables III and IV were obtained by interpolation. It was found that the changes in percentage levels, both with increasing n and f, ran most smoothly if the arguments $60/n$ and $60/f$ were used in place of n and f. On this basis, interpolation from the framework values was effected, using five and six-point Lagrangian formulae. Various checks were

* These levels correspond to deviations at which the ordinates cut off 2·5 and 0·5 % from each end of the t-distribution, but they are ter.ned by Fisher the 5 and 1 % levels.

TABLE II

Framework values for $q_{0\cdot05}$ and $q_{0\cdot01}$

n / f	5% points						1% points					
	2	3	4	6	10	20	2	3	4	6	10	20
5	3·64	4·60	5·22	6·03	7·00	8·21	5·70	6·99	7·83	8·94	10·26	11·95
10	3·15	3·88	4·34	4·92	5·60	6·47	4·48	5·27	5·77	6·42	7·21	8·22
20	2·95	3·58	3·97	4·45	5·01	5·71	4·02	4·63	5·01	5·50	6·08	6·82
30	2·89	3·49	3·86	4·31	4·82	5·47	3·89	4·45	4·78	5·23	5·75	6·40
∞	2·77	3·31	3·65	4·04	4·48	5·01	3·64	4·12	4·38	4·74	5·15	5·64

carried out, as for example that of comparing the values obtained by this method with the known true values in the case of $n = 2$. Finally, the figures were reduced to one place of decimals, and these are given in Tables III and IV.

4. Illustrative examples

In the following examples the range test is used as an alternative to the z-test; the latter is, on theoretical grounds, the more efficient of the two in the sense that it is the more likely to detect the presence of real differences if they exist. Both "Student" and L. H. C. Tippett have, however, held that situations may be met, particularly when dealing with industrial problems, where the gain in speed following the use of range justifies the relatively small loss in efficiency. No doubt other types of examples besides those illustrated below will occur to the reader.

Example A

Fisher (1937, p. 93) has given the results of a 6×6 Latin square experiment in which six different fertilizer treatments were applied to a crop of potatoes. Denoting these treatments by the letters $A, B, \ldots, F$, the mean yields per plot in lb. were as follows:

A	*B*	*C*	*D*	*E*	*F*
345·0	426·5	477·8	405·2	520·2	601·8

The analysis of variance table is as follows:

Sources of variation	Degrees of freedom	Mean squares
Rows	5	10,839·72
Columns	5	4,893·45
Treatments	5	49,635·98
Error	20	1,527·05

TABLE III

5 % points for $q = w/s$

f \ n	2	3	4	5	6	7	8	9	10	11	12	20
5	3·64	4·6	5·2	5·7	6·0	6·3	6·6	6·8	7·0	7·2	7·3	8·2
6	3·46	4·4	4·9	5·3	5·6	5·9	6·1	6·3	6·5	6·7	6·8	7·6
7	3·34	4·2	4·7	5·1	5·4	5·6	5·8	6·0	6·2	6·3	6·5	7·2
8	3·26	4·1	4·5	4·9	5·2	5·4	5·6	5·8	5·9	6·0	6·2	6·9
9	3·20	4·0	4·4	4·7	5·0	5·2	5·4	5·6	5·7	5·8	5·9	6·6
10	3·15	3·9	4·3	4·7	4·9	5·1	5·3	5·5	5·6	5·7	5·8	6·5
11	3·11	3·8	4·3	4·6	4·8	5·0	5·2	5·4	5·5	5·6	5·7	6·3
12	3·08	3·8	4·2	4·6	4·8	5·0	5·1	5·3	5·4	5·5	5·6	6·2
13	3·05	3·7	4·2	4·5	4·7	4·9	5·0	5·2	5·3	5·4	5·5	6·1
14	3·03	3·7	4·1	4·4	4·6	4·8	5·0	5·2	5·3	5·4	5·5	6·0
15	3·01	3·7	4·1	4·4	4·6	4·8	4·9	5·1	5·2	5·3	5·4	6·0
16	3·00	3·6	4·1	4·4	4·6	4·7	4·9	5·0	5·1	5·2	5·3	5·9
17	2·98	3·6	4·0	4·3	4·5	4·7	4·8	5·0	5·1	5·2	5·3	5·8
18	2·97	3·6	4·0	4·3	4·5	4·7	4·8	5·0	5·1	5·2	5·3	5·8
19	2·96	3·6	4·0	4·3	4·5	4·6	4·8	4·9	5·0	5·1	5·2	5·8
20	2·95	3·6	4·0	4·2	4·5	4·6	4·7	4·9	5·0	5·1	5·2	5·7
24	2·92	3·5	3·9	4·2	4·4	4·6	4·7	4·8	4·9	5·0	5·1	5·6
30	2·89	3·5	3·9	4·1	4·3	4·5	4·6	4·7	4·8	4·9	5·0	5·5
40	2·86	3·4	3·8	4·0	4·2	4·3	4·5	4·6	4·7	4·8	4·9	5·4
60	2·83	3·4	3·8	4·0	4·2	4·3	4·4	4·5	4·6	4·7	4·8	5·2
∞	2·77	3·31	3·65	3·87	4·04	4·18	4·29	4·39	4·48	4·55	4·62	5·01

TABLE IV

1 % points for $q = w/s$

f \ n	2	3	4	5	6	7	8	9	10	11	12	20
5	5·70	7·0	7·8	8·5	8·9	9·3	9·6	10·0	10·3	10·5	10·7	12·0
6	5·24	6·4	7·0	7·5	7·9	8·3	8·5	8·9	9·1	9·3	9·5	10·5
7	4·95	5·9	6·5	7·0	7·4	7·7	7·9	8·2	8·4	8·6	8·7	9·6
8	4·74	5·7	6·2	6·6	6·9	7·2	7·4	7·7	7·9	8·1	8·2	9·0
9	4·60	5·4	6·0	6·3	6·6	6·9	7·1	7·3	7·5	7·7	7·8	8·6
10	4·48	5·3	5·8	6·1	6·4	6·7	6·8	7·0	7·2	7·4	7·5	8·2
11	4·39	5·1	5·6	5·9	6·2	6·4	6·6	6·8	7·0	7·2	7·3	8·0
12	4·32	5·0	5·5	5·8	6·1	6·3	6·5	6·7	6·8	6·9	7·0	7·7
13	4·26	5·0	5·4	5·7	6·0	6·2	6·4	6·6	6·7	6·8	6·9	7·6
14	4·21	4·9	5·3	5·7	5·9	6·1	6·2	6·4	6·5	6·6	6·7	7·4
15	4·17	4·8	5·2	5·6	5·8	6·0	6·1	6·3	6·4	6·5	6·6	7·3
16	4·13	4·8	5·2	5·5	5·7	5·9	6·0	6·2	6·3	6·4	6·5	7·2
17	4·10	4·7	5·1	5·5	5·7	5·9	6·0	6·2	6·3	6·4	6·5	7·1
18	4·07	4·7	5·1	5·4	5·6	5·8	5·9	6·1	6·2	6·3	6·4	7·0
19	4·05	4·7	5·0	5·3	5·5	5·7	5·8	6·0	6·1	6·2	6·3	6·9
20	4·02	4·6	5·0	5·3	5·5	5·7	5·8	6·0	6·1	6·2	6·3	6·8
24	3·96	4·5	4·9	5·2	5·4	5·6	5·7	5·8	5·9	6·0	6·1	6·6
30	3·89	4·5	4·8	5·0	5·2	5·4	5·5	5·6	5·8	5·8	5·9	6·4
40	3·82	4·4	4·7	4·9	5·1	5·3	5·4	5·5	5·6	5·7	5·8	6·2
60	3·76	4·3	4·6	4·8	5·0	5·1	5·2	5·3	5·4	5·5	5·6	6·0
∞	3·64	4·12	4·38	4·59	4·74	4·87	4·98	5·07	5·15	5·22	5·28	5·64

To test the significance of treatment differences as a whole we find $z = 1{\cdot}7407$, while for degrees of freedom $f_1 = 5, f_2 = 20$, R. A. Fisher's tables give $z_{0{\cdot}01} = 0{\cdot}7058$. There are clearly, therefore, significant treatment differences present. The individual treatment means have been plotted in the accompanying figure, and the tabled significance levels for $q = w/s$ may be used, with discretion, as a foot-rule in investigating the situation.

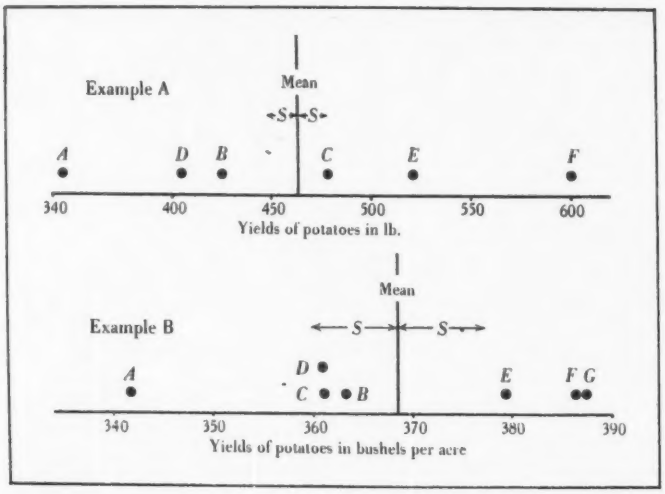


Fig. 1.

In the first place the appropriate pooled estimate, based on 20 degrees of freedom, of the standard error of a treatment mean is

$$s = \frac{39{\cdot}08}{\sqrt{6}} = 15{\cdot}95.$$

The range of the six treatment means is

$$w = 601{\cdot}8 - 345{\cdot}0 = 256{\cdot}8,$$

so that

$$q = \frac{w}{s} = 16{\cdot}1.$$

Table IV above gives for $n = 6, f = 20$ a 1 % level for q of 5·5, confirming the very significant scatter of treatment means brought out by the z-test.

We may now ask whether, if we were to exclude the most divergent treatment F, there is evidence of a significant difference among the remaining five treatments. We find that $w = 520{\cdot}2 - 345{\cdot}0 = 175{\cdot}2$, and, using the same estimate of standard

error, $s = 15{\cdot}95$, find that $q = 11{\cdot}0$. This value is still well beyond $q_{0{\cdot}01} = 5{\cdot}3$, the figure obtained from Table IV with $n = 5, f = 20$.

Making successive trials we find that:

(i) Omitting A and F, the range of the four treatments B, C, D, and E is still significant, since $q = 115{\cdot}0/15{\cdot}95 = 7{\cdot}2$, while for $n = 4$, $f = 20$ we have $q_{0{\cdot}01} = 5{\cdot}0$.

(ii) Omitting A, E, and F, the range of the three treatments B, C, and D is significant, at the 5 % but just not significant at the 1 % level. For in this case $q = 72{\cdot}6/15{\cdot}95 = 4{\cdot}5$, while for $n = 3, f = 20$ we find $q_{0{\cdot}05} = 3{\cdot}6$ and $q_{0{\cdot}01} = 4{\cdot}6$.

(iii) On the other hand, if we divided the six treatments into two groups, one consisting of A, D, B, and the other of C, E, F, we find that the value of q in both groups falls beyond $q_{0{\cdot}01} = 4{\cdot}6$.

We are therefore led to conclude that the high value of z obtained from the comprehensive test cannot be explained by one, or even two, treatments differing from the others. It is doubtful, even, whether any three treatments out of the six could be regarded as forming a homogeneous group.

Two final points should be noted. In the first place, after omitting successive treatments regarded as divergent, analysis of variance procedure could be applied to test for significant differences among the remaining treatments. The calculation would, however, not be as quick as that involved in the successive trials (i), (ii), and (iii) above, using q. Finally, as mentioned above, the method followed, whether z or q is used, must be employed with discretion, as is always the case when observations are rejected and a hypothesis tested using the selected data that remain.

Example B

A similar example has been taken from Snedecor (1937, p. 214). The following are seven treatment means expressed in terms of bushels per acre, obtained from a 7×7 Latin square experiment, again comparing the effect of different fertilizers on potatoes.

A	*B*	*C*	*D*	*E*	*F*	*G*
341·86	363·14	360·57	360·43	379·86	386·29	387·14

The appropriate estimate of the standard deviation of a mean of seven plots calculated from the error sum of squares of the analysis of variance table is $s = 9{\cdot}52$. Testing the significance of treatment differences as a whole, it is found that $z = 0{\cdot}5574$, while for degrees of freedom $f_1 = 6$, $f_2 = 30$, $z_{0{\cdot}05} = 0{\cdot}4420$ and $z_{0{\cdot}01} = 0{\cdot}6226$. The ratio $q = w/s$ for all seven treatments is $45{\cdot}3/9{\cdot}52 = 4{\cdot}7$ a value lying between $q_{0{\cdot}05} = 4{\cdot}5$ and $q_{0{\cdot}01} = 5{\cdot}4$ (entering Tables III and IV with $n = 7, f = 30$). Thus using either test we should conclude that there were probably significant treatment differences.

The seven treatment means have been plotted in the lower half of the figure. There is a suggestion that either if (i) treatments F and G or (ii) treatment A

were regarded as exceptional, the remaining treatments would form a homogeneous group.

This is confirmed on investigation:

(i) Removing F and G, we find $q = 38{\cdot}0/9{\cdot}52 = 4{\cdot}0$, which is just below the 5 % level ($q_{0{\cdot}05} = 4{\cdot}1$) for $n = 5, f = 3$.

(ii) Removing A, we find $q = 26{\cdot}7/9{\cdot}52 = 2{\cdot}8$, which is well below the 5 % level ($q_{0{\cdot}05} = 4{\cdot}3$) for $n = 6, f = 3$. The more homogeneous group appears to be left in the second case, i.e. on removing A alone. Beyond this indication we cannot go, as it would need a knowledge of the character of the treatments to draw more definite conclusions.

Example C

In cases where a number of duplicate observations are available, an estimate of variability may be obtained rapidly from summing the squares of the differences between pairs. Thus

$$s^2 = \sum_{t=1}^{k} (x_{t1} - x_{t2})^2/2k$$

and has k degrees of freedom. We may now compare the range in the means of pairs with s, in order to determine whether there is too much variation between pairs having regard to the variation within pairs. It may be noted that an estimate of σ may be obtained even more rapidly by calculating the mean range in pairs and multiplying by 0·8862,* so that

$$s' = 0{\cdot}8862 \times \sum_{t=1}^{k} |x_{t1} - x_{t2}|/k,$$

but since the sampling distribution of $(s')^2$ is not that of χ^2, we cannot justifiably take $q = w/s'$ and refer to the tables of percentage limits I have given.

Determinations of percentage fibre

Analyst	A	B	C	D	E	F	G	H	I	J
1st determination (x_{t1})	12·66	12·51	12·32	13·15	12·73	12·48	12·30	12·14	12·48	13·30
2nd determination (x_{t2})	12·47	12·62	12·55	12·93	12·43	12·46	12·73	12·03	12·44	12·68
Difference (d_t)	0·19	−0·11	−0·23	0·22	0·30	0·02	−0·43	0·11	0·04	0·62
Sum	25·13	25·13	24·87	26·08	25·16	24·94	25·03	24·17	24·92	25·98

* The reciprocal of 1·12838, since the expectation of range in a sample of two individuals is $1{\cdot}12838\sigma$.

The data shown on p. 28 above consist of ten duplicate determinations of the percentage fibre in carefully mixed samples taken from the same supply of Soya Cotton Cake, each pair of values being obtained by one analyst.* Ten different analysts were concerned. The problem is to determine whether these few observations provide any evidence of systematic differences in technique between the analysts.

The full analysis of variance is as follows:

	Sum of squares	D.F.	Mean square
Within pairs	0·411450	10	0·041145
Between pairs	1·352045	9	0·150227
Total	1·763495	19	

Testing for differences between analysts, we obtain $z = 0{\cdot}6475$, a value falling between the 1 % (0·7990) and 5 % (0·5527) levels.

Using the range method, we note that the estimate of the variance of the *sum* of two determinations is

$$\frac{1}{k}\sum_{i=1}^{k}(x_{i1}-x_{i2})^2 = \frac{1}{10}\sum_{i=1}^{10}(d_i^2) = 0{\cdot}08229,$$

giving an estimate of the standard error of a sum of 0·2868.† The range in the ten sums in the table is $w = 26{\cdot}08 - 24{\cdot}17 = 1{\cdot}91$, so that $q = 1{\cdot}91/0{\cdot}2868 = 6{\cdot}66$. For $n = 10$, $f = 10$, Tables III and IV show $q_{0{\cdot}05} = 5{\cdot}6$, $q_{0{\cdot}01} = 7{\cdot}2$. Thus the difference is significant at the 5 % level, a result similar to that found using the z-test.

If now we omit determinations of analyst D (who gave the highest readings) we find $q = (25{\cdot}98 - 24{\cdot}17)/0{\cdot}2868 = 6{\cdot}31$, and is still significant at the 5 % level, since for $n = 9$, $f = 10$, $q_{0{\cdot}05} = 5{\cdot}5$. If, however, we omit the determinations of analyst H (who gave the lowest readings) we find $q = (26{\cdot}08 - 24{\cdot}87)/0{\cdot}2868 = 4{\cdot}22$, and is no longer significant. We should conclude, therefore, that except possibly in the case of analyst H, there is no evidence on the data available of systematic differences in technique.

I should like to take this opportunity of thanking Prof. E. S. Pearson not only for suggesting the problem to me, but also for his kind help in putting this paper together.

* The figures have been taken from more extensive data made available through the kindness of Dr J. F. Tocher.

† The corresponding estimate obtained, as described on p. 28 above, from the mean difference between pairs is 0·291.

REFERENCES

FISHER, R. A. (1938). *Statistical Methods for Research Workers.* 7th ed. Edinburgh: Oliver and Boyd.

—— (1937). *The Design of Experiments*, 2nd ed. Edinburgh: Oliver and Boyd.

HARTLEY, H. O. (1938). *J.R. Statist. Soc. Suppl.* **5**, 80.

MCKAY, A. T. & PEARSON, E. S. (1933). *Biometrika*, **25**, 415.

PEARSON, E. S. (1926). *Biometrika*, **18**, 173.

—— (1932). *Biometrika*, **24**, 404.

PEARSON, E. S. & HAINES, JOAN (1935). *J.R. Statist. Soc. Suppl.* **2**, 83.

PEARSON, KARL (1922). *Tables of the Incomplete Gamma Function.* London: Biometrika Office.

—— (1931). *Tables for Statisticians and Biometricians*, Part II. London: Biometrika Office.

SNEDECOR, G. W. (1937). *Statistical Methods.* Iowa Collegiate Press, Inc.

"STUDENT" (1927). *Biometrika*, **19**, 151.

TIPPETT, L. H. C. (1925). *Biometrika*, **17**, 364.

ON A COMPREHENSIVE TEST FOR THE HOMOGENEITY OF VARIANCES AND COVARIANCES IN MULTIVARIATE PROBLEMS

By D. J. BISHOP

Department of Statistics, University College, London

1. Introduction

Now that satisfactory and probably final solutions have been obtained for a wide variety of statistical problems concerned with a single normally distributed variable, more and more attention has recently been given to the solution of multivariate problems. The multiple correlation methods of the old large sample theory have been replaced in many instances by others for which "studentized" test criteria are available, often having sampling distributions that are already familiar in univariate problems. In a recent paper on "The statistical utilization of multiple measurements", R. A. Fisher (1938*a*) has shown the connexion between certain of these methods: the D^2-statistic work of Mahalanobis, the discriminant function methods of the Galton Laboratory and the generalized "Student's" ratio of Hotelling. A similar very general problem was dealt with some time ago by S. S. Wilks (1932), while mention may also be made of two papers by D. G. Lawley (1938*a*, *b*) and a paper by P. L. Hsu (1938). The purpose of the methods put forward is to obtain information regarding the mean values of a number, say q, of correlated variables in one or more, say k, populations from which random samples have been drawn. If we denote by x_s a value of the sth variable ($s = 1, 2, \ldots, q$), then in all this work it has been assumed not only that x_s is normally distributed, but that it has the same variance σ_s^2 in every population sampled. Further, it is assumed that if x_u is a second variable the correlation coefficients ρ_{su} between x_s and x_u is the same in all populations. The estimates of variance and covariance required in order to "studentize" the function of the sample means are therefore obtained by pooling together the sums of squares and sums of products from all samples. While it is true that even if σ_s and ρ_{su} are not the same in all populations the error involved may not be very large, it is however important to have available some means of testing the basic hypothesis which assumes homogeneity throughout the populations.

Such a test has been derived by S. S. Wilks (1932) by an extension of Neyman & Pearson's likelihood ratio method of approach. Hitherto the somewhat lengthy computations required to obtain the moments of the sampling distribution of the test criterion have probably discouraged its use. The objects of the present paper are as follows:

(*a*) In the simple but commonly met case, where the k samples are of the same

size, to put Wilks' test into such a form that once the lengthy process of computing the kq variances and $\frac{1}{2}kq(q-1)$ covariances has been carried out, relatively little further labour is required to obtain a test criterion which may be referred with practical accuracy either to Fisher's z-tables or to the *Tables of the Incomplete Beta Function* (K. Pearson, 1934).

(*b*) In the case where the sample sizes differ, to suggest an alternative procedure which is accurate when dealing with large samples.

It is of course always open to question how far a single comprehensive criterion is satisfactory in a complex problem. Certain points should, however, be remembered. In the problem referred to above, dealing with the means only, the usefulness of a single criterion has been widely recognized. If, when applied to adequately large samples, a suitably chosen comprehensive criterion shows no significant evidence of lack of homogeneity among the variances and covariances, we are saved the lengthy process of making many individual comparisons. If, however, the criterion falls beyond the significance level, it will be necessary, as when dealing with the means, to make a more detailed analysis in order to locate the source of disturbance.

Finally, it may be noted that in the case of a single variate ($q = 1$) the problem is much simplified. A full discussion of two tests available in this case, the Neyman & Pearson L_1 test and the Bartlett μ test has recently been published elsewhere (Bishop & Nair, 1939).

2. Wilks' generalized likelihood criterion

It is proposed first to define this criterion in the simple case when the samples from each of the k populations are of the same size, n; to quote the sampling moments derived by Wilks; to give the form of a working test developed in the later pages; and to illustrate its use on an example.

It will be convenient to use the following notation. We shall consider k samples, each of size n, drawn from populations of q variates. That is to say, the total number of individuals sampled is $kn = N$ and for each individual q characters are measured, so that the total number of observations is Nq. Let x_{sti} be the observed value of the character s of the ith individual in the tth sample. Then $s = 1, 2, \ldots, q$; $t = 1, 2, \ldots, k$; and $i = 1, 2, \ldots, n$.

Let
$$x_{st.} = \frac{1}{n}\sum_{i=1}^{n} x_{sti}$$
be the mean value observed for the character s in the tth sample. Now put

$$v_{sut} = \frac{1}{n}\sum_{i=1}^{n}(x_{sti} - x_{st.})(x_{uti} - x_{ut.}),^{*} \qquad \ldots\ldots(1)$$

* The use of the divisor n instead of the more usual $n-1$ avoids the necessity for a multiplying constant in the expression for λ_1.

where $s, u = 1, 2, \ldots, q$. Then if $s = u$, v_{sut} is just the variance of observations of character $s = u$ within the tth sample, whereas if $s \neq u$, v_{sut} is the covariance of the characters s and u.

The generalized variance in the tth sample is defined by the symmetrical determinant

$$|v_{sut}| = \begin{vmatrix} v_{11t}, & v_{12t}, & \ldots, & v_{1qt} \\ v_{12t}, & v_{22t}, & \ldots, & v_{2qt} \\ \vdots & \vdots & & \vdots \\ v_{1qt}, & v_{2qt}, & \ldots, & v_{qqt} \end{vmatrix} \qquad \ldots\ldots(2)$$

Then if

$$v_{su} = \frac{1}{k}\sum_{t=1}^{k} v_{sut}, \qquad \ldots\ldots(3)$$

the likelihood criterion appropriate for testing the comprehensive hypothesis, say H_0, that all the corresponding variances and covariances in the k sampled populations are equal is

$$\lambda_1 = \frac{\prod_{t=1}^{k} |v_{sut}|^{\frac{1}{2}n}}{|v_{su}|^{\frac{1}{2}N}} \qquad \ldots\ldots(4)$$

It has been found convenient to study the sampling distribution of some fractional power of λ_1, rather than that of λ_1 itself, owing to the extreme skewness of the latter distribution. Within limits there is some choice in the power of λ_1 which may be selected to give a convenient sampling distribution. Throughout this paper we shall follow Pearson & Wilks (1933) by using the $1/N$th power, so that the criterion for use in practice will be

$$\lambda_1^{1/N} = l_1 = \sqrt{\frac{\prod_{t=1}^{k} |v_{sut}|^{1/k}}{|v_{su}|}}. \qquad \ldots\ldots(5^*)$$

If H_0, the hypothesis tested, be true, the hth moment about zero of the sampling distribution of l_1 has been given by Wilks (1932, p. 490) in the form

$$\mu'_h = k^{\frac{1}{2}qh}\left\{\frac{(\Gamma(\frac{1}{2}(n+h/k-s)))^k\,\Gamma(\frac{1}{2}(k(n-1)+1-s))}{(\Gamma(\frac{1}{2}(n-s)))^k\,\Gamma(\frac{1}{2}(k(n-1)+1+h-s))}\right\}. \qquad \ldots\ldots(6)$$

As a result of trial in a number of cases it seems probable that the Pearson Type I distribution in the form

$$p(l_1) = \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\,\Gamma(m_2)} l_1^{m_1-1}(1-l_1)^{m_2-1}, \text{ for } 0 \leqslant l_1 \leqslant 1, \qquad \ldots\ldots(7)$$

will give a good approximation to the distribution of l_1, if the parameters m_1 and

* Note that $\{l_1(q=1)\}^2 = L_1$, in the original notation of Neyman & Pearson (1931).

m_2 are so chosen that the first two moments of (7) agree with the true values as given by (6). If this is done it is found that

$$m_1 = \frac{\mu_1'(\mu_1' - \mu_2')}{\mu_2}, \qquad \text{.....(8)}$$

and

$$m_2 = \frac{(1-\mu_1')\,(\mu_1' - \mu_2')}{\mu_2}, \qquad \text{.....(9)}$$

where

$$\mu_2 = \mu_2' - (\mu_1')^2, \qquad \text{.....(10)}$$

μ_1' and μ_2' being respectively the first and second moments about zero of l_1. From the relation (6) it is seen that their values are

$$\mu_1' = k^{\frac{1}{2}q} \prod_{s=1}^{q} \left\{ \frac{(\Gamma(\frac{1}{2}(n+1/k-s)))^k\, \Gamma(\frac{1}{2}(k(n-1)+1-s))}{(\Gamma(\frac{1}{2}(n-s)))^k\, \Gamma(\frac{1}{2}(k(n-1)+2-s))} \right\} \qquad \text{.....(11)}$$

and

$$\mu_2' = k^q \prod_{s=1}^{q} \left\{ \frac{(\Gamma(\frac{1}{2}(n+2/k-s)))^k\, \Gamma(\frac{1}{2}(k(n-1)+1-s))}{(\Gamma(\frac{1}{2}(n-s)))^k\, \Gamma(\frac{1}{2}(k(n-1)+3-s))} \right\}. \qquad \text{.....(12)}$$

The hypothesis H_0 will be rejected when l_1 is exceptionally low. The probability level may be obtained from the *Tables of the Incomplete Beta Function*. Alternatively 5 % and 1 % levels for l_1 can be obtained from R. A. Fisher's z-tables (1938*b*, Table VI), by writing

$$l_1 = \frac{f_2}{f_2 + f_1 e^{2z}}, \qquad \text{.....(13)}$$

where z has degrees of freedom

$$f_1 = 2m_2, \quad f_2 = 2m_1.^* \qquad \text{.....(14)}$$

It is no doubt the labour involved in calculating the moments μ_1' and μ_2' that has discouraged the use of Wilks' test. As will be shown in the later sections of the paper, if n is not too small,† the following empirical relations may be used with sufficient accuracy for practical purposes to express the m_1 and m_2 of (14) directly in terms of (*a*) the number of variables q, (*b*) the sample size n and (*c*) the number of samples k:

$$m_1 = k(n-q) - 0{\cdot}01(k-1)\,(90 - 39q + 9q^2), \qquad \text{.....(15)}$$

$$m_2 = 0{\cdot}25(k-1)\,q(q+1). \qquad \text{.....(16)}$$

These formulae are found to give satisfactory results for all values of k, over the range considered for q, namely 1 to 6.

In the following sections of the paper we shall first give an example illustrating the practical use of the test and afterwards justify the approximations which have been made.

* Since in general f_1 and f_2 in (14) will not be integral, it is necessary to interpolate in the tables for fractional degrees of freedom.

† Certainly if $n \geqslant 20$, and probably if $n \geqslant 10$ provided $q < 5$.

3. ILLUSTRATIVE EXAMPLE

The data used were taken from the appendix of a craniological study by E. Pittard (1909) and consist of measurements of skulls of males found in different localities in the Rhone valley. The characters considered are:

(1) Length of skull in mm., denoted by L.
(2) Breadth of skull in mm., denoted by B.
(3) Height of skull in mm., denoted by H.
(4) Breadth of face in mm., denoted by F.

The skulls are divided into five groups of thirty skulls each, according to the neighbourhood in which they were found. The five groups are:

(1) Skulls found in the village of Biel.
(2) ,, ,, ,, ,, Naters.
(3) ,, ,, ,, ,, Viège.
(4) ,, ,, ,, ,, Rarogne.
(5) ,, ,, ,, ,, Sierre.

It is desired to investigate if the population variances and covariances of the measurements of the four characters may be taken as the same for each of the five populations sampled. This is equivalent to asking whether the population variances and covariances differ from village to village.

With the notation previously employed it is clear that in this case $n = 30$, $k = 5$ and $q = 4$. The sample variances and covariances, as given by (1), were calculated in the usual manner. Their values, together with the sample correlation coefficients, are shown in Table IA. Using these results, the values of the generalized sample variances $|v_{sut}|$, which are also given in Table IA, were calculated from equation (2). The substitution of these values of $|v_{sut}|$ into equation (5) gives $l_1 = 0{\cdot}725$.

The probability levels of the sampling distribution of l_1 are given by (13) and (14), where

$$m_1 = 126{\cdot}880 \quad \text{and} \quad m_2 = 20{\cdot}000$$

are determined by substituting $k = 5$, $n = 30$ and $q = 4$ in the empirical relations (15) and (16). The levels are found to be:

$$5\,\% \text{ level of } z = 0{\cdot}1830 \text{ and } 1\,\% \text{ level of } z = 0{\cdot}2573,$$

hence

$$5\,\% \text{ level of } l_1 = 0{\cdot}815 \text{ and } 1\,\% \text{ level of } l_1 = 0{\cdot}791.$$

The calculated value of l_1 being considerably below the 1 % level, the hypothesis that the variances and covariances are the same for each of the five populations sampled must be rejected.

Further tests may now be made in order to find where the lack of uniformity

TABLE IA

s	u	v_{su1}	v_{su2}	v_{su3}	v_{su4}	v_{su5}	v_{su}
L	*L*	39·2222	46·1333	46·9123	29·8267	34·0990	39·2387
B	*B*	25·4766	28·4989	22·5789	43·4622	21·7556	28·3544
H	*H*	29·6989	18·3733	23·9156	41·1289	32·4933	29·1220
F	*F*	32·6489	25·8767	31·4322	25·4900	16·6722	26·4240
L	*B*	22·0333	1·5000	10·5212	−2·4467	10·6444	8·4504
L	*H*	1·7444	8·9000	0·3689	19·4467	0·2600	6·1440
L	*F*	28·0556	7·2333	12·5922	0·3867	−0·3278	9·5880
B	*H*	1·8433	10·5867	−2·2689	16·0822	−7·2000	3·8087
B	*F*	20·7400	12·4567	7·3744	17·5067	3·2111	12·2578
H	*F*	8·9844	1·6467	−10·0004	15·3300	0·9000	3·3721
		$\lvert v_{su1} \rvert$	$\lvert v_{su2} \rvert$	$\lvert v_{su3} \rvert$	$\lvert v_{su4} \rvert$	$\lvert v_{su5} \rvert$	$\lvert v_{su} \rvert$
		138,521	307,934	519,165	422,517	296,115	585,487
s	u	r_{su1}	r_{su2}	r_{su3}	r_{su4}	r_{su5}	
L	*B*	0·69702	0·04137	0·32327	−0·06796	0·39081	
L	*H*	0·05111	0·30570	0·01101	0·55522	0·00781	
L	*F*	0·78401	0·20935	0·32792	0·01402	−0·01375	
B	*H*	0·06701	0·46265	−0·09764	0·38038	−0·27080	
B	*F*	0·71913	0·45871	0·27681	0·52597	0·16861	
H	*F*	0·28853	0·07552	−0·36475	0·47346	0·03867	

occurs. To decide whether the variances differ we may apply four single variate L_1 tests, treating the observations of each character separately. When this is done the results following in Table IB are obtained.

The tables of P. P. N. Nayer (1936) give the 5 % level for L_1, when $n = 30$ and $k = 5$, as 0·936, so that there is no evidence to suggest that the variances of measurements of any one character differ significantly from sample to sample.

The lack of uniformity must therefore occur in the correlations, so that it is necessary to examine the variation in each of the six sets of five correlation

TABLE IB

Character	L_1
L	0·985
B	0·967
H	0·964
F	0·974

coefficients. If r be a sample correlation coefficient calculated from n pairs of observations it is known that

$$z' = \tfrac{1}{2}(\log_e(1+r) - \log_e(1-r))$$

is approximately normally distributed with standard deviation $1/\sqrt{(n-3)}$, the mean of z' being a function of the population correlation coefficient and the sample size n. Consequently, we may test whether correlation coefficients $r_{sut}(t = 1, 2, \ldots, k)$, each based on n pairs of observations and obtained from k independent samples, differ only through chance fluctuations from some common population value ρ_{su}, by calculating

$$\chi^2 = (n-3)\sum_{t=1}^{k}(z'_t - \bar{z}')^2,$$

where

$$\bar{z}' = \frac{1}{k}\sum_{t=1}^{k} z'_t,$$

and entering the tables of the χ^2 integral with $k-1$ degrees of freedom. A significantly large value of χ^2 will indicate that the k samples cannot be considered as having been drawn from populations with a common value of ρ. This procedure was carried out with the results shown in Table Ic.

TABLE Ic

Correlation between characters	χ^2	Remarks
L and *B*	14·31	Significant
L ,, *H*	7·82	
L ,, *F*	20·47	Significant
B ,, *H*	11·67	Probably significant
B ,, *F*	8·81	
H ,, *F*	12·08	Probably significant

The probability levels for the distribution of χ^2 with $f = 4$ are: 5 % level = 9·49, 2 % level = 11·67 and 1 % level = 13·28. It is seen that all the six calculated values of χ^2 are above the expectation value of 4. There is significant variation in the correlation coefficients r_{LBl} and r_{LFl}, whilst it is largely a matter of personal opinion as to whether the suggestion of lack of uniformity in r_{BHl} and r_{HF} shall be judged significant or not.

To summarize: analysis of the data leads to the following conclusions:

(*a*) The comprehensive test shows evidence of significant variation from sample to sample of the variances and covariances.

(*b*) This lack of uniformity is not due to differences in the population variances.

(*c*) There is clear evidence of heterogeneity among some of the correlation coefficients, in particular for those between length and breadth of skull and between length of skull and breadth of face. Results of this kind are frequently met with when dealing with craniological data, the correlation coefficient often being subject to a considerable degree of instability. The meaning to be attached to these fluctuations in the correlations, when considered from a craniological viewpoint, seems to be somewhat obscure.

The above example may give the impression that a great deal of labour is required, even after the comprehensive test is used. If, however, the comprehensive test had provided no evidence for rejecting the hypothesis H_0, it would, of course, not have been necessary to make the four single-variate L_1 tests and the six correlation tests. Even when the hypothesis H_0 is rejected and it is necessary to apply separate tests to find the causes of the lack of uniformity, the only labour which will have been wasted is the relatively small amount involved in the calculation of the determinants $|v_{sut}|$. The really lengthy computation is that required to obtain the sums of squares and sums of products on which Table I A is based; this cannot be avoided if a detailed analysis is desired.

4. The adequacy of the Type I approximation

The work now falls into two stages, which will be concerned with

(*a*) the adequacy of the Type I curve of equation (7) to represent the unknown true distribution of l_1;

(*b*) the accuracy of the empirical formulae (15) and (16) for m_1 and m_2.

The hope that the Type I form of curve might give an adequate approximation was based on two main considerations:

(*a*) Values of l_1 are restricted and can only lie between zero and unity.

(*b*) On other occasions the use of the principle of likelihood has yielded criteria which are either exactly distributed in the Type I form, or are so distributed that a good approximation has been obtainable by the use of this kind of curve.

In order to compare the true distribution with the approximate form, the first four moments of both distributions have been calculated in a number of cases. The manner of choosing the approximate distribution ensures that its first two moments will agree with those of the true distribution, and an idea of the accuracy of the approximation may be gathered by comparing the third and fourth moments of the true distribution with the corresponding moments of the approximate form. The distribution is such that a comparison of moments calculated about the mean is easier than that obtained when moments taken about zero are used. The cases considered fall into two groups:

(i) $k = 5$; $n = 10$, 20, 40 and 50; $q = 1$, 2, 3, 4, 5 and 6

and (ii) $n = 30$; $k = 2$, 5 and 10; $q = 1$, 2, 3, 4, 5 and 6.

The process of calculation was as follows:

(*a*) The first four moments about zero of the true distribution were obtained by using 10-figure tables of logarithms of the Gamma function (E. S. Pearson, 1922). A large number of figures were required in the values of these moments, since the dispersion of many of the distributions was small.

(*b*) The values of μ_2, μ_3 and μ_4, the second, third and fourth moments about the mean, were then obtained in the usual way and the constants m_1 and m_2 were calculated by using (8) and (9).

(*c*) The third and fourth moments about zero of the Type I distribution (7) are given by

$$ {}_1\mu'_3 = \frac{m_1(m_1+1)\,(m_1+2)}{(m_1+m_2)\,(m_1+m_2+1)\,(m_1+m_2+2)} = \frac{(m_1+2)}{(m_1+m_2+2)}\mu'_2, \quad \text{.....(17)} $$

and

$$ {}_1\mu'_4 = \frac{m_1(m_1+1)\,(m_1+2)\,(m_1+3)}{(m_1+m_2)\,(m_1+m_2+1)\,(m_1+m_2+2)\,(m_1+m_2+3)} = \frac{(m_1+3)}{(m_1+m_2+3)}\,{}_1\mu'_3 \quad \text{.....(18)} $$

so that, using values of m_1 and m_2 already obtained, these may be calculated.

(*d*) From the values of ${}_1\mu'_3$ and ${}_1\mu'_4$ the corresponding moments about the mean, ${}_1\mu_3$ and ${}_1\mu_4$, were calculated. Comparing μ_3 and μ_4 with ${}_1\mu_3$ and ${}_1\mu_4$, respectively, gives an idea of the adequacy of the approximation.

Table II gives the results obtained; in addition, values of $\beta_1 = \mu_3^2/\mu_2^3$ and $\beta_2 = \mu_4/\mu_2^2$ are included as measures of the skewness and kurtosis of the distributions with which we are dealing.

It is seen from these tables that the distributions represented vary widely in shape, but as far as can be judged by comparison of third and fourth moments, the use of the Type I approximation would seem to be quite satisfactory in every case considered. The cases dealt with cover a fairly wide field and have not been chosen with a view to obtaining especially good agreement of the moments.

The inclusion of curves for which $q = 1$ may be questioned having regard to the fact, already stated, that the single variate problem has been fully treated. We must therefore remark that, as $L_1 = \{l_1(q=1)\}^2$, the fact that the distribution of L_1 may be represented by a Type I curve does not imply that this kind of approximation will work adequately for the distribution of $l_1 = L_1^{\frac{1}{2}}$. Moreover, some of the distributions obtained by putting $q = 1$ are abnormally skew and highly leptokurtic. As it happens, when $q = 1$ the Type I curve appears to give as good an agreement as for other values, but the exclusion of this set of distributions might have had the effect of making the approximation appear better than was really the case, in that it would have removed a number of the "more difficult" distributions. Thus, although the criterion L_1 will probably always be used in preference to $l_1(q=1) = L_1^{\frac{1}{2}}$, it seemed desirable to include distributions

TABLE II

Moment coefficients of l_1 in trial cases

k = no. of populations sampled, q = no. of variables, n = size of samples

q		$k=5$ $n=10$	$k=5$ $n=20$	$k=5$ $n=30$	$k=5$ $n=40$	$k=5$ $n=50$
1	μ_1'	0·9551,5592	0·9788,4625	0·9861,620246	0·9897,183906	0·9918,206271
	μ_2	0·0009,3729	0·0002,1661	0·0000,937517	0·0000,520392	0·0000,330404
	μ_3	−0·0000,3646	−0·0000,0429	−0·0000,012438	−0·0000,005188	−0·0000,002638
	$_1\mu_3$	−0·0000,3654	−0·0000,0430	−0·0000,012440	−0·0000,005184	−0·0000,002638
	μ_4	0·0000,0461	0·0000,0026	0·0000,000514	0·0000,000152	0·0000,000060
	$_1\mu_4$	0·0000,0462	0·0000,0026	0·0000,000508	0·0000,000154	0·0000,000067
	β_1	1·61	1·81	1·88	1·91	1·93
	β_2	5·2	5·6	5·9	5·6	5·5
2	μ_1'	0·8642,3289	0·9362,9104	0·9583,8364	0·9690,995805	0·9754,270546
	μ_2	0·0025,9457	0·0006,2664	0·0002,7472	0·0001,534270	0·0000,977634
	μ_3	−0·0000,8171	−0·0000,1137	−0·0000,0345	−0·0000,014671	−0·0000,007557
	$_1\mu_3$	−0·0000,8178	−0·0000,1137	−0·0000,0345	−0·0000,014677	−0·0000,007557
	μ_4	0·0000,2312	0·0000,0146	0·0000,0029	0·0000,000901	0·0000,000381
	$_1\mu_4$	0·0000,2312	0·0000,0146	0·0000,0029	0·0000,000912	0·0000,000383
	β_1	0·38	0·53	0·57	0·59	0·61
	β_2	3·4	3·7	3·9	3·8	4·0
3	μ_1'	0·7336,9741	0·8737,2072	0·9172,5148	0·9384,6566	0·9510,224795
	μ_2	0·0042,3560	0·0011,4861	0·0005,1975	0·0002,9459	0·0001,893284
	μ_3	−0·0000,8380	−0·0000,1767	−0·0000,0590	−0·0000,0262	−0·0000,013829
	$_1\mu_3$	−0·0000,8401	−0·0000,1769	−0·0000,0590	−0·0000,0262	−0·0000,013826
	μ_4	0·0000,5399	0·0000,0427	0·0000,0090	0·0000,0029	0·0000,001226
	$_1\mu_4$	0·0000,5403	0·0000,0428	0·0000,0090	0·0000,0029	0·0000,001214
	β_1	0·09	0·21	0·25	0·27	0·28
	β_2	3·0	3·2	3·3	3·4	3·4
4	μ_1'	0·5768,8665	0·7939,4237	0·8639,4783	0·8984,6442	0·9190,1507
	μ_2	0·0050,1034	0·0016,6539	0·0007,9361	0·0004,6061	0·0003,9008
	μ_3	−0·0000,2967	−0·0000,1968	−0·0000,0774	−0·0000,0369	−0·0000,0202
	$_1\mu_3$	−0·0000,3099	−0·0000,1973	−0·0000,0775	−0·0000,0369	−0·0000,0202
	μ_4	0·0000,7256	0·0000,0850	0·0000,0198	0·0000,0067	0·0000,0029
	$_1\mu_4$	0·0000,7262	0·0000,0850	0·0000,0198	0·0000,0067	0·0000,0029
	β_1	0·01	0·08	0·12	0·14	0·15
	β_2	2·9	3·1	3·1	3·2	3·2
5	μ_1'	0·4123,5896	0·7010,2826	0·8002,0843	0·8500,5217	0·8800,0945
	μ_2	0·0045,0478	0·0020,5609	0·0010,5521	0·0006,3313	0·0004,2033
	μ_3	+0·0000,3208	−0·0000,1587	−0·0000,0829	−0·0000,0437	−0·0000,0253
	$_1\mu_3$	+0·0000,2882	−0·0000,1606	−0·0000,0831	−0·0000,0438	−0·0000,0253
	μ_4	0·0000,5903	0·0000,1262	0·0000,0341	0·0000,0123	0·0000,0055
	$_1\mu_4$	0·0000,5897	0·0000,1262	0·0000,0339	0·0000,0124	0·0000,0055
	β_1	0·01	0·03	0·06	0·08	0·09
	β_2	2·9	3·0	3·1	3·1	3·1
6	μ_1'	0·2603,9120	0·5999,9719	0·7282,3688	0·7944,5984	0·8347,8956
	μ_2	0·0030,5059	0·0022,3214	0·0012,6522	0·0007,9285	0·0005,3942
	μ_3	+0·0000,4976	−0·0000,0779	−0·0000,0726	−0·0000,0446	−0·0000,0286
	$_1\mu_3$	+0·0000,4559	−0·0000,0823	−0·0000,0734	−0·0000,0451	−0·0000,0281
	μ_4	0·0000,2830	0·0000,1472	0·0000,0480	0·0000,0185	0·0000,0095
	$_1\mu_4$	0·0000,2807	0·0000,1472	0·0000,0481	0·0000,0191	0·0000,0089
	β_1	0·09	0·01	0·03	0·04	0·05
	β_2	3·0	3·0	3·0	2·9	3·3

TABLE II (*continued*)

q		$n = 30$ $k = 2$	$n = 30$ $k = 5$	$n = 30$ $k = 10$
1	μ_1'	0·9913,444323	0·9861,620246	0·9844,364077
	μ_2	0·0001,459732	0·0000,937517	0·0000,527977
	μ_3	−0·0000,047980	−0·0000,012438	−0·0000,003516
	${}_1\mu_3$	−0·0000,047992	−0·0000,012440	−0·0000,003516
	μ_4	0·0000,002947	0·0000,000514	0·0000,000136
	${}_1\mu_4$	0·0000,002947	0·0000,000508	0·0000,000124
	β_1	7·40	1·88	0·84
	β_2	13·8	5·9	4·9
2	μ_1'	0·9737,237167	0·9583,8364	0·9533,389957
	μ_2	0·0004,404662	0·0002,7472	0·0001,532142
	μ_3	−0·0000,141247	−0·0000,0345	−0·0000,009540
	${}_1\mu_3$	−0·0000,141253	−0·0000,0345	−0·0000,009535
	μ_4	0·0000,012312	0·0000,0029	0·0000,000813
	${}_1\mu_4$	0·0000,012309	0·0000,0029	0·0000,000787
	β_1	2·33	0·57	0·25
	β_2	6·4	3·9	3·5
3	μ_1'	0·9471,293171	0·9172,5148	0·9075,804094
	μ_2	0·0008,673760	0·0005,1975	0·0002,860673
	μ_3	−0·0000,264701	−0·0000,0590	−0·0000,015837
	${}_1\mu_3$	−0·0000,264137	−0·0000,0590	−0·0000,015851
	μ_4	0·0000,034381	0·0000,0090	0·0000,002570
	${}_1\mu_4$	0·0000,033682	0·0000,0090	0·0000,002566
	β_1	1·07	0·25	0·11
	β_2	4·6	3·3	3·1
4	μ_1'	0·9118,024417	0·8639,4783	0·8487,578437
	μ_2	0·0013,941771	0·0007,9361	0·0004,295258
	μ_3	−0·0000,391427	−0·0000,0774	−0·0000,019936
	${}_1\mu_3$	−0·0000,391348	−0·0000,0775	−0·0000,019982
	μ_4	0·0000,072591	0·0000,0198	0·0000,005613
	${}_1\mu_4$	0·0000,072561	0·0000,0198	0·0000,005636
	β_1	0·57	0·12	0·05
	β_2	3·7	3·1	3·0
5	μ_1'	0·8682,461741	0·8002,0843	0·7791,127138
	μ_2	0·0019,746075	0·0010,5521	0·0005,595144
	μ_3	−0·0000,493446	−0·0000,0829	−0·0000,020115
	${}_1\mu_3$	−0·0000,493538	−0·0000,0831	−0·0000,020241
	μ_4	0·0000,131257	0·0000,0341	0·0000,009414
	${}_1\mu_4$	0·0000,131262	0·0000,0339	0·0000,009434
	β_1	0·32	0·06	0·02
	β_2	3·4	3·1	3·0
6	μ_1'	0·8172,266960	0·7282,3688	0·7013,941584
	μ_2	0·0025,529007	0·0012,6522	0·0006,547817
	μ_3	−0·0000,543573	−0·0000,0726	−0·0000,016168
	${}_1\mu_3$	−0·0000,544455	−0·0000,0734	−0·0000,016440
	μ_4	0·0000,206107	0·0000,0480	0·0000,012833
	${}_1\mu_4$	0·0000,206344	0·0000,0481	0·0000,012845
	β_1	0·18	0·03	0·01
	β_2	3·2	3·0	3·0

TABLE II (*continued*)

	$q=1$ $k=2$ $n=10$	$q=1$ $k=2$ $n=30$	$q=1$ $k=2$ $n=50$
μ_1'	0·9719,147029	0·9913,444323	0·9948,854137
μ_2	0·0014,478736	0·0001,459732	0·0000,515182
μ_3	−0·0001,373323	−0·0000,047980	−0·0000,010221
$_1\mu_3$	−0·0001,376674	−0·0000,047992	−0·0000,010222
μ_4	0·0000,242507	0·0000,002947	0·0000,000375
$_1\mu_4$	0·0000,243786	0·0000,002947	0·0000,000376
β_1	6·21	7·40	7·64
β_2	11·6	13·8	14·1
	$q=2$ $k=2$ $n=10$	$q=2$ $k=2$ $n=30$	$q=2$ $k=2$ $n=50$
μ_1'	0·9122,379882	0·9737,237167	0·9845,495758
μ_2	0·0044,231601	0·0004,404662	0·0001,550848
μ_3	−0·0003,816694	−0·0000,141247	−0·0000,030326
$_1\mu_3$	−0·0003,818601	−0·0000,141253	−0·0000,030334
μ_4	0·0000,997748	0·0000,012312	0·0000,001570
$_1\mu_4$	0·0000,998432	0·0000,012309	0·0000,001584
β_1	1·68	2·33	2·47
β_2	5·1	6·4	6·5

It will be noted that there are sections common to the three parts of Table II. These have been included more than once in order to facilitate comparisons in any one part of the table.

for which $q = 1$ when discussing the adequacy of the Type I approximation to the distribution of l_1.

Consideration of an alternative form of approximation

For the larger values of n which have been considered it is seen that the distribution of l_1 is of small dispersion and is situated close to unity, the upper limit of possible values of l_1. In fact, in many of the cases dealt with, the mean is separated from the start of the curve by some twenty or thirty times the standard deviation. In these circumstances it seemed possible that a better approximation might be obtained by taking a Pearson Type I curve in the form

$$p(l_1) = \frac{\Gamma(m_1+m_2)}{(1-b)^{m_1+m_2-1}\,\Gamma(m_1)\,\Gamma(m_2)}(l_1-b)^{m_1-1}(1-l_1)^{m_2-1}, \text{ for } b \leqslant l_1 \leqslant 1. \quad \ldots(19)$$

There are now three assignable parameters; b, defining the start of curve, m_1 and m_2, which may be selected by equating the first three moments of the distribution (19) to those of the true distribution of l_1. As before, an idea of the degree of approximation may be obtained by comparing the fourth moment of l_1,

calculated from the mean, with that of the distribution (19). This procedure has been carried out in a few cases, the results obtained being set out in the upper part of Table III.

As far as can be judged by a comparison of moments, there is little difference between the approximations obtained by using a Type I curve in form (7), which is fitted by two moments, and the Type I curve in form (19), which is fitted by three moments. However, as we are primarily concerned with the probability levels of the distribution of l_1, the effect on these levels of the change in the method of approximation must be ascertained.

TABLE III

Comparison of distributions fitted by two and three moments

		$n=30$ $k=5$ $q=1$	$n=30$ $k=5$ $q=2$	$n=30$ $k=5$ $q=3$	$n=30$ $k=5$ $q=4$	$n=30$ $k=5$ $q=5$	$n=30$ $k=5$ $q=6$
$\mu_4 \times 10^8$	Of true distribution	5·14	29	90	198	341	480
	Of distribution (7)	5·08	29	90	198	339	481
	Of distribution (19)	5·08	29	90	198	339	480
5% level	Of distribution (7)	0·9674	0·9280	0·8767	0·8149	0·7446	0·6678
	Of distribution (19)	0·9674	0·9280	0·8767	0·8149	0·7446	0·6677
1% level	Of distribution (7)	0·9546	0·9110	0·8563	0·7915	0·7192	0·6411
	Of distribution (19)	0·9546	0·9110	0·8562	0·7915	0·7192	0·6413

The method of obtaining the 5 % and 1 % levels when the distribution is in form (7) has been given. When the second approximation is considered, the transformation

$$t = \frac{l_1 - b}{1 - b} \qquad \text{.....(20)}$$

gives the distribution of t as

$$p(t) = \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\,\Gamma(m_2)}\, t^{m_1-1}\,(1-t)^{m_2-1}, \text{ for } 0 \leqslant t \leqslant 1,$$

so that, as before, the tables of z may be utilized in calculating the levels. The lower part of Table III compares the levels given by the two methods of approximation.

It is seen that, for all practical purposes, the probability levels as given by the two forms of Type I curve may be taken as identical, so that nothing is to be gained by using the more complicated distribution (19). Furthermore, the agreement obtained strengthens the conviction that the Type I curve in form (7) really provides a good approximation to the true distribution. From now onward, therefore, we shall only be concerned with the approximate distribution given by (7).

5. Direct comparisons in cases where the true distribution of l_1 is known

When only two groups are considered the distribution of l_1 is known in the cases of one and two variates. These known distributions may be used to test the accuracy of the probability levels as obtained by using the Type I curve. If $q = 1$ we have seen that $l_1^2 = L_1$. P. P. N. Nayer (1936, p. 43) has shown that, when $k = 2$,

$$p(L_1) = \frac{\Gamma(n-1)}{2^{n-3}\left\{\Gamma\left(\frac{n-1}{2}\right)\right\}^2} L_1^{n-2}(1-L_1^2)^{-\frac{1}{2}}, \text{ for } 0 \leqslant L_1 \leqslant 1.$$

Now, $$P\{l_1 < l_1^0\} = P\{L_1 < (l_1^0)^2\} = \frac{\Gamma(n-1)}{2^{n-3}\left\{\Gamma\left(\frac{n-1}{2}\right)\right\}^2} \int_0^{(l_1^0)^2} L_1^{n-2}(1-L_1^2)^{-\frac{1}{2}}\, dL_1.$$

Hence, $$P\{l_1 < l_1^0\} = I_x(\tfrac{1}{2}(n-1), \tfrac{1}{2}), \text{ where } x = (l_1^0)^4. \qquad \text{.....(21)}$$

When $q = 2$ and $k = 2$, Pearson & Wilks (1933) have proved that

$$P\{l_1 < l_1^0\} = \frac{\Gamma(2n-3)}{\Gamma(n-1)\,\Gamma(n-2)\,2^{2n-5}} \left\{(l_1^0)^{2n-4} \log_e\left(\frac{1+\sqrt{\{1-(l_1^0)^2\}}}{l_1^0}\right)\right\} + I_x(n-2, \tfrac{1}{2}),$$

where $x = (l_1^0)^2$. .....(22)

Thus if $k = 2$, relations (21) and (22) enable the probability integrals of $l_1\,(q = 1)$ and $l_1\,(q = 2)$ to be obtained by using *Tables of the Incomplete Beta Function*. In this manner the true probability associated with each of the 5 % and 1 % limits, obtained by using the Type I approximation, may be calculated. Results obtained are given in Table IV.

For the values of n considered it is seen, from Table IV, that the true probabilities are very close to the desired values 0·05 and 0·01. It should be pointed out that, as is shown in Table II, the six distributions considered above are unusually skew and leptokurtic compared with the remainder of the distributions, and moreover the agreement of true and approximate distribution, as judged by the moments, is not noticeably better for these curves than for the curves in general. Hence, for distributions which, judging from the values of β_1 and β_2, are certainly not in any way favoured, the limits set by using the approximation are found to give true probabilities near enough to the desired values for all practical purposes.

To summarize the salient points in the preceding sections, we have seen that:

(*a*) For a wide range of distributions close agreement between the third and fourth moments of the true and approximate distributions is obtained.

(*b*) Type I curves in form (19) fitted by three moments, which might be expected to give a better approximation, lead to probability levels which are for practical purposes the same as those given by the Type I curve fitted by two moments.

(c) In cases where the true distribution is known, the limits set by the approximation give values of the true probability near to those desired.

We may therefore say with some confidence that, having regard to the above considerations, the use of the Type I curve in form (7) as an approximation to the distribution of l_1 seems to be amply justified.

TABLE IV

	$q=1$ $k=2$ $n=10$	$q=1$ $k=2$ $n=30$	$q=1$ $k=2$ $n=50$
Type I 5% limit	0·8936	0·9669	0·9804
True probability	0·05005	0·05008	0·04950
Type I 1% limit	0·8235	0·9435	0·9664
True probability	0·00998	0·01001	0·01002
	$q=2$ $k=2$ $n=10$	$q=2$ $k=2$ $n=30$	$q=2$ $k=2$ $n=50$
Type I 5% limit	0·7814	0·9324	0·9600
True probability	0·05005	0·04997	0·04979
Type I 1% limit	0·6989	0·9034	0·9425
True probability	0·00999	0·00996	0·00998

6. THE LIMITING FORM OF THE DISTRIBUTION OF l_1

As was the case in the single variate problem (Neyman & Pearson, 1931), the distribution of l_1 in large samples may be obtained approximately from that of χ^2. $M_h(\lambda_1)$, the hth moment of the sampling distribution of $\lambda_1 = l_1^N$, may be obtained from (6) by replacing h by Nh. Using Stirling's approximation,

$$\Gamma(x) \sim \sqrt{(2\pi)}\, x^{x-\frac{1}{2}} e^{-x},$$

it may readily be shown that $M_h(\lambda_1)$ tends uniformly to $(1+h)^{-\frac{1}{2}f}$ as n tends to ∞, for all $h \geqslant 0$, where

$$f = \tfrac{1}{2}(k-1)\, q(q+1). \qquad \text{.....(23)}$$

The distribution of χ^2 with f degrees of freedom is

$$p(\chi^2) = \{2^{\frac{1}{2}f}\, \Gamma(\tfrac{1}{2}f)\}^{-1}\, (\chi^2)^{\frac{1}{2}f-1}\, e^{-\frac{1}{2}\chi^2}, \text{ for } 0 \leqslant \chi^2 \leqslant \infty. \qquad \text{.....(24)}$$

If we put $y = e^{-\frac{1}{2}\chi^2}$ (so that $0 \leqslant y \leqslant 1$), the hth moment of y is

$$M_h(y) = \int_0^1 y^h p(y)\, dy = \{2^{\frac{1}{2}f}\, \Gamma(\tfrac{1}{2}f)\}^{-1} \int_0^\infty (\chi^2)^{\frac{1}{2}f-1}\, e^{-\frac{1}{2}\chi^2(1+h)}\, d(\chi^2)$$
$$= (1+h)^{-\frac{1}{2}f}.$$

Thus the corresponding moments of the distribution of y and the limiting form of the distribution of λ_1 are equal. The range of possible values of the variables being finite, i.e. from zero to unity, the equality of moments is sufficient to ensure that the distribution of y and the limiting distribution of λ_1 are identical.

Therefore, in large samples, $-2\log_e\lambda_1 = -2N\log_e l_1$ will be approximately distributed as χ^2 with f degrees of freedom. Thus if

$$l_1 = e^{-\chi^2/2N}, \qquad \text{.....(25)}$$

it follows that the probability levels for l_1 may be obtained by inserting in (25) the corresponding levels for χ^2. It had been hoped that this method of calculating the probability levels might prove satisfactory for moderately large values of n, so that the labour of calculating the first two moments of l_1 could have been avoided. In actual fact, however, this hope is not realized, as is shown by the results given in Table V.

TABLE V

q		$k=2$ $n=30$		$k=5$ $n=30$		$k=5$ $n=40$		$k=5$ $n=50$		$k=10$ $n=30$	
		5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
1	(1) Level from Type I	0·9669	0·9435	0·9674	0·9546	0·9757	0·9662	0·9807	0·9731	0·9709	0·9629
	(2) Level from χ^2	0·9685	0·9462	0·9689	0·9567	0·9766	0·9673	0·9812	0·9738	0·9722	0·9645
	Difference (2)−(1) divided by $\sqrt{(\mu_2)}$	0·13	0·22	0·15	0·22	0·12	0·15	0·10	0·12	0·18	0·22
2	(1) Level from Type I	—	—	0·9280	0·9110	0·9463	0·9336	0·9573	0·9470	—	—
	(2) Level from χ^2	—	—	0·9323	0·9163	0·9488	0·9366	0·9588	0·9489	—	—
	Difference (2)−(1) divided by $\sqrt{(\mu_2)}$	—	—	0·26	0·32	0·20	0·24	0·15	0·19	—	—
3	(1) Level from Type I	0·8913	0·8576	0·8767	0·8563	0·9079	0·8922	0·9265	0·9138	0·8783	0·8643
	(2) Level from χ^2	0·9004	0·8693	0·8857	0·8665	0·9130	0·8981	0·9298	0·9176	0·8867	0·8736
	Difference (2)−(1) divided by $\sqrt{(\mu_2)}$	0·31	0·40	0·39	0·45	0·30	0·34	0·24	0·28	0·50	0·55
4	(1) Level from Type I	—	—	0·8149	0·7915	0·8610	0·8428	0·8887	0·8737	—	—
	(2) Level from χ^2	—	—	0·8304	0·8087	0·8699	0·8528	0·8945	0·8804	—	—
	Difference (2)−(1) divided by $\sqrt{(\mu_2)}$	—	—	0·55	0·61	0·41	0·47	0·33	0·38	—	—
5	(1) Level from Type I	0·7884	0·7476	0·7446	0·7192	0·8067	0·7867	0·8446	0·8281	—	—
	(2) Level from χ^2	0·8119	0·7750	0·7684	0·7448	0·8206	0·8018	0·8537	0·8380	—	—
	Difference (2)−(1) divided by $\sqrt{(\mu_2)}$	0·53	0·62	0·73	0·79	0·55	0·60	0·44	0·48	—	—
6	(1) Level from Type I	—	—	0·6678	0·6411	0·7462	0·7244	0·7935	0·7761	—	—
	(2) Level from χ^2	—	—	0·7014	0·6769	0·7665	0·7463	0·8083	0·7913	—	—
	Difference (2)−(1) divided by $\sqrt{(\mu_2)}$	—	—	0·94	1·01	0·72	0·78	0·64	0·65	—	—

It is evident from inspection of Table V that, as would be expected, the agreement between the levels, as given by the Type I and χ^2 approximations, improves with increase of n, whereas it slowly becomes worse as k is increased. However, as q increases there is such a rapid deterioration in the accuracy of the approximation that this method cannot be justifiably employed, except for large values of n.

It may however be usefully noted that the levels obtained from the χ^2 transformation are always above those set by the Type I curve, so that if the hypothesis is not rejected when the level given by χ^2 is employed it certainly would not be rejected if the more accurate Type I level had been calculated.

7. EMPIRICAL RELATIONS FOR m_1 AND m_2

When using the test it is found that the calculation of the first and second moments of l_1 accounts for the major part of the labour of computation which is involved. It therefore seemed desirable to attempt to obtain empirical relations giving m_1 and m_2 in terms of n, k and q, such that the use of these values will lead to probability levels of l_1 sufficiently accurate for practical purposes.

We have seen that $M_h(\lambda_1)$ tends uniformly to $(1+h)^{-\frac{1}{2}f}$ as n tends to ∞. Put $l_1' = \lambda_1^{1/M}$, where M is any positive number. Then $M_h(l_1')$, the hth moment of l_1' tends uniformly to $(1+h/M)^{-\frac{1}{2}f}$ as $N \to \infty$. If a Type I curve be fitted to the distribution of l_1', in a way similar to the one employed when approximating to the distribution of l_1, the exponent of the power of $(1-l_1')$ will be given by

$$m_2' = \frac{\{1-M_1(l_1')\}\{M_1(l_1')-M_2(l_1')\}}{M_2(l_1')-\{M_1(l_1')\}^2},$$

and this may be considered as a function $\psi(M, N)$, of M and N.

Now,

$$\lim_{N\to\infty} \psi(M, N) = \lim_{N\to\infty} m_2' = \frac{\{1-(1+1/M)^{-\frac{1}{2}f}\}\{(1+1/M)^{-\frac{1}{2}f}-(1+2/M)^{-\frac{1}{2}f}\}}{(1+2/M)^{-\frac{1}{2}f}-(1+1/M)^f} = g(M),$$

the limit being approached uniformly for all positive M, and $\lim_{M\to\infty} g(M) = \frac{1}{2}f$.

Hence $$\lim_{N\to\infty} \psi(N, N) = \lim_{N\to\infty} m_2 = \tfrac{1}{2}f = 0{\cdot}25(k-1)\,q(q+1).$$

Values of m_1 and m_2 which have been calculated in a number of cases are given in Table VI. The values of m_2 indicate that the limit $\frac{1}{2}f$ is approached rapidly with increase of n, and it seems probable that, for $n \geqslant 20$, no error of practical importance will be made if we substitute $\frac{1}{2}f$ for m_2.

Methods of approximating to m_1 must now be considered. U. S. Nair (1938, p. 285) has shown that the distribution of l_1 may be reduced to the form

$$p(l_1) = p_1(l_1)\, p_2(l_1),$$

where $$p_1(l_1) = \text{const}\ l_1^{\frac{1}{2}k(n-q)-1}$$

and $$p_2(l_1) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} l_1^{-t}\, k^{\frac{1}{2}qt} \prod_{j=1}^{q}\left\{\frac{(\Gamma(\frac{1}{2}(q-j+t/k)))^k}{\Gamma(\frac{1}{2}(k(q-1)+1-j+t))}\right\}dt,$$

TABLE VI

Values of m_1 and m_2

	$q=1$	$q=2$	$q=3$	$q=4$	$q=5$	$q=6$
$k=2$ $n=10$	17·351 0·501	15·599 1·501	— —	— —	— —	— —
$k=2$ $n=30$	57·282 0·500	55·588 1·500	53·732 2·999	51·683 4·999	49·432 7·501	46·997 10·511
$k=2$ $n=50$	97·269 0·500	95·586 1·500	— —	— —	— —	— —
$k=5$ $n=10$	42·697 2·005	38·219 6·004	33·111 12·018	27·527 20·189	21·769 31·022	16·178 45·953
$k=5$ $n=20$	92·591 2·001	88·188 6·000	83·053 12·004	77·199 20·036	70·758 30·177	63·912 42·609
$k=5$ $n=30$	142·559 2·000	138·177 6·000	133·033 12·001	127·094 20·014	120·439 30·071	113·187 42·239
$k=5$ $n=40$	192·543 2·000	188·178 6·000	183·014 12·000	177·043 20·008	170·285 30·038	162·830 42·127
$k=5$ $n=50$	242·532 2·000	238·175 6·000	233·020 12·001	227·017 20·005	220·191 30·023	212·600 42·075
$k=10$ $n=30$	284·689 4·501	275·836 13·501	265·205 27·006	252·811 45·049	238·861 67·719	223·649 95·214
$k=25$ $n=30$	711·207 12·004	688·901 36·007	— —	— —	— —	— —

In each cell m_1 is the upper and m_2 the lower number.

so that $p_2(l_1)$ is independent of n. Now if $\frac{1}{2}f$ replaces m_2, the Type I distribution (7) takes the form

$$p(l_1) = \text{const}\, l_1^{m_1-1}(1-l_1)^{\frac{1}{2}f-1},$$

so that it is expressed as the product of two factors $l_1^{m_1-1}$ and $(1-l_1)^{\frac{1}{2}f-1}$, the second of which is independent of n. In these circumstances it seemed possible that m_1 might perhaps be replaced by $k(n-q)$. Therefore putting

$$m_1 = k(n-q) \qquad \text{.....(26)}$$

and

$$m_2 = \tfrac{1}{2}f = 0{\cdot}25(k-1)\,q(q+1), \qquad \text{.....(16) } bis$$

the 5 % and 1 % levels of l_1 were calculated in a number of cases and compared with those given by using the values of m_1 and m_2 obtained from the moments. The results given in Table VII indicate that, although the agreement in cases where $q = 1$ is satisfactory, for larger values of q the discrepancies become too large to enable this method of approximation to be used in practice.

TABLE VII

Comparison of levels given by Method 1 and Method 2

		$q=1$		$q=3$		$q=5$	
	Method	5%	1%	5%	1%	5%	1%
$k=2$ $n=30$	1	0·967	0·944	0·891	0·858	0·788	0·748
	2	0·967	0·944	0·892	0·858	0·791	0·750
$k=5$ $n=10$	1	0·896	0·857	0·621	0·570	0·304	0·263
	2	0·901	0·864	0·635	0·585	0·346	0·304
$k=5$ $n=20$	1	0·950	0·931	0·814	0·784	0·624	0·590
	2	0·952	0·933	0·818	0·788	0·640	0·607
$k=5$ $n=30$	1	0·967	0·955	0·877	0·856	0·745	0·719
	2	0·968	0·955	0·878	0·858	0·752	0·728
$k=5$ $n=40$	1	0·976	0·966	0·908	0·892	0·807	0·787
	2	0·976	0·967	0·909	0·893	0·811	0·792
$k=5$ $n=50$	1	0·981	0·973	0·927	0·914	0·845	0·828
	2	0·981	0·973	0·927	0·915	0·848	0·831
$k=10$ $n=30$	1	0·971	0·963	0·878	0·864	—	—
	2	0·970	0·962	0·880	0·866	—	—

Method 1. m_1 and m_2 obtained from the first two moments of l_1.
Method 2. m_1 and m_2 given by the empirical relations

$$m_1 = k(n-q), \quad m_2 = 0{\cdot}25(k-1)\,q(q+1).$$

It is clear that the deviations are due almost entirely to an inadequate approximation to m_1, although the expression does not appear to differ from m_1 in such a marked manner that its use as a basis of approximation must be rejected entirely. Thus it was thought that some slight correcting term might be subtracted from $k(n-q)$ to provide a better approximation to m_1. With this end in view values of the necessary correction $C = k(n-q)-m_1$ were calculated and are given in Table VIII.

TABLE VIII

Values of correction $C = k(n-q) - m_1$

	$q=1$	$q=2$	$q=3$	$q=4$	$q=5$	$q=6$
$k=2,\ n=10$	0·65	0·40	—	—	—	—
$k=2,\ n=30$	0·72	0·41	0·27	0·32	0·57	1·00
$k=2,\ n=50$	0·73	0·41	—	—	—	—
$k=5,\ n=10$	2·30	1·72	1·89	2·47	3·23	3·82
$k=5,\ n=20$	2·41	1·81	1·95	2·80	4·24	6·09
$k=5,\ n=30$	2·44	1·82	1·97	2·91	4·66	6·81
$k=5,\ n=40$	2·46	1·82	1·99	2·96	4·71	7·17
$k=5,\ n=50$	2·47	1·82	1·98	2·98	4·81	7·40
$k=10,\ n=30$	5·31	4·16	4·79	7·19	11·14	16·35
$k=25,\ n=30$	13·79	11·10	—	—	—	—

When $n \geqslant 20$, the variation of C with n, for fixed k and q, is small; so that in seeking a simple empirical relation for C, functions of k and q only were considered. If, for constant q, C be plotted against k, it is found that the relation between them is nearly linear and; moreover, C may be taken as proportional to $(k-1)$. Assuming therefore that $C = (k-1)\theta_q$, values of θ_q for different q may be calculated. On plotting θ_q against q, the form of the resulting curve suggests that, over the range of values of q which are considered, θ_q will be given with sufficient accuracy by a quadratic form in q. Making this assumption the coefficients of the quadratic were adjusted by trial and error until the expression which resulted seemed likely to give satisfactory results in most cases. The correcting term was thus tentatively obtained in the form

$$C = 0{\cdot}01(k-1)(90-39q+9q^2).$$

The final replacement for m_1, as previously given, is therefore

$$m_1 = k(n-q) - 0{\cdot}01(k-1)(90-39q+9q^2), \qquad \text{.....(15) } bis$$

and as before

$$m_2 = 0{\cdot}25(k-1)\,q(q+1). \qquad \text{.....(16) } bis$$

It is realized that the reader may have some doubts as to the validity of the empirical methods by which this approximation for m_1 was obtained. In order therefore to establish this approximation as a satisfactory practical method, it is necessary to calculate for a wide range of cases the 5 % and 1 % levels given by the approximation. These may then be compared with the levels obtained when values of m_1 and m_2 calculated from the moments are used. This has been done in a number of cases, the results being given in Table IX.

Inspection of this table shows that when $n \geqslant 20$ the levels obtained by using values of m_1 and m_2 as given by (15) and (16) agree with those resulting

TABLE IX

Comparison of levels given by Method 1 *and Method* 3

			$q=1$		$q=2$		$q=3$		$q=4$		$q=5$		$q=6$	
k	n	Method	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
2	10	1	0·894	0·824	0·781	0·699	—	—	—	—	—	—	—	—
		3	0·894	0·824	0·780	0·698	—	—	—	—	—	—	—	—
2	30	1	0·967	0·944	0·932	0·903	0·891	0·858	0·843	0·805	0·788	0·748	0·728	0·685
		3	0·967	0·944	0·932	0·903	0·891	0·857	0·842	0·804	0·786	0·745	0·725	0·681
2	50	1	0·980	0·966	0·960	0·943	—	—	—	—	—	—	—	—
		3	0·980	0·966	0·960	0·943	—	—	—	—	—	—	—	—
5	10	1	0·896	0·857	0·772	0·724	0·621	0·570	0·458	0·409	0·304*	0·263*	0·174*	0·145*
		3	0·896	0·857	0·771	0·723	0·619	0·567	0·454	0·405	0·292	0·251	0·146	0·118
5	20	1	0·950	0·931	0·891	0·865	0·814	0·784	0·724	0·691	0·624	0·590	0·521	0·488
		3	0·950	0·931	0·890	0·865	0·814	0·784	0·723	0·690	0·624	0·590	0·520	0·486
5	30	1	0·967	0·955	0·928	0·911	0·877	0·856	0·815	0·792	0·745	0·719	0·668	0·641
		3	0·967	0·955	0·928	0·911	0·877	0·856	0·815	0·791	0·745	0·719	0·668	0·641
10	30	1	0·971	0·963	0·931	0·920	0·878	0·864	0·813	0·797	—	—	—	—
		3	0·971	0·963	0·931	0·920	0·878	0·864	0·813	0·797	—	—	—	—
25	30	1	0·975	0·970	0·936	0·930	—	—	—	—	—	—	—	—
		3	0·975	0·970	0·936	0·929	—	—	—	—	—	—	—	—

Method 1. m_1 and m_2 obtained from the first two moments of l_1.

Method 3. m_1 and m_2 given by the empirical relations

$$m_1 = k(n-q) - 0{\cdot}01(k-1)\,(90-39q+9q^2), \quad m_2 = 0{\cdot}25(k-1)\,q(q+1).$$

from the use of values of m_1 and m_2 calculated from the moments with sufficient accuracy for practical purposes. Even when n is as small as 10 the only serious discrepancies, marked with an asterisk, occur when there are as many as five or six variables.

The range of values of m_1 and m_2 of the distributions considered in the last table may be found by consulting Table VI. From the practical viewpoint, it must be noted that when both m_1 and m_2 exceed 60 it becomes impossible to interpolate in the z-tables without running the risk of making appreciable errors. In these cases an approximation, due to Fisher (1938*b*, § 41), may be used. When z has degrees of freedom f_1 and f_2 which are both large and $f = 2f_1f_2/(f_1+f_2)$ is their harmonic mean, the 5 % level of z is given approximately by

$$z_{\cdot 05} = \frac{1{\cdot}6449}{\sqrt{(f-1)}} - 0{\cdot}7843\left(\frac{1}{f_1}-\frac{1}{f_2}\right)$$

and the 1 % level by

$$z_{\cdot 01} = \frac{2{\cdot}3263}{\sqrt{(f-1)}} - 1{\cdot}235\left(\frac{1}{f_1}-\frac{1}{f_2}\right).$$

8. Some consideration of the case of unequal samples

We have so far considered k equal samples each of size n, but in the general case there may be k samples of sizes $n_t (t = 1, 2, \ldots, k)$. The appropriate criterion in this case is

$$l_1 = \lambda_1^{1/N} = \sqrt{\left(\prod_{t=1}^{k} \left\{\frac{|v_{sut}|}{|v_{su}|}\right\}^{n_t/N}\right)},$$

where now $$v_{sut} = \frac{1}{n_t}\sum_{i=1}^{n_t}(x_{sti} - x_{st.})(x_{uti} - x_{ut.}), \quad v_{su} = \frac{1}{N}\sum_{t=1}^{k} n_t v_{sut}$$

and $$x_{st.} = \frac{1}{n_t}\sum_{i=1}^{n_t} x_{sti}, \quad N = \sum_{t=1}^{k} n_t.$$

It is seen that l_1^2 is the ratio of a weighted geometric mean of the generalized variances to a determinant of order q in which the element in the sth row and uth column is v_{su}, the weighted arithmetic mean of $v_{su1}, v_{su2}, \ldots v_{suk}$, the weight given in each case being the corresponding sample size. When considering single-variate criteria, Bartlett (1937) suggested that a better test will be obtained if each variance is weighted with the number of degrees of freedom it possesses rather than with the sample size, and more recent unpublished work confirms this contention. Thus there is a possibility that in the general case, where more than one variate is considered, some adjustment of the weighting will yield a criterion which will more frequently detect the falsehood of the hypothesis H_0 when in fact some alternative hypothesis is true. That is to say, the modified test might prove to be more powerful, in the sense of Neyman & Pearson (1936), with regard to a certain set of alternative hypotheses. However, it is likely that such a modification will only have an appreciable effect when some, at least, of the samples are small.

The preceding reasoning concerning the limiting form of the distribution of l_1 may be shown to apply whether the samples are of the same size or not. The distribution of l_1, as approximated to by means of the χ^2 transformation, depends only on N, k and q, and not on the size of individual samples. It therefore seems possible that with large samples, which although unequal do not vary greatly in size, a reasonable approximation to the true levels may be obtained by using the χ^2 transformation of equation (25).

The accuracy of the χ^2 method of approximation may be improved by following a procedure similar to that used by Bartlett (1937) when dealing with a single variate. In the general case, the hth moment of λ_1 is given by

$$M_h(\lambda_1) = \prod_{t=1}^{k}\left\{\left(\frac{N}{n_t}\right)^{\frac{1}{2}qhn_t}\prod_{i=1}^{q}\frac{\Gamma\left(\frac{n_t(1+h)-i}{2}\right)}{\Gamma\left(\frac{n_t-i}{2}\right)}\right\}\cdot\prod_{t=1}^{q}\left\{\frac{\Gamma\left(\frac{N-k+1-i}{2}\right)}{\Gamma\left(\frac{N(1+h)-k+1-i}{2}\right)}\right\}.$$

In the limit $-2\log_e\lambda_1$ is distributed as χ^2 with f degrees of freedom and our purpose is to find a correcting factor G, a function of n_t, k and q which only involves terms of order $(n_t)^{-1}$, such that $-2G^{-1}\log_e\lambda_1$ has a mean value differing

from f, the mean value of χ^2, by terms of order $(n_t)^{-2}$. Thus Gf must be identical with the sum of terms of order $(n_t)^{-1}$ in the coefficient of h in the expansion, in powers of h, of $\log_e\{M_{-2h}(\lambda_1)\}$. Using Stirling's approximation we obtain

$$G = 1+\frac{1}{f}\left\{\sum_{t=1}^{k}\sum_{i=1}^{q}\left(\frac{i^2}{2n_t}+\frac{i}{n_t-i}+\frac{n_t}{3(n_t-i)^2}\right)\right.$$
$$\left.-\sum_{i=1}^{q}\left(\frac{(k+i-1)^2}{2N}+\frac{k+i-1}{N-k+1-i}+\frac{N}{3(N-k+1-i)^2}\right)\right\}.$$

If now $-2G^{-1}\log_e\lambda_1$ be referred to the χ^2 distribution with f degrees of freedom, approximate levels for l_1 may be obtained from the corresponding levels for χ^2 by using the relation

$$l_1 = \exp\left(-\frac{G\chi^2}{2N}\right). \qquad \text{.....(27)}$$

The degree of accuracy provided by this method is indicated in Table X, which compares, for some typical cases, the levels given by (27) with those obtained by fitting a Type I curve.

TABLE X

			$q=1$	$q=2$	$q=3$	$q=4$	$q=5$	$q=6$
$k=5$, $n_t=10$ ($t=1,\ldots,5$)	5% level	From Type I	0·896	0·772	0·621	0·458	0·304	0·174
		From relation (27)	0·896	0·774	0·628	0·473	0·328	0·206
	1% level	From Type I	0·857	0·724	0·570	0·409	0·263	0·145
		From relation (27)	0·857	0·726	0·577	0·425	0·288	0·176
$k=5$, $n_t=20$ ($t=1,\ldots,5$)	5% level	From Type I	0·950	0·891	0·814	0·724	0·624	0·521
		From relation (27)	0·950	0·890	0·813	0·724	0·628	0·528
	1% level	From Type I	0·931	0·865	0·784	0·691	0·590	0·488
		From relation (27)	0·931	0·866	0·783	0·692	0·594	0·495

It is apparent from Tables V and X that the crude χ^2 approximation has been considerably improved by the use of the correcting factor G. Although examples in which the k samples are of *unequal* sizes have not been given in Table X, it is probable that, if the smallest sample contains at least twenty individuals, the application of the above method will give a fairly satisfactory result.

Mention may be made in passing of a similar approximation which is found to give close agreement with the distribution of the comprehensive likelihood-ratio criterion for testing one of the hypotheses concerning the values of the means of populations referred to in the introduction to this paper. It is assumed that the hypothesis H_0 regarding the equality of variances and covariances is true, and it is desired to test whether the corresponding means of the q characters are

the same in all k populations. In the case of two samples ($k = 2$), the test is equivalent to use of Hotelling's generalized T. If

$$a_{su} = \frac{1}{N}\sum_{t=1}^{k}\sum_{i=1}^{n_t}(x_{sti} - x_{s..})(x_{uti} - x_{u..}),$$

where

$$x_{s..} = \frac{1}{N}\sum_{t=1}^{k}\sum_{i=1}^{n_t} x_{sti},$$

the appropriate comprehensive likelihood criterion was given by Wilks in the form

$$W = \frac{|v_{su}|}{|a_{su}|}.$$

The distribution of W may be expressed simply in the following three cases:

(*a*) When $q = 1$, $p(W) = \left\{B\left(\frac{N-k}{2}, \frac{k-1}{2}\right)\right\}^{-1} W^{\frac{1}{2}(N-k-2)}(1-W)^{\frac{1}{2}(k-3)}$.*

(*b*) When $q = 2$, $p(\sqrt{W}) = \{B(N-k-1, k-1)\}^{-1}(\sqrt{W})^{N-k-2}(1-\sqrt{W})^{k-2}$.

(*c*) When $k = 2$, $p(W) = \left\{B\left(\frac{N-q-1}{2}, \frac{q}{2}\right)\right\}^{-1} W^{\frac{1}{2}(N-q-3)}(1-W)^{\frac{1}{2}(q-2)}$.†

In other cases, following the previous procedure, it may be shown that approximate levels for W may be obtained by using the relation

$$W = \exp\left(-\frac{G'\chi^2}{N}\right),$$

where

$$G' = 1 + \frac{1}{q(k-1)}\sum_{i=1}^{q}\left\{\frac{i(k-2)}{N-i} + \frac{N(3(k-1)^2-2)}{6(N-i)^2} + \frac{k+i-1}{N-k+1-i} + \frac{N}{3(N-k+1-i)^2}\right\},$$

and χ^2 has $q(k-1)$ degrees of freedom. Calculations which have been made for certain examples show that this method of approximation will give the levels of W with sufficient accuracy for most practical purposes if the ratio N/k is not less than 10.

Summary

The main object of this paper has been to put into easier form for application the likelihood ratio criterion l_1 for testing the hypothesis that all the corresponding variances and covariances in a number, k, of multivariate q-variable normal populations are identical. In the case where the samples from each population are of the same size ($n_t = n$, $t = 1, \ldots, k$) empirical formulae have been given, using which 5 % and 1 % probability levels for l_1 may be obtained readily from the levels of R. A. Fisher's z; alternatively the *Tables of the Incomplete Beta-*

* When $q = 1$, W is just the ratio of two sums of squares, familiar in the analysis of variance.

† $W = \left(1 + \frac{T^2}{N-2}\right)^{-1}$.

Function may be used. The conditions under which these formulae may be safely employed are discussed.

If the samples are large and $N = \sum_{t=1}^{k} n_t$, $-2N \log_e l_1$ is distributed approximately as χ^2 with degrees of freedom $f = \frac{1}{2}(k-1)\,q(q+1)$. The test is then independent of the individual values of n_t, which may now differ. If there are many variables the sample sizes must, however, be very large for this transformation to be justifiable.

A more accurate approximation, of the type suggested by M. S. Bartlett, in the case of a single variable ($q = 1$), has also been considered. The corrective term is, however, rather troublesome to calculate unless again $n_t = n$ ($t = 1, \ldots, k$).

In conclusion I wish to thank Prof. E. S. Pearson for much assistance in the preparation of this paper.

Additional note

U. S. Nair has recently obtained, on a theoretical basis, an approximation to the distribution of l_1, in the case of k equal samples, somewhat similar to the one given as Method 3 on pp. 50–51.

He finds that the distribution of l_1 may be taken approximately as

$$p(l_1) = \text{const}\; l_1^{k(n-q)-C'-1}\,(1-l_1)^{\frac{1}{2}f-1},$$

where
$$C' = \frac{1}{24}(k-1)\,(3q^2-13q+28) - \frac{1}{6}(2q-5) - \frac{k+1}{3(q+1)}.$$

As far as we have been able to check, this approximation and Method 3 seem to be adequate for the same values of n and appear to give levels which are substantially the same.

REFERENCES

BARTLETT, M. S. (1937). *Proc. Roy. Soc.* A, **160**, 268–82.
BISHOP, D. J. & NAIR, U. S. (1939). *J.R. Statist. Soc., Suppl.* **6**, 89–99.
FISHER, R. A. (1938*a*). *Ann. Eugen., Lond.*, **8**, 376–86.
—— (1938*b*). *Statistical Methods for Research Workers*, 7th edition. London: Oliver and Boyd.
HSU, P. L. (1938). *Ann. Math. Statist.* **9**, 231–43.
LAWLEY, D. G. (1938*a*). *Biometrika*, **30**, 180–87.
—— (1938*b*). *Biometrika*, **30**, 467–9.
NAIR, U. S. (1938). *Biometrika*, **30**, 274–94.
NAYER, P. P. N. (1936). *Statist. Res. Mem.* **1**, 38–51.
NEYMAN, J. & PEARSON, E. S. (1931). *Bulletin de l'Académie Polonaise des Sciences et des Lettres*, A, 460–81.
—— —— (1936). *Statist. Res. Mem.* **1**, 1–37.
PEARSON, E. S. (1922). *Tracts for Computers*, **8**. Cambridge University Press.
PEARSON, E. S. & WILKS, S. S. (1933). *Biometrika*, **25**, 353–78.
PEARSON, K. (1934). *Tables of the Incomplete Beta-Function*. Biometrika Office, University College, London.
PITTARD, E. (1909). *Crania Helvetica I. Les Cranes Valaisans de la Vallée du Rhône*. Extrait du tome 20 des Mémoires de l'Institut national genevois.
WILKS, S. S. (1932). *Biometrika*, **24**, 471–94.

OBSERVED AND THEORETICAL RATIOS IN MENDELIAN INHERITANCE

By E. ROBERTS, W. M. DAWSON* and MARGARET MADDEN*
University of Illinois

According to the present conception of Mendelian inheritance genes are located on chromosomes, and if not located on the same chromosome are independent in inheritance. During the formation of reproductive cells the number of chromosomes is reduced to one-half the number common to the species or one-half the number found in the somatic cells. As a result of fertilization the original number is restored.

An allelomorphic pair of genes are genes occupying the same relative loci on homologous chromosomes, one of which came from the father and the other from the mother. The members (alleles) of an allelomorphic pair under usual conditions separate, going into different reproductive cells or gametes. When located on different chromosomes the result is as many different kinds of germ cells as the number of possible combinations of the genes, except that members of an allelomorphic pair are not found in the same gamete. For any number of allelomorphic pairs the number of different kinds of gametes is 2^n, in which n is the number of allelomorphic pairs. In case of dominance and recessiveness the number of visible classes or phenotypes formed among the offspring as the result of random fertilization is also 2^n.

The purpose of this paper is to put on record the ratios obtained in several mammalian crosses made in genetic studies in the Laboratory of Animal Genetics, College of Agriculture, University of Illinois, during a period of several years.

Daily inspection for birth of young was made and number of young recorded. Usually two classifications were made, one at 12–18 days of age and the second at the time of weaning, which in the case of the mice and rats was at about 28 days. Rabbits were weaned at an age of about 8 weeks. Colours and colour patterns can be easily classified as soon as the pelage is well formed. Dark eye and pink eye are distinguishable at birth and were recorded when the birth records were made. In hypotrichotic rats the hair is normal in appearance until between 16 and 20 days of age when depilation begins, first noticeable around the head.

In our records litters were classified as complete and incomplete. A complete litter is one in which all the young were classified and an incomplete litter one in which one or more had died before classification. Ratios among incomplete litters did not differ significantly from ratios among complete litters, and for this reason they are combined in the tables.

* Formerly Assistant in Animal Husbandry.

The symbols used and the characters which they designate are as follows:

Rats

A = agouti, **a** = non-agouti.

H = self colour (entirely coloured), **h** = hooded.

R = dark eye, **r** = red eye.

C = colour, **c** = albinism.

$\mathbf{H_y}$ = haired, $\mathbf{h_y}$ = hypotrichosis (hairless).

D = intense colour, **d** = dilute colour.

Mice

A = agouti, **a** = non-agouti.

$\mathbf{P_1}$ = dark eye, $\mathbf{p_1}$ = pink eye.

$\mathbf{P_2}$ = dark eye, $\mathbf{p_2}$ = pink eye (a second gene causing pink eye).

B = black, **b** = brown.

Y = yellow, **y** = non-yellow (black or brown).

Rabbits

A = agouti, **a** = non-agouti.

E = extension, **e** = non-extension (yellow).

For obtaining probabilities of ratios with two classes the formula

$$E = \pm 0{\cdot}6745\sqrt{(npq)},$$

in which n is the number of individuals, p the observed proportion of one class and q the proportion of the other class, was used. For ratios having three or more terms, the fit of the observed to the theoretical ratio was determined by Pearson's formula

$$\chi^2 = S\left\{\frac{(m_1' - m_r)^2}{m_r}\right\};$$

m_1' is the observed number in a class, m_r the theoretical or expected number, and S is the sum. P is obtained from Pearson's *Tables for Statisticians and Biometricians*.

1. Matings of the type **Aa** × **aa**. Theoretical ratio of 1 : 1.

$$\left.\begin{array}{l}\mathbf{Aa} \rightarrow \mathbf{A} + \mathbf{a} \\ \mathbf{aa} \rightarrow \mathbf{a} + \mathbf{a}\end{array}\right\} \text{gametes.}$$

$$\overline{\mathbf{Aa} + \mathbf{aa}} \text{ zygotes.}$$

TABLE I

Mating		Dominant	Recessive	Deviation	Probable error	*D/E*
Rats						
Aa × **aa**	O	690	633	28·5	12·26	2·32
	E	661·5	661·5			
Hh × **hh**	O	552	448	52·0	10·61	4·90
	E	500	500			
Rr × **rr**	O	418	401	8·5	9·65	0·88
	E	409·5	409·5			
Cc × **cc**	O	168	177	4·5	6·26	0·72
	E	172·5	172·5			
$\mathbf{H_yh_y} \times \mathbf{h_yh_y}$	O	745	720	12·5	12·90	0·97
	E	732·5	732·5			
Dd × **dd**	O	669	576	46·5	11·86	3·92
	E	622·5	622·5			
Mice						
Aa × **aa**	O	1906	1828	39·0	20·61	1·89
	E	1867	1867			
$\mathbf{P_1p_1} \times \mathbf{p_1p_1}$	O	1898	1836	31·0	20·61	1·50
	E	1867	1867			
Bb × **bb**	O	1842	1892	25·0	20·61	1·21
	E	1867	1867			
$\mathbf{P_2p_2} \times \mathbf{p_2p_2}$	O	374	316	29·0	8·83	3·28
	E	345	345			
Yy × **yy**	O	259	270	5·5	7·76	0·71
	E	264·5	264·5			
Rabbits						
Ee × **ee**	O	27	24	1·5	2·40	0·62
	E	25·5	25·5			
Aa × **aa**	O	40	32	4·0	2·84	1·41
	E	36	36			

2. Matings of the type **Aa** × **Aa**, producing two phenotypes in the ratio of 3 dominants to 1 recessive.

$$\left.\begin{matrix}\mathbf{Aa}\rightarrow\mathbf{A}+\mathbf{a}\\ \mathbf{Aa}\rightarrow\mathbf{A}+\mathbf{a}\end{matrix}\right\}\text{gametes.}$$

$$\mathbf{1AA}+\mathbf{2Aa}+\mathbf{1aa}\ \text{zygotes.}$$

Since **A** is dominant a ratio of 3 : 1 is expected.

TABLE II

Mating		Dominant	Recessive	Deviation	Probable error	*D/E*
Rats						
Aa × **Aa**	O	607	185	13·00	8·03	1·62
	E	594	198			
Hh × **Hh**	O	622	227	14·75	8·70	1·70
	E	636·75	212·25			
Rr × **Rr**	O	389	128	1·25	6·62	0·19
	E	387·75	129·25			
Cc × **Cc**	O	946	324	6·50	10·47	0·62
	E	952·5	317·5			
Hyhy × **Hyhy**	O	1305	435	0·00	12·18	0·00
	E	1305	435			
Dd × **Dd**	O	605	204	1·75	8·33	0·21
	E	606·75	202·25			
Mice						
$P_2P_2 \times P_2P_2$	O	471	141	12·00	7·03	1·71
	E	459	153			
Rabbits						
Ee × **Ee**	O	125	33	6·50	3·46	1·88
	E	118·5	39·5			

3. Matings of the type **AaBb × aabb** resulting in a theoretical ratio of 1 : 1 : 1 : 1.

TABLE III

Matings		Phenotypes				χ^2	P
Rats							
AaHyhy × aahyhy	O	**AHy** 187	**Ahy** 175	**aHy** 165	**ahy** 174	1·396	0·711
	E	175·25					
HhHyhy × hhhyhy	O	**HHy** 194	**Hhy** 187	**hHy** 146	**hhy** 158	9·219	0·027
	E	171·25					
RrHyhy × rrhyhy	O	**RHy** 146	**Rhy** 150	**rHy** 145	**rhy** 147	0·095	0·99 +
	E	147					
DdHyhy × ddhyhy	O	**DHy** 158	**Dhy** 166	**dHy** 144	**dhy** 138	3·241	0·360
	E	151·5					
CcHyhy × cchyhy	O	**CHy** 13	**Chy** 13	**cHy** 16	**chy** 12	0·668	0·880
	E	13·5					
AaDd × aadd	O	**AD** 181	**Ad** 164	**aD** 187	**ad** 159	2·967	0·398
	E	172·75					
HhDd × hhdd	O	**HD** 187	**Hd** 149	**hD** 133	**hd** 133	12·937	0·005
	E	150·5					
RrDd × rrdd	O	**RD** 184	**Rd** 160	**rD** 175	**rd** 161	2·364	0·507
	E	170					
AaRr × aarr	O	**AR** 150	**Ar** 146	**aR** 147	**ar** 143	0·172	0·982
	E	146·5					
HhRr × hhrr	O	**HR** 167	**Hr** 159	**hR** 134	**hr** 131	6·543	0·090
	E	147·75					
AaHh × aahh	O	**AH** 175	**Ah** 136	**aH** 164	**ah** 147	5·720	0·128
	E	155·5					
Mice							
$AaP_1p_1 \times aap_1p_1$	O	**AP_1** 937	**Ap_1** 969	**aP_1** 961	**ap_1** 867	6·911	0·075
	E	933·5					
AaBb × aabb	O	**AB** 935	**Ab** 971	**aB** 907	**ab** 921	2·428	0·495
	E	933·5					
$BbP_1p_1 \times bbp_1p_1$	O	**BP_1** 942	**Bp_1** 900	**bP_1** 956	**pb_1** 936	1·829	0·612
	E	933·5					

4. Matings of the type **AaBb × AaBb**. Expected ratio of 9 : 3 : 3 : 1.

TABLE IV

Mating		Phenotypes				χ^2	P
Rats		**AHy**	**Ahy**	**aHy**	**ahy**		
AaHyhy × AaHyhy	O	309	94	93	37	1·526	0·681
	E	299·81	99·94	99·94	33·31		
		HHy	**Hhy**	**hHy**	**hhy**		
HhHyhy × HhHyhy	O	377	135	128	34	2·186	0·539
	E	379·12	126·38	126·38	42·12		
		RHy	**Rhy**	**rHy**	**rhy**		
RrHyhy × RrHyhy	O	66	17	20	6	1·044	0·791
	E	61·31	20·44	20·44	6·81		
		DHy	**Dhy**	**dHy**	**dhy**		
DdHyhy × DdHyhy	O	111	42	24	14	5·438	0·145
	E	107·46	35·82	35·82	11·94		
		CHy	**Chy**	**cHy**	**chy**		
CcHyhy × CcHyhy	O	427	136	140	46	0·232	0·972
	E	421·31	140·44	140·44	46·81		
		AD	**Ad**	**aD**	**ad**		
AaDd × AaDd	O	41	19	16	5	1·456	0·697
	E	45·56	15·19	15·19	5·06		
		RD	**Rd**	**rD**	**rd**		
RrDd × RrDd	O	195	52	69	18	3·119	0·376
	E	187·83	62·61	62·61	20·87		

5. Matings of the type **AaBbCc × aabbcc**. Theoretical ratio of 1 : 1 : 1 : 1 : 1 : 1 : 1 : 1.

TABLE V

Mating		Phenotypes								χ^2	P
Rats											
AaDdHyhy × aaddhyhy	O	**ADHy** 72	**ADhy** 71	**AdHy** 70	**Adhy** 62	**aDHy** 70	**aDhy** 77	**adHy** 58	**adhy** 66	3·714	0·810
	E	68·25									
HhDdHyhy × hhddhyhy	O	**HDHy** 81	**HDhy** 85	**HdHy** 70	**Hdhy** 65	**hDHy** 61	**hDhy** 63	**hdHy** 58	**hdhy** 63	9·810	0·201
	E	68·25									
RrDdHyhy × rrddhyhy	O	**RDHy** 67	**RDhy** 78	**RdHy** 69	**Rdhy** 60	**rDHy** 75	**rDhy** 70	**rdHy** 59	**rdhy** 68	4·389	0·733
	E	68·25									
HhRrHyhy × hhrrhyhy	O	**HRHy** 79	**HRhy** 80	**HrHy** 81	**Hrhy** 76	**hRHy** 64	**hRhy** 67	**hrHy** 60	**hrhy** 66	6·671	0·465
	E	71·625									
AaHhHyhy × aahhhyhy	O	**AHHy** 88	**AHhy** 77	**AhHy** 62	**Ahhy** 67	**aHHy** 73	**aHhy** 80	**ahHy** 64	**ahhy** 67	7·742	0·357
	E	72·25									
AaHhRr × aahhrr	O	**AHR** 83	**AHr** 83	**AhR** 67	**Ahr** 63	**aHR** 81	**aHr** 75	**ahR** 66	**ahr** 68	6·519	0·482
	E	73·25									
AaHhDd × aahhdd	O	**AHD** 81	**AHd** 75	**AhD** 65	**Ahd** 58	**aHD** 89	**aHd** 63	**ahD** 62	**ahd** 68	11·307	0·127
	E	70·125									
AaRrHyhy × aarrhyhy	O	**ARHy** 69	**ARhy** 77	**ArHy** 79	**Arhy** 65	**aRHy** 74	**aRhy** 70	**arHy** 62	**arhy** 77	3·684	0·813
	E	71·625									
AaRrDd × aarrdd	O	**ARD** 68	**ARd** 69	**ArD** 76	**Ard** 64	**aRD** 79	**aRd** 62	**arD** 69	**ard** 64	3·613	0·820
	E	68·875									
HhRrDd × hhrrdd	O	**HRD** 88	**HRd** 63	**HrD** 78	**Hrd** 72	**hRD** 57	**hRd** 66	**hrD** 67	**hrd** 55	12·242	0·094
	E	68·25									
Mice											
AaDdBb × aaddbb	O	**ADB** 475	**ADb** 462	**AdB** 460	**Adb** 509	**aDB** 467	**aDb** 494	**adB** 440	**adb** 427	10·626	0·157
	E	466·75									

6. Matings of the type **AaBbCcDd × aabbccdd**. Theoretical ratio of sixteen classes in equal numbers.

TABLE VI (Rats)

AaRrDdHyhy × aarrddhyhy			**HhRrDdHyhy × hhrrddhyhy**			**AaHhDdHyhy × aahhddhyhy**			**AaHhRrHyhy × aahhrrhyhy**			**AaHhRrDd × aahhrrdd**		
Phenotypes	O	E	Phenotypes	O	E	Phenotypes	O	E	Phenotypes	O	E	Phenotypes	O	E
ARDHy	29	34·438	**HRDHy**	42	34·438	**AHDHy**	41	34·438	**AHRHy**	40	36·125	**AHRD**	41	34·750
ARDhy	39		**HRDhy**	47		**AHDhy**	40		**AHRhy**	42		**AHRd**	36	
ARdHy	39		**HRdHy**	34		**AHdHy**	43		**AHrHy**	49		**AHrD**	41	
ArDHy	44		**HrDHy**	40		**AhDHy**	32		**AhRHy**	31		**AhRD**	29	
ARdhy	30		**HRdhy**	31		**AHdhy**	32		**AHrhy**	34		**AHrd**	39	
ArDhy	32		**HrDhy**	39		**AhDhy**	31		**AhRhy**	35		**AhRd**	33	
ArdHy	32		**HrdHy**	37		**AhdHy**	28		**AhrHy**	30		**AhrD**	35	
Ardhy	32		**Hrdhy**	35		**Ahdhy**	30		**Ahrhy**	31		**Ahrd**	25	
aRDHy	40		**hRDHy**	27		**aHDHy**	41		**aHRHy**	41		**aHRD**	50	
aRDhy	39		**hRDhy**	31		**aHDhy**	46		**aHRhy**	39		**aHRd**	29	
aRdHy	31		**hRdHy**	36		**aHdHy**	28		**aHrHy**	32		**aHrD**	38	
arDHy	31		**hrDHy**	35		**ahDHy**	30		**ahRHy**	34		**ahRD**	30	
aRdhy	31		**hRdhy**	30		**aHdhy**	34		**aHrhy**	43		**aHrd**	34	
arDhy	39		**hrDhy**	32		**ahDhy**	32		**ahRhy**	32		**ahRd**	34	
ardHy	27		**hrdHy**	22		**ahdHy**	30		**ahrHy**	30		**ahrD**	32	
ardhy	36		**hrdhy**	33		**ahdhy**	33		**ahrhy**	35		**ahrd**	30	
χ^2	10·626			15·621			14·459			12·948			16·086	
P	0·778			0·408			0·492			0·606			0·377	

7. Matings of the type **AaBbCcDdEe × aabbccddee**. Theoretical ratio of thirty-two classes in equal numbers.

TABLE VII (RATS)

AaHhRrDdHyhy × aahhrrddhyhy		
Phenotypes	O	E
AHRDHy	17	17·219
AHRDhy	23	
AHRhHy	20	
AHrDHy	24	
AhRDHy	12	
AHRdhy	16	
AHrdHy	23	
AHrDhy	17	
AhrDHy	20	
AhRDhy	16	
AhRdHy	19	
AHrdhy	16	
AhrdHy	9	
AhRdhy	14	
AhrDhy	15	
Ahrdhy	16	
aHRDHy	25	
aHRDhy	24	
aHRdHy	14	
aHrDHy	16	
ahRDHy	15	
aHRdhy	15	
aHrdHy	14	
aHrDhy	22	
ahrDHy	15	
ahRDhy	15	
ahRdHy	17	
aHrdhy	19	
ahrdHy	13	
ahrDhy	17	
ahRdhy	16	
ahrdhy	17	
χ^2	25·64	
P	0·728	

8. When albinism is involved, as in a mating of heterozygous coloured self (**CcHh** × **CcHh**), any animal which is albino (**cc**) will not show self or hooded though genetically present. The same is true for agouti and non-agouti. The theoretical ratio is 9 coloured self : 3 coloured hooded : 4 albino.

TABLE VIII (RATS)

Mating		Phenotypes			χ^2	P
		CH	**Ch**	**c**		
CcHh × **CcHh**	O	358	127	162	0·364	0·835
	E	363·94	121·31	161·75		
		CA	**Ca**	**c**		
CcAa × **CcAa**	O	362	123	162	0·034	0·983
	E	363·94	121·31	161·75		

9. Matings of rats heterozygous for colour, agouti, and self (**CcAaHh** × **CcAaHh**) will give a theoretical ratio of 27 coloured, agouti, self : 9 coloured, agouti, hooded : 9 coloured, non-agouti, self : 3 coloured, non-agouti, hooded : 16 albino.

TABLE IX

Mating		Phenotypes						χ^2	P
		CAH	**CAh**	**CaH**	**Cah**	**c**			
CcAaHh × **CcAaHh**	O	278	95	79	39	161		4·263	0·375
	E	275·06	91·69	91·69	30·56	163			

10. The theoretical ratio among progeny from matings of rats heterozygous for colour, agouti, and hair (**CcAaH$_y$h$_y$** × **CcAaH$_y$h$_y$**) or heterozygous for colour, self and hair (**CcHhH$_y$h$_y$** × **CcHhH$_y$h$_y$**) is 27 : 9 : 9 : 3 : 12 : 4.

TABLE X

Mating		Phenotypes						χ^2	P
		CAH$_y$	**CAh$_y$**	**CaH$_y$**	**Cah$_y$**	**cH$_y$**	**ch$_y$**		
CcAaH$_y$h$_y$ × **CcAaH$_y$h$_y$**	O	252	78	79	29	104	38	0·867	0·971
	E	244·69	81·56	81·56	27·19	108·75	36·25		
		CHH$_y$	**CHh$_y$**	**ChH$_y$**	**Chh$_y$**	**cH$_y$**	**ch$_y$**		
CcHhH$_y$h$_y$ × **CcHhH$_y$h$_y$**	O	263	93	102	26	114	43	2·935	0·710
	E	270·42	90·14	90·14	30·05	120·19	40·06		

11. A mating of heterozygous coloured, agouti, self, haired rats (**CcAaHhH$_y$h$_y$ × CcAaHhH$_y$h$_y$**) would give a theoretical ratio of 81 coloured, agouti, self, haired : 27 coloured, agouti, self, hairless : 27 coloured, non-agouti, self, haired : 27 coloured, agouti, hooded, haired : 9 coloured, agouti, hooded, hairless : 9 coloured, non-agouti, self, hairless : 9 coloured, non-agouti, hooded, haired : 3 coloured, non-agouti, hooded, hairless : 48 albino, haired : 16 albino, hairless.

TABLE XI

CcAaHhH$_y$h$_y$ × CcAaHhH$_y$h$_y$		
Phenotypes	O	E
CAHH$_y$	189	183·5
CAHh$_y$	60	61·2
CAhH$_y$	63	61·2
CaHH$_y$	50	61·2
CAhh$_y$	18	20·4
CaHh$_y$	24	20·4
CahH$_y$	29	20·4
Cahh$_y$	5	6·8
cH$_y$	104	108·8
ch$_y$	38	36·2
χ^2	7·612	
P	0·574	

Among the sixty-five ratios given in Tables I–XI, five depart significantly from the theoretical expectation. D/E for the ratio obtained from **Hh × hh** is 4·90, for **Dd × dd**, 3·92 and for **P$_2$p$_2$ × p$_2$p$_2$**, 3·28 (Table I). For **Hh H$_y$h$_y$ × hhh$_y$h$_y$**, P is 0·027, and for **Hh Dd × hhdd**, P is 0·005 (Table III). Four of these five ratios involve the genes for hooded and dilution. Wherever these genes appear in backcrosses, a deficiency in the phenotypes showing these genes always occurs.

Among thirteen crosses giving a theoretical ratio of 1 : 1, ten have the recessive class smaller than the expected, but only three significantly so. The sum of all these monohybrid backcrosses is 9588 dominants : 9153 recessives D/E = 4·71. For the crosses of the type **Aa × Aa** and which give a theoretical ratio of 3 : 1, a total of 5070 dominants : 1677 recessives was obtained. In this case D/E = 0·407, which is a very close fit.

For the ratios with four classes expected in equal number, P = 0·0076, and for the crosses giving a theoretical ratio of 9 : 3 : 3 : 1, P = 0·804. For ratios with eight equal classes, P = 0·0014, and for sixteen classes, P = 0·0278.

When the differences between the observed and theoretical classes are all (or nearly all) in the same direction, the larger the number of such divergent cases which are added together the greater will be the significance of the departure from the theoretical expectation.

NOTE ON THE PRECEDING ANALYSIS OF MENDELIAN SEGREGATIONS

By J. B. S. HALDANE, F.R.S.

The data summarized in Table VII are, I think, unique. However the authors' analysis of them can, I believe, be slightly improved. There are thirty-two classes whose expectation is equal, giving thirty-one degrees of freedom. Now each of these can be specified, and the appropriate value of χ^2 calculated. This is done in my Table I. The first five degrees of freedom are the segregations for single pairs of genes. Thus there were 277 **A** and 274 **a** rats. Hence

$$\chi^2 = \frac{(277-274)^2}{551} = 0{\cdot}0163.$$

The next ten correspond to potential linkages. Thus the degree denoted as **AH** is due to the dichotomy of the total into 281 **AH** and **ah** rats, and 270 **Ah** and **aH**, giving $\chi^2 = 0{\cdot}2196$. Ten more degrees are obtained by considering the associations of gene pairs three at a time. Thus there were 270 (**AHR** + **Ahr** + **aHr** + **ahR**) rats and 281 (**aHR** + **AhR** + **AHr** + **ahr**), giving $\chi^2 = 0{\cdot}2196$. This degree is denoted by **AHR**. Five degrees are obtained by considering four pairs at a time. Thus the degree **AHRD** is given by 274 (**AHRD** + **AHrd** + **AhRd** + **AhrD** + **aHRd** + **aHrD** + **ahRD** + **ahrd**) and 277 (**aHRD** + **AhRD** + **AHrD** + **AHRd** + **Ahrd** + **aHrd** + **ahRd** + **ahrD**) rats, giving $\chi^2 = 0{\cdot}0163$. Finally the degree **AHRDHy** is given by

278 (**AHRDHy** + **AHRdhy** + **AHrDhy** + ... + **Ahrdhy** + ...)

and 273 (**aHRDHy** + ... + **ahrDHy** + ... + **ahrdhy**), giving $\chi^2 = 0{\cdot}0454$.

In each case the principle is the same. We consider the genes 2, 3, 4 or 5 at a time, and divide the rats into two groups, one containing an even number of dominant genes, the other an odd.

Biologically the first five degrees of freedom represent differences of viability between dominants and recessives.

The next ten could represent linkages. However, as there is good reason to think that the five genes concerned are in different chromosomes, deviations greater than could be accounted for by sampling would probably be due to the fact that the effects of the different genes on viability were not additive. For example, Roberts, Dawson and Madden's Table I leaves little doubt that **h** and **d** have an adverse effect on viability, presumably before birth. We further notice that the degree of freedom **HD** has a large, though not significant, χ^2, due to an excess of **HD** and **hd** rats. The actual numbers are 168 **HD**, 137 **Hd**, 125 **hD**, 121 **hd**. If the viabilities were as $1 : 1-\alpha : 1-\beta : 1-\alpha-\beta$ we should expect, on the basis of the single-factor segregations, to find 161·25 **HD**, 143·75 **Hd**,

TABLE I

Analysis of χ^2 in Roberts et al., Table VII

Degree of freedom	Difference	χ	χ^2	P
A	+ 3	+0·1278	0·0163	0·90
H	+59	+2·5135	6·3172	0·012
R	+ 5	+0·2130	0·0454	0·83
D	+35	+1·4911	2·2232	0·14
Hy	− 5	−0·2130	0·0454	0·83
AH	+11	+0·4686	0·2196	0·64
AR	−11	−0·4686	0·2196	0·64
AD	−13	−0·5538	0·3067	0·58
AHy	+27	+1·1502	1·3158	0·25
HR	+ 1	+0·0426	0·0018	0·97
HD	+27	+1·1502	1·3158	0·25
HHy	+ 7	+0·2982	0·0889	0·77
RD	− 3	−0·1278	0·0163	0·90
RHy	+ 5	+0·2130	0·0454	0·83
DHy	− 5	−0·2130	0·0454	0·83
AHR	−11	−0·4686	0·2196	0·64
AHD	−25	−1·0650	1·1343	0·29
AHHy	+19	+0·8094	0·6552	0·42
ARD	−23	−0·9798	0·9419	0·33
ARHy	−31	−1·3206	1·7459	0·19
ADHy	− 9	−0·3834	0·1470	0·70
HRD	37	+1·5762	2·4664	0·12
HRHy	−15	+0·6290	0·4083	0·53
HDHy	−13	−0·5538	0·3067	0·58
RDHy	−31	−1·3206	1·7459	0·19
AHRD	− 3	−0·1278	0·0163	0·90
ARDHy	−31	−1·3206	1·7459	0·19
AHDHy	−13	−0·5538	0·3067	0·58
AHRHy	−23	−0·9798	0·9419	0·33
HRDHy	+17	+0·7242	0·5245	0·47
AHRDHy	+ 5	+0·2130	0·0454	0·83
Total 31	− 7	−0·2130	25·639	0·76

131·75 **hD**, 114·25 **hd**, which would give $\chi^2 = 0$ for the degree of freedom **HD**. If the viabilities were as $1 : 1-\alpha : 1-\beta : (1-\alpha)(1-\beta)$, which is perhaps a more plausible hypothesis, we should expect 162·19 **HD**, 143·81 **Hd**, 131·81 **hD**, 114·19 **hd**, giving $\chi^2 = 0{\cdot}001$. So the two hypotheses are indistinguishable except in enormous samples.

The remaining degrees of freedom represent similar biological possibilities as to the non-additive character of the effects of various genes on differential viability. Fisher (1925), who was the first to point out the method here employed for the analysis of χ^2, states that these degrees have "no simple biological meaning". However, the positive value for the degree **HRD** could be mainly due, for example, to the fact that **hRD** rats, of which there were only 58 as against an expectation of 68·875, are more inviable than was to be expected from the effects

of the genes one at a time, which would give an expectation of 66. Such interactions have, of course, been observed by Gonsalez (1923), Timofeeff-Ressovsky (1934) and others.

In my Table I the values of P in the fifth column are read off from those of χ in the third by means of a table of the probability integral, except the last entry, which is calculated by Wilson and Hilferty's (1931) theorem. There are no suspiciously high values of P, and only one (for the degree **H**) which is significantly low. Actually a glance at Roberts, Dawson and Madden's Table I shows that had their Table VII been based on a larger sample the values for **H** and **D** would almost certainly have been significantly low.

In the earlier tables some rats are included which do not figure in Tables VII and XI. Hence for a full discussion of the data each table would have to be analysed separately.

We can now criticize Roberts, Dawson and Madden's final analysis of the data. They point out that the total χ^2 for segregations involving eight equal classes, i.e. three genes at a time, is unduly high. We can see why this is so. Supposing we extracted from Table VII the data on segregations of three genes at a time we should obtain ten tables with seventy degrees of freedom. If these were analysed into their components we should find that we had counted the ten degrees of which **AHR** is typical once each, those of which **AH** is typical three times each, and those of which **A** is typical (i.e. single gene-pair segregations) six times each. We have thus given a quite undue weight to those degrees of freedom which actually show the greatest deviations. Actually each determination of χ^2 by the authors involves one degree of freedom not dealt with in the earlier tables, while the remainder refer to segregations already considered, and often on larger samples, in their earlier tables.

Thus, to take an example, the rat mating **AaHhDd** × **aahhdd** of Table V has a χ^2 of 11·31 for seven degrees of freedom. But the only new information in Table V relates to the difference between **AHD** + **Ahd** + **aHd** + **ahD** and **aHD** + **AhD** + **AHd** + **ahd**, giving $\chi^2 = 1{\cdot}941$ for one degree of freedom. All the other information is already given in Table III, along with some more, for example, concerning rats segregating for **A** and **D**, but not for **H**. The correct method of collating the data is therefore to add up the values of χ^2 for the last degree of freedom in each case.

The expression for the last degree of freedom in a F_2 mating such as **AaDd** × **AaDd** can easily be shown to be $\frac{(\mathbf{AD}-3\mathbf{Ad}-3\mathbf{aD}+9\mathbf{ad})^2}{9(\mathbf{AD}+\mathbf{Ad}+\mathbf{aD}+\mathbf{ad})}$. This expression has $\beta_1 = \frac{16}{9n}$, and a nearly normal distribution. Similar expressions can be written down for larger F_2's. Where the mating involves epistacy the formulae for the last degree of freedom are similar. For example, the first mating of Table VIII involves two degrees of freedom, the degree **C** for the segregation of **C** and **c** rats and the degree **H**(**C**) for the segregation of **H** and **h** among **C** rats.

TABLE II

Values of χ^2 for single degrees of freedom, all data

Animal	Degree of freedom	χ^2
Rat	A	2·456
,,	H	10·816
,,	R	0·353
,,	C	0·191
,,	Hy	0·427
,,	D	6·947
Mouse	A	1·630
,,	P_1	1·024
,,	B	0·670
,,	P_2	0·901
,,	Y	0·032
Rabbit	E	0·177
,,	A	0·889
Rat	AA	1·138
,,	HH	1·367
,,	RR	0·016
,,	CC	0·177
,,	HyHy	0·000
,,	DD	0·020
Mouse	P_2P_2	1·255
Rabbit	EE	1·426
1 factor	21	31·912
Rat	AHy	0·629
,,	HHy	0·527
,,	RHy	0·061
,,	DHy	0·323
,,	CHy	0·296
,,	AD	0·175
,,	HD	2·532
,,	RD	1·471
,,	AR	0·001
,,	HR	0·387
,,	AH	0·778
Mouse	AP_1	4·252
,,	AB	0·130
,,	BP_1	0·130
Rat	A_2Hy_2	1·368
,,	H_2Hy_2	1·852
,,	R_2Hy_2	0·081
,,	D_2Hy_2	0·885
,,	C_2Hy_2	0·025
,,	A_2D_2	0·657
,,	R_2D_2	0·012
2 factors	21	16·572
Rat	ADHy	0·118
,,	HDHy	0·264
,,	RDHy	2·117
,,	HRHy	0·141
,,	AHHy	0·692
,,	AHR	0·246
,,	AHD	1·941
,,	ARHy	2·934
,,	ARD	1·134
,,	HRD	2·930
Mouse	ADB	2·786
3 factors	11	15·303
Rat	ARDHy	1·746
,,	HRDHy	0·525
,,	AHDHy	0·307
,,	AHRHy	0·692
,,	AHRD	0·065
4 factors	5	3·335
Rat	AHRDHy	0·045
Rat	$H_2(C)$	0·362
,,	$A_2(C)$	0·008
,,	$A_2H_2(C)$	2·536
,,	$A_2Hy_2(C)$	0·906
,,	$H_2Hy_2(C)$	1·790
,,	$A_2H_2Hy_2(C)$	2·578
Epistacy	6	8·180
Grand total	65	75·35

This would be affected by linkage. Its $\chi^2 = \frac{4(CH-3Ch)^2}{9(CH+Ch+c)}$. Similarly, for Table IX the last degree of freedom involving the association of A and H among C animals is $\chi^2 = \frac{4(CAH-3CAH-3CaH+9Cah)^2}{27(CAH+CAh+CaH+Cah+c)}$.

The last component degrees of the sixty-five segregations of Tables I–XI are given in my Table II. Whereas Roberts, Dawson and Madden, for the reason given above, find a significantly large total χ^2, I do not. The probability of $\chi^2 = 75{\cdot}44$ for sixty-five degrees of freedom is $P = 0{\cdot}18$. The whole excess is due to the single-factor segregations, and of these, one gives a significantly large χ^2, while another probably does so. Of the forty-four degrees of freedom involving more than one factor the largest is 4·252, and one value as large as this is to be expected. The order of the degrees is taken from Roberts, Dawson and Madden. If we include the eight cases involving epistacy with the others involving the same number of factors (gene pairs) we find the results summarized in my Table III.

TABLE III

Summary of χ^2 values

Number of factors segregating	Number of degrees of freedom	Total χ^2
1	21	31·912
2	23	16·942
3	14	20·535
4	6	5·913
5	1	0·045
Total	65	75·35

It will be seen that, except for the single-factor ratios, there is no evidence of deviation from Mendelian expectations, and on the whole the fit is very much better than would appear from the analysis of Roberts, Dawson and Madden.

REFERENCES

FISHER, R. A. (1925). *Statistical Methods for Research Workers.* Edinburgh.

GONSALEZ, B. (1923). "Experimental studies on the duration of life. VIII. The influence upon duration of life of certain mutant genes of *Drosophila melanogaster*." *Amer. Nat.* **57**, 289.

TIMOFEEFF-RESSOVSKY, N. W. (1934). "Über die Vitalität Genmutationen und ihrer Kombinationen bei *Drosophila funebris* und ihre Abhängigkeit vom 'genotypischen' und vom äusseren Milieu." *Z. indukt. Abstamm.- u. VererbLehre*, **66**.

WILSON, E. B. & HILFERTY, M. M. (1931). "The distribution of Chi-square." *Proc. Nat. Acad. Sci., Wash.*, **17**, 684.

THE USE OF STATISTICAL METHODS IN THE INVESTIGATION OF PROBLEMS OF CLASSIFICATION IN ANTHROPOLOGY

PART I. THE GENERAL NATURE OF THE MATERIAL AND THE FORM OF INTRARACIAL DISTRIBUTIONS OF METRICAL CHARACTERS

By G. M. MORANT

Every naturalist who has had the misfortune to undertake the description of a group of highly varying organisms, has encountered cases (I speak after experience) precisely like that of man; and if of a cautious disposition, he will end by uniting all the forms which graduate into each other, under a single species; for he will say to himself that he has no right to give names to objects which he cannot define.

CHARLES DARWIN, *The Descent of Man*, 1871.

1. INTRODUCTION

THE main aim of physical anthropology is to unravel the course of human evolution, and it may be taken for granted to-day that the proper study of the natural history of man is concerned essentially with the mode and the path of his descent. The general method employed is to compare the physical characters of suitably chosen groups of individuals, and to discover the interrelationships of these groups from an interpretation of the differences found between them. Difficulties are encountered at the outset owing to the nature of the continuum of which the component parts have to be compared. The groups which are most suitable for the purpose in view can easily be recognized in a general way, but it is difficult to define them with precision. There is a lamentable lack of agreement among anthropologists to-day regarding the way in which populations suitable for the purpose of investigating mass descent can best be discriminated. Darwin stressed the difficulty of the problem, and it would be idle to hope that any simple solution of it has been overlooked.

This paper provides a discussion of the statistical approach to anthropological taxonomy. The general thesis maintained in it is that the comparatively new method can be used in a systematic way to discriminate suitable groups, to reveal their interrelationships, and hence to disclose the course of racial history in so far as adequate evidence is available.

It is now more than forty years since Karl Pearson first applied to racial

material the statistical methods which he established and greatly extended.* Recognition of the value of his procedure was slowly won, but the methods are almost universally accepted to-day. It may even be said now that the advantages of their use in physical anthropology are generally taken for granted. There is no longer a need to emphasize the value of measurements, or to point out repeatedly the futility of attempting to derive any useful conclusions from evidence as scanty as that on which many earlier theories were based.

Karl Pearson's teaching in this field has been most widely accepted in so far as methods of reducing group data are concerned, but it is one thing to describe and another to interpret in anthropological terms the situation observed. With regard to such interpretation, he indicated the lines along which he expected that progress would be most profitable, but he did not codify a system or lay down any rigid rules for the guidance of those who wished to follow him. His ideas with regard to this matter could only be grasped by observation of the ways in which he treated particular sets of material. Meanwhile, different anthropologists were drawing deductions from statistical evidence in a variety of ways, some of which are entirely at variance with what came to be known as "biometric" practice, and many of which are irreconcilable *inter se*. The position became chaotic, and it is still in this state.

The view advocated in this paper is that the ways in which statistical methods may be used to supply valid and useful anthropological conclusions can only be determined from wide application of these methods to suitable material. No *a priori* considerations are likely to be of much help here: the nature of the situation has to be examined thoroughly before it is possible to decide on the best ways of treating it to give results of the kind required. The short history of the subject bears clear witness to the fact that the empirical test is always the crucial one. The nature of anthropological groups has now been sufficiently explored, and certain methods of interpretation have now been sufficiently applied, to make possible a just assessment of the value of a particular procedure due primarily to Karl Pearson. He repeatedly stressed the need for adapting statistical theory to practice, and we may follow him in deciding that the value of any new descriptive methods, or modes of interpretation, suggested must be judged from a sufficiently wide application of them.

The statistical nature of anthropological groups will be discussed first, and certain simple but important generalizations which are often ignored by

* The first general treatment of the topic was given in a paper by Cicely D. Fawcett and others (1902) which Karl Pearson edited and arranged. He had previously applied statistical methods to anthropological material in *The Chances of Death and other Studies in Evolution* (1897) and in a series of papers in the *Proceedings* and *Transactions* of the Royal Society of London. Quetelet (chiefly in 1835 and 1871), Galton (in various publications), Stieda (1883) and Witt (1879) had previously used statistical methods in discussing anthropological problems, but they were either not concerned with problems of racial differentiation or else they considered such problems only in a cursory way.

anthropologists can be formulated at this stage. In the second place, different methods of reduction will be considered, and, lastly, the anthropological conclusions which may legitimately be derived from the data reduced in that way will be discussed. In this Part I of the whole paper topics discussed concern the selection of anthropological samples (§ 2), general considerations regarding the treatment of the samples chosen (§ 3), and the forms of intraracial distributions (§ 4).

2. The selection of anthropological samples

Anthropology has been defined as the study of groups, and this definition is appreciated at once by the statistician, since his methods are essentially designed for the treatment of group data. In a consideration of anthropological material the nomenclature of the statistician may be used with advantage; the nature of the processes of description and analysis is thereby made clearer and certain ambiguous biological terms are avoided.

A *population* is defined to be any assemblage of individuals considered and treated as a single group. It may be large or small, and there may be little or much justification for treating it as a single group. The term is a general, but unambiguous, one which can be conveniently used in practice. A *sample* is made up by a number of individuals selected from a population, and the selection is said to be *random* when it is believed that the population as a whole is fairly represented, so that there is no bias favouring any special section of it. A sample of individuals may be said to form a *series*, and these two terms can often be interchanged without loss of perspicuity.

The general method of the physical anthropologist in dealing with new material is: (*a*) to select a sample at random from a particular population of a suitable kind, (*b*) to describe the characters of the individuals comprising the sample, (*c*) to infer from these observations, with greater or less accuracy according to the size of the sample, certain characteristics of the population sampled, (*d*) to make comparisons between the evidence so obtained and that available for other populations described in the same way, and (*e*) to deduce from these comparisons the biological relationships of the new population. It is necessary to make clear distinctions between these successive processes, and failure to do so has sometimes led to confusion.

The description in statistical terms of the first process, i.e. that of sampling, is perfectly precise providing that there is some means of ensuring that the samples are taken at random, but in applying the process to his material the anthropologist is faced with another difficulty. How is he to distinguish the populations from which suitable samples may be taken? In general very little consideration has been given to this question. In comparisons made between samples selected in different ways and representing different kinds of populations, differences between the kinds of sources from which they were derived have

generally been ignored. But in fact such differences are of vital importance, and half the difficulty of the whole process of the statistical analysis of anthropological material is overcome if an effective way of dealing with the problem of selecting suitable samples can be devised. The contention made here is that owing to diversity in the modes of selection of the samples commonly used by anthropologists, there is often little justification for applying statistical methods to them in a rigid way.

For the anthropologist the ideal population would be one made up by a number of individuals having a common descent. In the most favourable circumstances such communities cannot be distinguished at all exactly, however, and in the initial stage of his enquiry the anthropologist is generally quite unable to delimit with any approach to precision groups defined in such an abstract way. In practice far less stringent conditions have to be accepted. It may be said that a community made up by a number of individuals whose ancestors—or the majority of them, at least—are believed to have intermarried for a considerable number of generations will be a suitable one to accept as a unit group from which a sample may be taken. Such groups have to be chosen as carefully as possible, having regard to any relevant evidence available. But populations of very different sizes will satisfy the condition stated. The total population of a province and also a small parochial community forming a special part of this total might be considered, and it is possible that these are different in nature and that different conclusions would be derived from them. A further condition, then, which should be satisfied wherever possible, is that the population considered is a large one of the regional rather than of the parochial kind. Experience, discussed below, has shown that large communities are always more suitable for consideration than small ones when the purpose in view is the analysis of more remote origins. But if the only sample which can be obtained does represent a population of the parochial kind—as may be the case if a series of skeletons from a single cemetery is the only material available—then it should be fully recognized that this may have been a special part of a larger population which might more profitably be considered as a unit group, and allowance should be made for the peculiarity of the source.

Having selected a number of samples from what appear to be suitable populations, the anthropologist may compare their statistical features, such as the forms of distributions, measures of variation and correlations they supply. If the majority of the samples are found to possess certain particular characteristics, while departures from the rule are occasionally met, then it may be legitimate to conclude that the peculiarity of the exceptional cases is due to the fact that the populations they represent are unsuitable for the purpose in view. The empirical investigation of samples known to represent unsuitable populations can aid examination of the matter. The practical procedure which appears to be most profitable is this: samples believed to be of the right kind are selected

and then certain tests, derived from experience of the statistical characteristics customarily exhibited by such samples, are applied in order that abnormal ones may be revealed and rejected as unsuitable for further use.

The initial process of selecting samples must be considered rather more fully. In practice it may be advisable to take into account information of several kinds. The most important factor is usually geographical position, but the separation of series representing different social classes may be desirable, and archaeological, historical or linguistic evidence may suggest that the partitioning should be carried out in a particular way. In dealing with skeletal material the time factor may be an important one to consider; it may be well to treat a pagan and a Christian Anglo-Saxon series separately, for example, and not to assume that the two represent indistinguishable populations. The general rule is that the total series for which data are available should be split up in a natural way into as many subseries representing distinct populations of a considerable size as possible, providing that each subseries selected is large enough to give comparisons of value. The way in which this can best be done depends on circumstances which are peculiar to each set of material. The anthropologist is thus obliged to plan his survey of a particular region with evidence of several different kinds in view. The majority of these do not relate to physical characters, but he has a perfect right to consider, as relevant to his biological enquiry, any evidence which gives some indication of the subgroups of the total population which are such that intermarriage normally takes place, or took place, within rather than between them.

The procedure described is necessarily rather vague and of an arbitrary nature: any subdivision adopted initially is experimental. In practice choice is largely controlled by the amount of information available. Suppose, for example, that the total sample for which measurements are available represents people coming from all parts of a particular country. If there are enough individuals they may be divided into subsamples on a regional basis, or the population of each region may be thought of as subdivided into county groups, say, and each of these may be further subdivided into parochial communities, which may be split up into small groups of closely interrelated people. A hierarchy of groups within groups can thus be imagined, but in practice it will not be possible to carry the process of subdivision beyond a certain stage, as further dissection would lead to subsamples too small to yield comparisons of value. There can be no assurance that the divisions based on geographical or other considerations will be the most effective for the purpose in view, but some such divisions have to be adopted in order to disclose the nature of the total population.

At the outset of his survey of new material relating to the population of a particular region, the anthropologist thus requires a knowledge of the size of the smallest series which can be used profitably for the purpose of the classification

in view. This minimum requirement cannot be determined from any *a priori* considerations: the point has to be settled after an experience has been gained from the comparison of series of different sizes. The conclusions reached in this way are discussed below.

However ample his material is, the anthropologist engaged in investigating remoter origins and relationships should choose his unit groups, *whenever possible*, in such a way that they relate to populations of a considerable size. Family and parochial groups should certainly be considered too small. The need for restricting comparisons to larger groups than these if possible is evident on account of both *a priori* and *a posteriori* considerations, but it is frequently overlooked.* It should be anticipated that series representing small communities are likely to be biased, not random, samples of regional populations. The existence of local variants of a widespread population derived from a single source has to be recognized, and hence special precautions have to be taken in drawing deductions from samples representing local groups. In conformity with these expectations, it is commonly found in treating a total group of the national kind that samples representing small subgroups of it tend to differ more than samples representing large subgroups. It is customary to find in such a case that up to a certain point there is greater uniformity in the statistical attributes of the samples—quite apart from fluctuations due to random sampling—according as the groups represented by the samples become larger. Greater diversity has to be expected when smaller subgroups are compared. If the anthropologist concerned with the broader taxonomic problems of his subject treats groups of the family or parochial kind, he will thus be in danger of losing his way among the trees when he should be taking a bird's-eye view of the wood as a whole.

It is unlikely that an example of the statistical conception of a perfectly homogeneous population is ever encountered in dealing with anthropological material. The kind of continuum which has to be analysed by making comparisons between its component parts is of a peculiar nature. It would be an advantage if the subsections of it considered for the purpose of racial classification were always populations of a considerable size, and for many sets of anthropometric

* The writer had overlooked it at one time, and he was accordingly admonished in a letter from Karl Pearson dated 17 July 1924, in which the following passage occurs:

"May I give you an analogy? In a little Yorkshire village there are two manors—two small squires, little better than yeomen—and they kept on intermarrying their family members. About one-third of the churchyard contains, for at least two centuries, the graves of these folk. You could get at least 30, and possibly 100, crania from that third of the churchyard which would differ very sensibly from the skulls in the remainder of the yard. It would arise solely from the fact that we are dealing with an inbred population, which possessed certain characters which raised it above, or at least differentiated it from, the remaining population. I don't suppose there was any racial difference between these little squires and their neighbours, they were all ultimately of Danish descent. But simply one or two interbreeding families were buried in one earth. Now my analogy has for its bearing the danger of picking out 20 crania and saying these differ from the rest."

data this condition is satisfied. Not infrequently, however, and particularly in the case of excavated skeletal material, the only samples available for a particular regional group are of the parochial kind.

The Anglo-Saxon skeletons preserved in museums actually come from a number of scattered cemeteries, and there is no long series from a single cemetery. The fact that they were dispersed should be considered a real advantage when the object in view is to determine the characters of Anglo-Saxons in general, or of the larger subsections of this total population. If all the specimens had come from a single cemetery it would have been difficult not to assume that the series, particularly if it were a large one, could be accepted—*faute de mieux*—as representing the total Anglo-Saxon population, and to draw conclusions from a comparative study on this assumption. In fact there is good reason to suspect that the physical characters of the people forming any small Anglo-Saxon community differed appreciably in their averages from those of any large section of the total population.

In practice it is seldom possible to exercise much choice in selecting samples. In many instances the only material available representing a past population of the national kind consists of one or more series of skeletons from one or two cemeteries. The anthropologist cannot be expected to neglect all series which are not ideally suitable, but he should remember that several he is obliged to use are not ideal for the purpose in view. Allowance has to be made for known differences between the samples used dependent on the diverse ways in which they were selected.

3. The statistical treatment of anthropological samples

In treating his problems of classification the anthropologist starts by selecting series of individuals representing different populations which appear to be more or less suitable for the purpose. Data have been recorded for large numbers of such samples relating to living peoples in different parts of the world and to extinct peoples represented by series of skeletons. It is recognized that these samples are taken from groups of different kinds, and that allowance will have to be made for this fact in comparing them. Apart from this diversity which is appreciated, there is no guarantee that all the samples are suitable for the purpose of examining group relationships. Some may be entirely unsuitable because they represent populations of exceptional kinds, and their peculiarity in origin has to be detected by examination of the samples themselves. For example, a certain number may be found to have peculiar characteristics which suggest that they should not be used, while the majority conform to a particular type. In practice it is necessary to appeal to experience derived from the treatment of a large number of series in order to justify the use of certain tests employed to discriminate between suitable and unsuitable material.

A question which might be asked is: What is the nature of anthropological

samples selected in the way described?—what features are common to all of them and in what ways do they differ? The statistician—who is accustomed to thinking of problems in terms of populations and samples—points out at once that, though this question may be of the right kind, yet it is not put in the right way. He reminds the anthropologist that his ultimate concern is with populations, and that the samples are really of interest only in so far as they supply information regarding populations. A moment's consideration shows that this is, or should be, the view of the anthropologist. If he collects information relating to 100 Greenland Eskimos, say, it is only in order to arrive at generalizations regarding either all Greenland Eskimos, or else some section of them which is still a population many times larger than the sample observed. Hence the question should be: What generalizations regarding populations can be deduced from samples selected in the way described?—what features are common to all the populations and in what ways do they differ?

The fact that his ultimate concern is with populations and not with samples of them is not likely to be entirely overlooked by the anthropologist, but there is a real danger that he may assume in particular instances that the characteristics of samples are precisely the same as those of the populations they represent. Statistical treatment has a great advantage in this connexion, since it keeps the distinction referred to continually in view and provides a systematic and—as far as circumstances permit—precise method of reaching the generalizations required.

In this paper the only kind of information regarding samples which will be considered is that provided by measurements, whether of series of living people or skeletons. The general problem can be viewed in the same way if non-metrical characters are dealt with, but a different statistical treatment is required for them. All the individuals referred to will be supposed adult, and for such the measurements can be supposed, in general, to be unaffected by the age of the individual. The data for males and females have to be considered separately, but the generalizations reached are the same for the two sexes. Data relating to numerous series are available for a particular set of characters commonly recorded by anthropologists: nearly all of these measurements concern either absolute size (chords and arcs) or shape (indices and angles). In the case of the skull, for example—the data for it being more abundant than those for all other bones of the skeleton put together—the measurements are designed to give a description of the size and shape of the skeleton of the head considered as a whole and of all its principal parts. The majority of the anthropometric data for living people commonly recorded provide indirect measures of the size and proportions of different parts of the skeleton.

With the object of obtaining information regarding the distribution of a particular character in the population represented, the following features of the distribution provided by a sample will be considered:

(i) its form,

(ii) its scatter, that is to say the variation exhibited by the readings for the individuals composing the sample,

(iii) its central tendency, that is to say the value of the character (or *variate*) about which the readings can most conveniently be considered to be scattered.

These three features of the distributions are first considered in the case of characters dealt with singly. It is also necessary to gain information regarding the ways in which different characters are associated in individuals, and hence it is necessary to examine:

(iv) the correlations of different pairs or groups of characters in individuals.

A sample of the kind so far considered is made up by a number of individuals believed to belong to a population such that its members have been chiefly intermarrying with one another for a number of generations. This is called an *intraracial* sample—a rather unsatisfactory term to use at this stage since the concept of race has yet to be defined. An examination of the forms of the distributions and of their variabilities in the case of metrical characters considered singly, and for a considerable number of intraracial samples, shows that the only comparisons of them likely to give results of any anthropological interest must be of a certain kind defined in later sections of this paper. Statistical treatments of other kinds are seen to be unprofitable. The same evidence makes it clear that the most appropriate measure of the central tendency of any one of the distributions to use is the arithmetic mean, or average.

Attention is thus focused on mean measurements, and different ways of treating them have to be considered. The conception is introduced of samples for which the units are not actual individuals—as for intraracial samples—but abstract beings such that each has metrical characters equal to the averages for the particular group which it represents. This idea of *l'homme moyen* is not new in anthropology: it was clearly enunciated more than 100 years ago by Quetelet (1835). An assemblage made up by a number of *hommes moyens* representing different populations is called an *interracial* sample, and in the case of a particular character the mean measurements for a number of samples representing different populations will provide an interracial distribution.

In order to gain a more complete knowledge of the statistical nature of anthropological samples, required for the purpose of determining the ways in which they can best be treated in practice, it is necessary to take cognizance of other features of them in addition to those listed above. Interracial considerations are involved in examining:

(v) the ways in which the averages for different characters distinguish certain sets of populations,

(vi) the forms and variabilities of interracial distributions, and

(vii) the interracial correlations of characters.

In the course of the investigation of these topics it is possible to obtain estimates of the minimum sizes of samples which are required, under different

conditions, in order to provide intergroup comparisons of value. One of the advantages of the statistical approach to the problem of classification is that it makes it possible to determine with some precision the least amount of evidence which must be available in order to yield useful conclusions. The history of physical anthropology bears clear witness to the fact that some control of this kind is a vital need.

In the following section of this paper the forms that occur in practice of the intraracial distributions of anthropometric characters are discussed. It is hoped that this will be followed in later parts by discussions of the other topics listed above—viz. other intraracial characteristics of distributions and characteristics of interracial distributions—and, finally, by a discussion of methods of comparing samples.

4. THE FORMS OF INTRARACIAL DISTRIBUTIONS

It can be stated categorically that the distributions of measurements for the vast majority of samples that occur in anthropological practice tend to conform closely to the normal curve. Quetelet's suggestion (1871) that this is so has been confirmed by data relating to numerous series of living people and skeletons from all parts of the world. In general, the samples from a particular population tend to give a closer and closer approximation either to the normal or to a very similar form of continuous curve according as their sizes are increased. Hence it is safe to infer that the characteristic type is unimodal and symmetrical. This generalization applies to all absolute measurements and, in spite of a theoretical qualification mentioned below, also to all measurements of shape (indices and angles) for which the matter has been adequately investigated.

There has been considerable confusion regarding interpretation of the forms of frequency distributions in the discussions of this topic provided by some anthropologists to whom statistical conceptions were unfamiliar. In the early days of biometry the fallacy of drawing conclusions of certain kinds from peculiarities of the curves provided by samples was repeatedly stressed, but this warning is still occasionally ignored. The chief errors made with regard to the matter have been in supposing (*a*) that a small sample—made up by fewer than 100 individuals, say—is capable of giving an adequate estimate of the form of the distribution in the population sampled, and (*b*) that mere inspection of the diagram for a sample is sufficient to reveal the information required. In this connexion numerous examples may be found of the fallacy of supposing that the features of a sample are precisely the same as those of the population from which the sample was taken. The general appreciation by anthropologists of the perfectly clear implications of a simple sampling experiment such as the one to be described would still save much fruitless discussion.

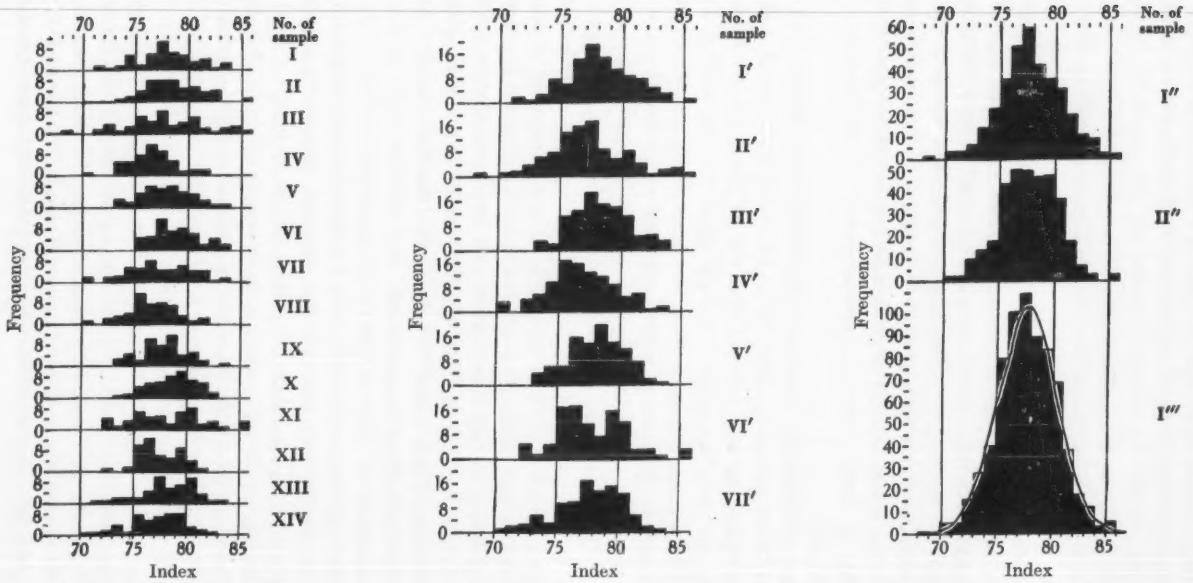
The head lengths and breadths of 1314 soldiers who were natives of Lanarkshire have been given by Tocher (1924). The order in which the first 700 men are arranged in the table appears to have been entirely random as far as the

measurements are concerned.* The cephalic indices were calculated for this sample and Fig. 1 A shows the distributions obtained by taking successive groups of 50 from the first (nos. 1–50) to the fourteenth (nos. 651–700). Each one of these subsamples provides an estimate of the distribution of the cephalic index for Lanarkshire men in general, and any one of these subsamples might have been the only one available relating to the total population. The distributions in Fig. 1 A clearly exhibit a great diversity of forms. They are all alike in showing a majority of individuals with indices between 75 and 80, and in either covering a continuous range or showing a few outlying values detached from the main body. Otherwise, there is little agreement between them. Some show a more or less gradual rise to a maximum frequency and then a gradual fall (as no. X), while others appear to have two (nos. XII and XIII) or more (nos. III and XI) distinct peaks. The fact that inconsistencies are found in these respects makes it obvious that the series are too small to provide any reliable information as to the existence of such features in the distribution for the population they all represent.

Fig. 1 B shows the same 700 measurements treated in successive subsamples of 100, so that the first series (I′) is the sum of the first two (I and II) in Fig. 1 A, and so on. The distributions for these larger series show more uniformity—the maximum frequency being for the range 77–78 in four cases out of the seven, for example—but they still differ very appreciably among themselves in some respects, and one (VI′) appears to be clearly of a bimodal form. It is evident that the measurements of more than 100 individuals are required to give any reliable indication of the details of the distribution of the cephalic index in the parent population.

The top and the middle distributions in Fig. 1 C relate to the samples made up by the first 350 (I″) and the second 350 (II″) Lanarkshire men, respectively. Both distributions appear to be of symmetrical form, but the first shows an outstanding maximum frequency while the second is "flat-topped" (platykurtic). The conclusion must be that it is unsafe to draw deductions regarding some features of the distribution for the parent population by merely inspecting the distributions of samples, even if they are as large as these two. The bottom diagram in Fig. 1 C relates to the total sample of 700 men considered. It appears to be slightly asymmetrical, but comparison with the corresponding areas of the

* After some number a little above 700 the order in which the individuals are arranged is clearly not random. They were evidently taken in successive small groups such that each group had a restricted range of head lengths, and consequently the distributions of the cephalic index for samples of 50 (nos. 701–50, and so on) are very peculiar. Dr Tocher has kindly answered my enquiries with regard to this matter, and he informs me that the departure from randomness was not appreciated when the measurements were taken. It may have been due to the fact that at one depot the officers arranged the privates measured in batches having hats of the same size, in the belief that this would help the anthropologists! Departures from a random selection which may affect the distributions of characters are easily overlooked.



A. Successive samples of 50.

B. Successive samples of 100.

C. Successive samples of 350 (I″ and II″) and the total sample (I‴) fitted with a normal curve.

Fig. 1. Distributions of the cephalic index for subsamples of a total sample of 700 men from Lanarkshire.

normal curve fitted to it shows that the divergences of the histograms from their theoretical values on the hypothesis of normality are all small relative to the sizes of these frequencies.

This example is sufficient to demonstrate that it is entirely unsafe to attach any significance to the peculiarities of the distributions for small samples—any made up by fewer than 200 individuals, say—and that mere inspection of the forms of distributions for samples which would usually be considered adequate for most statistical purposes is liable to mislead. The anthropologist who deduces theories of racial mixture or relationship from the "peaks" of small distributions is deceiving himself by building on a statistical foundation which is unsound owing to the inadequacy of the evidence. He is trying to get more information from his material than it can possibly provide with any assurance of correctness, and forgetting that a sample only provides estimates of the features of the population represented.

It is found in anthropological practice that distributions for small samples frequently appear to be markedly skew, or to be bi- or multimodal, but that those for large samples scarcely ever exhibit any of these peculiarities. The writer is unable to give any examples of a distribution, for the kind of sample now considered, which relates to more than 300 individuals and which fails to show a close approach to the form of a unimodal and symmetrical curve, with the exception of two discussed in the appendix below. The occurrence of more than one mode which appears to be definitely outstanding, or of any appreciable degree of skewness, is apparently never exhibited by the distributions of characters for large samples of the kind encountered in anthropological practice. The conclusion must be that when such peculiarities are found for the distributions of small samples they merely demonstrate that small samples are subject to large "errors" of random sampling. The "peaks" can have no anthropological or genetical significance. Their unstable nature can often be demonstrated by merely grouping the individual measurements in a different way, as this may change the appearance of the distribution to an appreciable extent.

If it can be demonstrated adequately that the distributions which occur in practice almost invariably indicate that the parental distributions must be closely similar in form to the normal curve, then this is obviously a conclusion of great anthropological importance, since it implies that the kind of statistical analysis likely to be at all profitable is severely restricted owing to the nature of the material. Evidence of a more adequate statistical kind regarding the forms of distributions will now be considered.

Values are given below of probabilities (χ^2, P test) obtained by comparing various distributions with the theoretical frequencies obtained by fitting normal curves to them. Those for the cephalic indices of series of Lanarkshire men for samples of 100 or more (see Fig. 1, B and C) are in Table I. The lowest P found is 0·03, indicating that for samples of the size (100) actually drawn at random

from a normally distributed population, 1 in 33 would be expected to give a less good correspondence with the theoretical distribution than that exhibited by the sample in question. All the other P's are much higher, indicating that the estimates provided by the samples bear a much closer resemblance to the normal form in these cases.

TABLE I

Probabilities (χ^2, P test) that the distributions of the cephalic index for different subsamples of 700 Lanarkshire men indicate that the character was normally distributed in the parent population

	Successive samples of 100 individuals*						
	1st	2nd	3rd	4th	5th	6th	7th
P	0·62	0·34	0·80	0·88	0·79	0·03	0·23

	First 350 individuals†	Second 350 individuals†	700 individuals‡
P	0·77	0·64	0·70

* Distribution in 9 groups. † Distribution in 13 groups. ‡ Distribution in 15 groups.

Table II gives the P's for distributions of the cephalic index in the case of 15 series of male skulls of the kinds which normally occur in anthropological practice, each series being made up by more than 100 specimens. These were selected merely because they happened to be the ones of the size required most easily accessible to the writer. The first 11 are believed to be homogeneous in the sense that any subseries of them likely to be chosen for anthropological purposes would not show significant differences in any statistical constants. All but one of the P's for these are high enough to provide excellent support for the hypothesis that the cephalic index was normally distributed in the parent population. The single exception is for an Egyptian series (3) which is known from other evidence to have statistical features which are typical for "homogeneous" samples, so the low P found for the cephalic index may well be attributed to the vagaries of chance selection.

Series 12 and 13 are known to be heterogeneous to some extent, since the means of the cephalic index and some other characters for their subseries show differences which are significant though small. The P's for these two are both high. Series 14 and 15 are known to represent populations which are unsuitable for the purpose of investigating group relationships, since they are exceptionally

variable. This is clearly indicated by the standard deviations for the cephalic index given in the right-hand column of Table II, and also by those for certain

TABLE II

Probabilities (χ^2, P test) that the distributions of cephalic indices provided by various series of male skulls indicate that the character was normally distributed in the parent populations

No. of series	Series*	No. of skulls	Mean	No. of groups	P†	$\sigma \pm$ S.E.
	(*a*) Series believed to be homogeneous					
1	Guanche	245	76·0	11	0·998	2·30 ± 0·10
2	Eskimo (St Lawrence Is.)	156	77·1	10	0·904	2·39 ± 0·14
3	Egyptian (26th–30th dyn.: Gizeh)	866	75·0	16	0·011	2·65 ± 0·06
4	New British	114	72·2	11	0·763	2·74 ± 0·18
5	English (Camb. dissecting rooms)	118	75·8	11	0·157	2·93 ± 0·19
6	Eskimo (Greenland)	190	71·3	13	0·694	3·00 ± 0·15
7	Czech	105	82·9	11	0·470	3·17 ± 0·22
8	English (17th cent.: Whitechapel)	131	74·3	7	0·965	3·26 ± 0·20
9	German (Reihengräber)	220	73·8	13	0·876	3·35 ± 0·16
10	English (17th cent.: Farringdon St.)	135	75·4	11	0·137	3·48 ± 0·21
11	Egyptian (18th–21st dyn.: Thebes)	167	75·0	13	0·283	3·55 ± 0·19
	(*b*) Series showing small but significant differences between component subseries					
12	Egyptian (predynastic: Naqada)	165	72·7	13	0·666	2·80 ± 0·17
13	Swiss (Valais)	453	83·9	19	0·423	4·01 ± 0·13
	(*c*) Heterogeneous series					
14	French (mediaeval and modern)	1000	79·6	22	0·000‡	4·32 ± 0·10
15	English (Bronze Age)	151	78·8	12	0·660	5·42 ± 0·31
	(*d*) Artificial mixture of two series					
2 + 12	Eskimo + Egyptian	321	74·8	16	0·297	3·43 ± 0·14

* The series are the ones described in the following sources, which give the individual measurements in the majority of cases:

(1) Guanche—Hooton (1925).
(2) Eskimo—Hrdlička (1924).
(3) Egyptian—Pearson & Davin (1924).
(4) New British—Müller (1905).
(5) English—Duckworth (1917).
(6) Eskimo—Fürst & Hansen (1915).
(7) Czech—Schiff (1912).
(8) English—Macdonell (1904).
(9) German (pooled)—Morant (1928).
(10) English—Hooke (1926).
(11) Egyptian—Schmidt (1886).
(12) Egyptian—Fawcett (1902).
(13) Swiss—Pittard (1909–10).
(14) French—Topinard (1885, p. 388).
(15) English (pooled)—Morant (1926).

† The P's were found from Table XII in *Tables for Statisticians and Biometricians*, Part I, the n' used being the number of groups, less 2 (i.e. degrees of freedom were taken as number of groups, less 3).

‡ 0·00003.

other characters. One of these heterogeneous series (14) gives a P so low as entirely to disallow for practical purposes the hypothesis that the index was

normally distributed in the parent population, but the P for the other (15) is higher than several of those in section (*a*) of the table. The test thus fails entirely to distinguish between all suitable samples, on the one hand, and all unsuitable ones, on the other. Another example of its failure in this respect is recorded in section (*d*) of the table, the data there relating to an artificial mixture of two series having the very considerable difference of 4·4 units in their mean cephalic indices. The P of 0·3 obtained gives no indication of the strange origin of this sample.

It is easier to point to series which are clearly unsuitable for purposes of classification than to any which would confidently be expected to be entirely satisfactory. A distribution which appears as likely as any to reveal the unsatisfactory nature of a series of the former kind has been provided by Herskovits (1930, p. 151). It relates to the thickness of the lips of 959 male adult "Negroes" in the United States. While a certain proportion of these men are believed to be of pure African origin, the majority represent varying grades of miscegenation of Negroes and members of other ethnic groups, with European ancestors predominating but some admixture of American Indian blood. The character is obviously one which makes a marked distinction between the types of the two major groups of parental populations involved. The distribution for it gives a P of 0·536 (13 groups). Examples of the same kind relating to samples which are obviously unsuitable for purposes of group classification—such as mixed "white" samples in the United States, or artificial mixtures of series which differ markedly in some respects—might be multiplied indefinitely, and it is quite usual in examining such material to find distributions which give high P's when fitted with normal curves. Trevor (1938) has observed this situation for all the characters he examined when dealing with data for living populations in different parts of the world derived from the crossing of European and non-European peoples.

It is actually found that the vast majority of the distributions of measurements that occur in anthropological practice satisfy a test which shows the hypothesis that the character was normally distributed in the parent population to be not at all unacceptable. The high values of P usually found when comparisons are made with normal curves fitted to the data do not demonstrate that the characteristic type of distribution for all but exceptional, and presumably unsuitable, populations is absolutely normal, but they are quite good enough evidence to suggest that the population distributions must be closely similar to unimodal and symmetrical forms.

This generalization applies almost equally well, as far as is known, to all measurements, whether of shape or size, of series of living people or skeletons. Furthermore, it apparently applies equally well to series from groups of any size which are likely to be considered as populations by the anthropologist. Of the series of skulls dealt with in section (*a*) of Table II, some came from single

cemeteries while others were made up by combining specimens from several scattered cemeteries. Samples from small communities of the parochial kind usually provide distributions which approximate to the form of the normal curve neither more nor less closely than do samples of the same size selected at random either from the populations of provinces or from those of countries. The form of the distributions of metrical characters fails entirely to differentiate populations of very different sizes.

The fact that a close approach to normality is almost invariably found suggests that this condition can safely be accepted as one which must be satisfied by the distributions provided by samples before they can be accepted as suitable for group comparisons. It has been seen, however, that the distributions of samples which are known to be unsuitable for purposes of classification may also satisfy the condition. Hence the test must be supposed a necessary but by no means a crucial one. In the case of a particular sample the theoretical test might be that all characters should show reasonably high probabilities that the distributions in the population were normal. In practice, of course, the question can never be examined for more than a small number of characters—very seldom greater than 50—but it is important to appreciate that the test should be applied, in theory at least, to all characters for which data are available. The demonstration that a single distribution indicates a close approach to normality is no evidence that any other distribution for the same sample will do so.

In general, useful anthropological results of a statistical nature can only be derived from the conjoint investigation of data for several characters. Different characters are quite likely to suggest different conclusions, and the kind of information required has to be obtained by taking the evidence of a sufficient number into consideration. To take a hypothetical example, it may be assumed that in the case of the crossing of two populations (A and B) the one derived from them (C) will only be likely to show any departure from the usual form of distribution—such as bimodality—in the case of those characters which show a clear difference between the averages for A and B. But experience shows that the two parental groups are likely to show clear differences in their averages for only a small proportion of the characters compared, even if A and B belong to quite distinct ethnic groups. Hence peculiarity in the distributions for the hybrid population would be expected only in the case of a small proportion of the characters examined, and it is necessary to examine as many as possible in order to ensure that such a peculiarity has not been overlooked. Furthermore, it should be appreciated that the chance of departures from normality being found is very different for different characters. This point is discussed below.

A test which might be applied in practice could specify that the distributions of all metrical characters recorded for a particular sample examined should give P's, when compared with normal curves fitted to them, greater than some

arbitrarily chosen value—0·001, say. If one or more of the distributions were found to give values less than the limit then the sample might be considered so peculiar that it should not be used for purposes of classification. It is known, however, that many unsuitable samples would pass such a test, and hence it is of little practical value. Its value is lessened further by the fact that when a clear departure from normality is found the peculiarity of the distribution can nearly always be detected more easily on account of its abnormally large variation: this is so for series 14 in Table II.

Much labour has been expended in making a detailed analysis of the distributions of characters provided by anthropological samples which are adequate in length for statistical purposes. Significant but slight degrees of skewness have sometimes been recorded. It has not been shown, however, that such examples provide any conclusions which are of the kind needed by anthropologists. Usually there is no guarantee that the samples were chosen absolutely at random, and the slightly peculiar but erratic characteristics sometimes observed may well be due to biased selection.

It has been shown that if the components of an index (A/B) are normally distributed, then the distribution of the index conforms more closely to a Pearson Type IV than to the normal curve. This topic has been discussed by Merrill (1928) and Fieller (1932). A Type IV curve is unimodal and may be almost symmetrical and very similar in form to the normal. In fitting normal curves to a series of distributions it is generally not possible to observe any tendency for indices to give lower values of P than absolute measurements. Elderton & Woo (1932) made a detailed study of the forms of the distributions for a single long series of Egyptian crania in the case of an unusual set of measurements relating to cranial bones considered singly. They concluded: "that the distributions of characters measured on the individual bones of the skull are not of normal type, but rather that the skewness and kurtosis of such distributions are peculiar to the individual measurement." This is a point of considerable theoretical interest, though all the departures from normality observed were in fact slight. The conclusion that the population distributions almost invariably bear a close similarity to the form of the normal curve in the case of all kinds of measurements appears to be sufficiently exact for almost all practical purposes.

This is a result which anthropologists are very loath to accept in spite of the clear evidence of a large amount of material which supports it. The belief that markedly skew and bi- or multimodal distributions must be found was very generally held thirty years ago, and it has not been abandoned yet. The fact that expectations of this kind have been categorically denied to him is a matter of prime importance to the anthropologist, as it suggests that certain methods of analysis are quite impracticable on any valid statistical lines. He still lives in hope of discovering a method of dissecting a sample in such a way as to disclose

the sources from which it was derived. Judging largely from the prevalence of unimodal and symmetrical distributions which offer no scope for dissection, but also to some extent from other characteristics of samples to be considered, the statistician says that it is idle to entertain such a hope in dealing with metrical data. He suggests that statistical tests may profitably be used to distinguish suitable from unsuitable samples, but that those selected for further use must be kept intact and somehow be compared with one another as entities in order that conclusions may be deduced regarding the relationships and descent of the populations represented.

The difference between the two points of view is fundamental. Repetition of the assertion that the vast majority of the samples with which they deal in practice do not indicate either appreciable skewness or more than one mode in the populations represented has had little effect in modifying the methods of anthropologists in general, or even of those who commonly use statistical methods. It is possible to go further, however, and to offer an explanation of what appears at first to be a surprising uniformity in one characteristic of diverse kinds of material.

It is an observed fact that the variabilities of all samples which can be accepted for purposes of classification are very considerable and that they tend to be similar in the case of all populations for which adequate data are available, the earliest of these having existed in Egypt about 5000 B.C. Absolute equality in the populations is not indicated, but intraracial variabilities all lie within a fairly restricted range in the case of any particular character. Evidence in support of this statement will be provided in the following section of this paper.

In the case of a mixture of two populations, a distribution would only be expected to be bimodal if the difference of the means for the two populations happened to be appreciable compared with the two intragroup variabilities. An estimate of the chance that the difference in question is likely to be appreciable will be given by the ratio of interracial to intraracial variabilities, and this is likely to be different for different characters. It has usually been assumed that the differences between the types of populations defined by average measurements are often large compared with the differences between individuals belonging to the same population, but though this situation may be true for skin colour it is not observed in the case of any metrical character for which data are available. With the evidence of skin colour in view, anthropologists have always been inclined to over-estimate the interracial variability and under-estimate the intraracial variability of other characters. The evidence of measurements corrects this tendency and shows that for metrical characters the situation is markedly different from that which has often been postulated.

Examples relating to particular characters will now be considered. Fig. 2 shows the distributions of stature for three series of men. The first relates to a tribe of Congo pygmies (Efé) recorded by Šebesta & Lebzelter (1933): it has a

mean of 1430 mm., which is very close to the smallest given for any people in the world, and a standard deviation of 51·4. The third relates to the Dinka tribe of Nilotic negroes, from unpublished data of A. MacTier Pirrie,* and its mean of 1804 mm. appears to be the largest on record, the standard deviation being 79·7. The second distribution relates to men from a province of Japan recorded by Matsumura (1925), and it has a mean of 1622 mm., which is almost midway between those for the extreme series, and a standard deviation of 53·1.

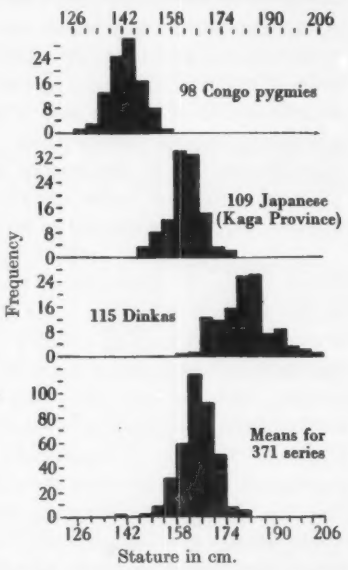


Fig. 2. Intraracial distributions of statures representing an extremely short, an intermediate and an extremely tall group, and an interracial distribution.

There is seen to be an absolute separation between the ranges of the statures for the pygmies and the Dinkas, the tallest member of the former group being shorter than the shortest of the latter, though it is probable that the distributions for larger samples would overlap to some extent. A mixed series made up by taking 100 pygmies and 100 Dinkas would obviously be bimodal. A distribution which would clearly be bimodal would also be provided if the Japanese men and the pygmies, or the Japanese men and the Dinkas, were taken together. If mixtures were made up by taking pairs of series at random from all populations in the world, however, the chance of getting a particular pair with means differing by as much as 182 mm. (= Dinka-Japanese mean) is quite small.

This point is illustrated by the interracial distribution given at the bottom of Fig. 2, which was compiled by taking all the means for series of 30 or more men collected by Deniker (1926) and representing populations in all parts of the world. The mean of the distribution is 1646 mm. and the standard deviation 58·8. It can be shown that if pairs of values (i.e. means for series) are drawn at random from it, then the probability that they will differ by less than 80 mm. is 0·56, by less than 120 mm. 0·80, by less than 160 mm. 0·91, and by less than 200 mm. 0·96. It is also known from experience that a mixed distribution of statures made up by combining two samples of equal sizes from populations in which the character is normally distributed, and which differ in their means by 80 mm. is quite likely to give a P on comparison with a normal curve fitted to it which would be high enough to make the incorrect hypothesis thus tested acceptable if

* I am indebted to Dr Otto Samson, who is working on the material, for the statures of the Dinkas. The records were kindly lent by Prof. J. C. Brash.

the source of the material were unknown. With any appreciably wider separation of the means for the two component series all P's found would probably be so low as to indicate clear departure from normality. The position may be summed up roughly in this way in the case of stature: if each of a number of samples of unknown origin was actually a mixture of two samples of equal sizes representing pairs of populations chosen at random from all in the world, then *the form of the distribution alone*, in the case of about half of them, would not be sufficiently peculiar to reveal clearly their distinction from samples representing single populations, but in the case of the other half peculiarity in origin would be indicated. In the same circumstances distributions which were definitely bimodal would be expected to occur occasionally.

Actually, however, such forms are scarcely ever found in practice, and hence it must be inferred that the condition postulated is of an artificial kind. In fact the anthropologist is not at all likely to choose a sample so badly that it represents what may be called a purely "mechanical" mixture of two distinct populations which differ appreciably in their averages for one or more characters. He is far more likely to come across a sample representing a population derived from the partial or complete blood admixture of two groups which were originally distinct. In such cases it is known that the mean for the mixed population will lie between those for the parental groups. Trevor (1938) found that the distributions for groups derived from the crossing of distinct ethnic stocks usually give high P's when compared with normal curves fitted to them. Samples occurring in anthropological practice may relate to diverse kinds of conglomerations, as it were, of mechanical and blood intermixture of populations which would best be considered singly in examining group relationships. In such cases the forms of the distributions of metrical characters will usually fail entirely to indicate the composite nature of the assemblage. A sample of the "white" population of the United States consisting of recruits was found to give a distribution of stature which could be adequately represented by a normal curve (Hoffman, 1918).

Anthropologists find it hard to accept the conclusion that their frequency distributions for heterogeneous material never give any clear indication of the component parts of a particular sample. This situation is observed because the interracial variation of metrical characters is of the same order as, or smaller than, intraracial variation. The situation has been considered in the case of stature, and for this character the interracial distribution gives a standard deviation of the same order as those normally found for intraracial samples. These questions have been investigated most fully for certain measurements of the skull which are more reliable than the vast majority of the measurements available for series of living people. Data similar to those illustrated in Fig. 2 for stature are given in Table III for three cranial characters. The material collected relates to series of adult male skulls made up by 30 or more specimens and

representing populations, believed to be suitable for purposes of classification, which have existed in various parts of the world from about 5000 B.C. to modern times. In the case of each of the three characters considered the distribution for the series with the lowest mean is given first, followed by the distribution of racial means and then that for the series with the highest mean.

TABLE III

Distributions of three cranial measurements, each for an intraracial sample with an extremely low mean (L) and one with an extremely high mean (H), and for an interracial sample (I): male skulls

Cephalic index				Horizontal circumference				Foraminal index			
Central value	L*	I	H†	Central value	L‡	I	H§	Central value	L‖	I	H¶
65	7	—	—	450	1	—	—	63·75	1	—	—
67	11	—	—	458	1	—	—	66·25	2	—	—
69	11	3	—	466	3	—	—	68·75	0·5	—	—
71	4	12·5	—	474	8·5	—	—	71·25	0·5	—	—
73	2	13·5	—	482	5·5	—	—	73·75	8	—	—
75	—	33	0·5	490	7	6	1	76·25	7·5	—	—
77	—	26	1·5	498	7	14	1	78·75	5·5	1	1·5
79	—	19	2	506	6	17	5	81·25	2	11	3·5
81	—	12	3	514	3	42	19	83·75	6	51·5	7
83	—	22·5	9	522	—	28	19	86·25	1	19·5	5·5
85	—	7·5	14	530	—	8	34	88·75	1	4	9·5
87	—	1	10	538	—	1	36·5	91·25	0	—	5
89	—	—	7·5	546	—	—	22·5	93·75	2	—	6
91	—	—	6	554	—	—	11	96·25	1	—	4
93	—	—	5	562	—	—	10·5	98·75	—	—	1·5
95	—	—	1·5	570	—	—	4·5	101·25	—	—	1·5
Total	35	150	60		42	116	164		38	87	45
Mean	68·1	77·5	86·4		488·8	512·8	534·6		79·5	84·2	89·2
σ	2·23	4·14	4·11		15·1	10·1	15·7		7·05	1·69	5·63

* Loyalty Islander (Sarasin & Roux, 1916–22).
† Telenghite (Reicher, 1913).
‡ Baining (New Britain) (Bauer, 1915).
§ Reihengräber (Morant, 1928).
‖ Tanganyika (Ried, 1915).
¶ Venezuelian (Marcano, 1890).

The situation is seen to be much the same for the cephalic index as for stature, the interracial variability being rather larger than that for nearly all samples which can be accepted as suitable for purposes of classification. The distributions for the series with extreme means cover contiguous ranges, and there is no overlap. Of all the cranial characters for which adequate data are available the cephalic index is quite outstanding in this respect. For most of them the situation is very similar to that illustrated by the data in Table III for

the horizontal circumference of the skull. In this case the standard deviation for the interracial distribution is appreciably less than any intraracial value found, and the extreme distributions show considerable overlap. The limit in the other direction is exhibited by the index expressing the breadth of the foramen magnum as a percentage of its length. In this case interracial variation is much smaller than intraracial variation, and the interracial distribution falls entirely within the range for any intraracial sample. For most cranial characters the averages for all populations in the world are quite likely to fall within the range provided by any population chosen at random.

It is evident that the chances that a heterogeneous sample will exhibit a distribution diverging appreciably from the normal form are very different for different metrical characters. As far as is known, stature and the cephalic index are the two most likely to show departures from the rule in the case of mixed samples, since the ratio of inter- to intraracial variation is largest for them. For this reason examples relating to the two characters have been given above. Most cranial measurements are decidedly less likely to reveal heterogeneity by the forms of distributions obtained for them, as the difference between the means of any two samples inadvertently combined compared with the two intra-group variabilities will be, on the average, appreciably less for them. For some cranial characters interracial variation is so small compared with intraracial variation that no heterogeneous samples will be at all likely to provide peculiar distributions.

In view of the circumstances discussed above, it is not surprising that the distributions which occur in anthropological practice scarcely ever show any appreciable departure from a symmetrical and unimodal form. Detailed statistical analysis of the slight peculiarities of the distributions sometimes observed is never likely to be profitable in the case of small samples—composed of fewer than 300 individuals, say—and in general it does not lead in the case of adequately large samples to any conclusions which aid the classification of the groups represented.

The normal curve may be accepted as the one which almost invariably gives an adequate description of the distributions in the populations sampled, even though some of these populations are unsuitable for the purpose in view. It is usually found to effect this purpose remarkably well, in spite of the fact that the theoretical curve can never be the correct one as it is of unlimited range. In this application it is safest not to draw any inferences regarding the composition of a population from the observed fact that its distributions are normal in form. Very occasionally a clear departure from normality can be taken to indicate that a peculiar and unsuitable population is represented.

For practical purposes the most important conclusion to be derived from the forms of the distributions of anthropometric characters is that dissection of the samples treated into subsamples which might be supposed to represent groups

of different origins is impracticable. Anthropologists in general are unwilling to accept this conclusion. Other characteristics of the samples, such as their variabilities and correlations, also have a bearing on the question and hence discussion of it will be deferred for a later part of this paper.

The writer is indebted to Miss M. L. Tildesley, Prof. E. S. Pearson and Prof. G. von Bonin for criticism which led to the improvement of this first part.

APPENDIX

CERTAIN DISTRIBUTIONS OF CEPHALIC INDICES PROVIDED BY FELIX VON LUSCHAN

As far as the writer is aware, the only published distributions for any anthropometric character which show a clear departure from normality, and which relate to unselected samples made up by more than 100 individuals, are three provided by Felix von Luschan. These are often referred to as evidence that such departures do occur in practice, though usually without comment on the rareness of such an occurrence. All the distributions in question are for the cephalic index, and they all relate to groups of men native to the Near East.

The first is for 179 men who called themselves Greeks measured in Lycia and neighbouring localities in the south of Asia Minor, including a few islands. The figures for the cephalic index were published (von Luschan, 1890). A frequency diagram in the same paper does not agree with these figures, as it shows one index of 95 while the highest in the table is 94. This error was perpetuated when the distribution was reproduced in two later publications (von Luschan, 1911, 1927). The frequencies are shown as histograms in the top diagram of our Fig. 3A.

The second series is made up by 756 Turks from the south of Asia Minor and the north of Syria. The figures for 187 of these men are available (1890) but not those for the remainder. A frequency distribution "reduced to one-third" was given (1911, 1927). It is not possible to read off the individual frequencies from this with absolute accuracy, but the middle diagram of Fig. 3A reproduces them as closely as possible. With reference to this distribution, von Luschan says (1911, p. 236) that it ranges from 69 to 96, but the highest value given in his diagram is 92.

The third series is of 1222 Jews: "52 % of these were Sephardim, whom I measured at Smyrna, at Constantinople, at Makri, and in Rhodes; the rest were Ashkenazim measured by myself...at Vienna, Austria" (1911, p. 226). A frequency distribution "reduced to one-fifth" is given and this is reproduced as accurately as possible in the bottom diagram of Fig. 3A.

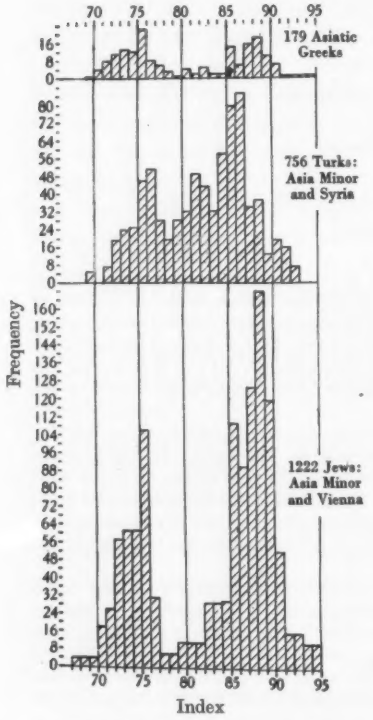
The first and third of these three distributions are clearly bimodal in form. That for the 756 Turks provides less clear evidence of the existence of more than one mode in the population distribution, but it certainly suggests that this must have been clearly asymmetrical. The three distributions provided by von Luschan relate principally to populations of Asia Minor, and their peculiarities might be supposed due to the fact that this region is inhabited by peoples who are racially heterogeneous to an unusual extent. In fact, however, other series from it do not appear to be at all distinguished by any unusual forms of

their metrical characters. Measurements for three series of men which represent the populations discussed by von Luschan as closely as any available are given by:

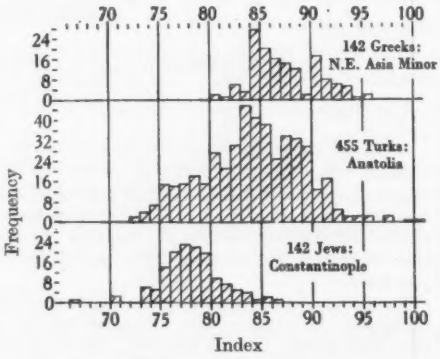
(i) Néophytos (1891): 142 Greeks in the town and neighbourhood of Kiresun in the north-east of Asia Minor;

(ii) Hauschild & Wagenseil (1931): 455 Turks from a number of localities in Anatolia;

(iii) Wagenseil (1925): 142 Jews measured in Constantinople, of whom 107 belonged to that town, 13 came from various countries of Eastern Europe and the remainder were from Asia Minor and Syria.



A. Three distributions provided by F. von Luschan.



B. Three distributions provided by other anthropologists.

Fig. 3. Distributions of the cephalic index for series of men measured in the Near East.

The frequency distributions of the cephalic index for these three series are given in Fig. 3 B, and they all differ markedly in form from the corresponding distributions given by von Luschan's data. That for the 142 Greeks is irregular, but little significance can be attached to this fact as the sample is not a large one: it shows no indices below 80, while more than half of von Luschan's Greeks had indices below this value. The measurements for Anatolian Turks and Constantinople Jews do not suggest that the population distributions showed any clear departure from the normal form.

The corresponding pairs of series are also distinguished clearly by their variabilities. The following standard deviations are found:

	Greeks	Turks	Jews
Von Luschan's series	$6{\cdot}96 \pm 0{\cdot}37$	$5{\cdot}20 \pm 0{\cdot}13$	$6{\cdot}72 \pm 0{\cdot}14$
Other series	$3{\cdot}21 \pm 0{\cdot}19$	$4{\cdot}76 \pm 0{\cdot}16$	$2{\cdot}90 \pm 0{\cdot}17$

In comparison with other material, the abnormally high standard deviations of the cephalic index for von Luschan's three series show at once that the samples are of a peculiar kind unsuitable for racial comparisons. This material is altogether exceptional and it does not affect the general conclusion that the distributions of metrical characters for samples treated in anthropological practice almost invariably indicate that the population distributions do not diverge appreciably from the normal form.

REFERENCES

BAUER, L. (1915). "Beiträge zur Kraniologie der Baining (Neu-Pommern)." *Arch. Anthrop., Braunschw.*, N.F. **14**, 145.

DENIKER, J. (1926). *Les Races et les Peuples de la Terre*, 2nd ed. Paris.

DUCKWORTH, W. L. H. (1917). "Notes on some measurements made on subjects in the dissecting-room." *J. Anat., Lond.*, **51**, 167.

ELDERTON, E. M. & WOO, T. L. (1932). "On the normality or want of normality in the frequency distributions of cranial measurements." *Biometrika*, **24**, 45.

FAWCETT, C. D., LEE, A. *et al.*: edited and arranged by PEARSON, K. (1902). "A second study of the variation and correlation of the human skull, with special reference to the Naqada crania." *Biometrika*, **1**, 408.

FIELLER, E. C. (1932). "The distribution of the index in a normal bivariate population." *Biometrika*, **24**, 428.

FÜRST, C. M. & HANSEN, F. C. C. (1915). *Crania Groenlandica*. Copenhagen.

HAUSCHILD, M. W. & WAGENSEIL, F. (1931). "Anthropologische Untersuchungen anatolischen Türken." *Z. Morph. Anthr.* **29**, 193.

HERSKOVITS, M. J. (1930). "The anthropometry of the American negro." *Col. Univ. Cont. Anthrop.* **11**.

HOFFMAN, F. L. (1918). *Army Anthropometry and Medical Rejection Statistics*. Newark, N.J.

HOOKE, B. G. E. (1926). "A third study of the English skull with special reference to the Farringdon Street crania." *Biometrika*, **18**, 1.

HOOTON, E. A. (1925). "The ancient inhabitants of the Canary Islands." *Harv. Afr. Stud.* **7**.

HRDLIČKA, A. (1924). "Catalogue of human crania in the United States National Museum Collections." *Proc. U.S. Nat. Mus.* (12), **63**.

LUSCHAN, F. VON (1890). "Die Tachtadschy und andere Überreste der alten Bevölkerung Lykiens." *Arch. Anthrop., Braunschw.*, **19**, 31.

—— (1911). "The early inhabitants of Western Asia." *J. Roy. Anthr. Inst.* **41**, 221.

—— (1927). *Völker, Rassen, Sprachen*. Berlin.

MACDONELL, W. R. (1904). "A study of the variation and correlation of the human skull, with special reference to English crania." *Biometrika*, **3**, 208.

MARCANO, G. (1890). *Ethnographie Précolombienne du Venezuela. Région des Raudals de l'Orénoque*. Paris.

MATSUMURA, A. (1925). "On the cephalic index and stature of the Japanese and their local differences." *J. Fac. Sci. Tokyo Univ.* Sec. 5, Anthrop. (1), **1**.

MERRILL, A. S. (1928). "Frequency distribution of an index when both the components follow the normal law." *Biometrika*, **20** A, 53.

MORANT, G. M. (1926). "A first study of the craniology of England and Scotland from Neolithic to early historic times, with special reference to the Anglo-Saxon skulls in London museums." *Biometrika*, **18**, 56.

—— (1928). "A preliminary classification of European races based on cranial measurements." *Biometrika*, **20** B, 301.

MÜLLER, W. (1905). "Beiträge zur Kraniologie der Neu-Britannier." *Mitt. Mus. Völkerk. Hamburg*, **23**.

NÉOPHYTOS, A. G. (1891). "Le grec du nord-est de l'Asie Mineure au point de vue anthropologique." *Anthropologie, Paris*, **2**, 25.

PEARSON, KARL (1897). *The Chances of Death and Other Studies in Evolution*. London.

PEARSON, K. & DAVIN, A. G. (1924). "On the biometric constants of the human skull." *Biometrika*, **16**, 328.

PITTARD, E. (1909–10). "Crania Helvetica. 1. Les crânes valaisans de la vallée du Rhône." *Mém. Inst. nat. genev.* **20**.

QUETELET, A. (1835). *Sur l'Homme et le Développement de ses Facultés ou Essai de Physique Sociale*. Paris.

—— (1871). *Anthropométrie ou Mesure des Différentes Facultés de l'Homme*. Brussels.

REICHER, M. (1913). "Untersuchungen über die Schädelform der alpenländischen und mongolischen Brachycephalen." *Z. Morph. Anthr.* **15**, 421.

RIED, H. A. (1915). "Zur Anthropologie des abflusslosen Rumpfschollenlandes im nordöstlichen Deutsch-Ostafrika." *Abh. Hamburg. Kolon. Inst.* **31**.

SARASIN, F. & ROUX, J. (1916–22). *Nova Caledonia: Forschungen in Neu-Caledonien und auf den Loyalty-Inseln*. C. *Anthropologie*. Berlin.

SCHIFF, F. (1912). "Beiträge zur Kraniologie der Czechen." *Arch. Anthrop., Braunschw.*, N.F. **11**, 253.

SCHMIDT, E. (1886). *Die anthropologischen Sammlungen Deutschlands*. Leipzig Cat.

ŠEBESTA, P. & LEBZELTER, V. (1933). "Anthropology of the central African pygmies in the Belgian Congo." *Czech. Acad. Sci. Arts*, 2. Class, *Anthropologica*.

STIEDA, L. (1883). "Über die Anwendung der Wahrscheinlichkeitsrechnung in der anthropologischen Statistik." *Arch. Anthrop., Braunschw.*, **14**, 167.

TOCHER, J. F. (1924). "Anthropometric observations on samples of the civil populations of Aberdeenshire, Banffshire, and Kincardineshire", and "A study of the chief physical characters of soldiers of Scottish nationality, etc." *The William Ramsay Henderson Trust Reports*, 2 and 3.

TOPINARD, P. (1885). *Eléments d'Anthropologie Générale*. Paris.

TREVOR, J. (1938). "Some anthropological characteristics of hybrid populations." *Eugen. Rev.* **30**, 21.

WAGENSEIL, F. (1925). "Beiträge zur physischen Anthropologie der spaniolischen Juden und zur jüdischen Rassenfrage." *Z. Morph. Anthr.* **23**, 1925.

WITT, H. (1879). "Die Schädelform der Esten." Diss. Dorpat.

A STUDY OF THE CRANIAL AND OTHER HUMAN REMAINS FROM PALESTINE EXCAVATED AT TELL DUWEIR (LACHISH) BY THE WELLCOME-MARSTON ARCHAEOLOGICAL RESEARCH EXPEDITION

By D. L. RISDON
Crewdson Benington Student

CONTENTS

		PAGE
1.	Introduction	99
2.	The general nature of the remains and remarks on the origin of the ossuaries	102
3.	Remarks on the condition and anomalies (other than dental) of the Lachish crania	106
4.	Remarks on the condition and anomalies of the jaws of the Lachish skulls	117
5.	The statistical nature of the material	121
6.	The relationships of the Lachish and Ancient Egyptian populations judged from comparisons of mean measurements	128
7.	Comparisons of the Lachish and Ancient Egyptian series for characters considered singly	143
8.	The contours of the Lachish series	145
9.	Comparisons of the Lachish series with series of crania and living people other than Egyptian	158
10.	Mandibles and long bones of the Lachish series	159
11.	Summary and conclusions	160
	Appendices I. Definitions of skull measurements taken	162
	II. Tables of individual measurements	*to face p.* 162
	List of Plates	163
	References	165
	Plates I–XXIII	*to face p.* 166

Note that tissues of Figs. 4–9 are contained in end pocket

1. Introduction

The human skeletal remains which form the subject of this report were excavated by the Wellcome (later the Wellcome-Marston) Archaeological Research Expedition to the Near East from 1933 to 1936. They were collected by the late Mr J. L. Starkey, who was then the Director of the Expedition, and it was owing to his enthusiasm and untiring efforts in the field that so much material is now available for study. His tragic death in 1938 deprived archaeology and physical anthropology alike of one who was chiefly responsible for the collection of some of the most valuable evidence extant relating to the early history of the peoples of the eastern Mediterranean.

The human remains from Lachish were brought to England, and the greater part of the work on them was carried out at the Galton Laboratory, University College, London, under the supervision of Dr G. M. Morant. I am indebted to the Trustees of the late Sir Henry Wellcome for a grant which enabled me to carry out this work over a period of rather more than three years, and also for a subsidy which has made possible the complete presentation of the illustrative and tabular matter collected for the report. I also owe a very great debt of gratitude to Dr Morant for his untiring help and advice in all the stages of its preparation. A complete report of the excavations is in course of preparation, and I have to thank Mr Charles Inge, the present Director of the Expedition, for most of the following particulars relating to the discovery of the bones.

The only published account of them is a note by Mr Starkey (1936), prefaced to a description of three trepanned skulls by Dr Wilson Parry. He reports that a roughly circular chamber (No. 107), which contained a deposit of human remains much damaged by fire, was opened in 1934. At the same time an adjoining and larger rectangular cavern (No. 120) was located. Starkey writes:

"The top layer consisted of many animal bones, mostly pig, and this refuse should be ascribed to the latter half of the Judean kingdom....The lower or main deposit consisted of a mass of human bones, the remains of at least 1500 bodies. As they were pitched in through the hole in the broken roof, the skulls rolled down from the apex of the pile to the sides of the chamber."

Sherds of pottery intermixed with the bones can all be assigned to the seventh and eighth centuries B.C.

"Some bones were partially calcined, suggesting that they were abstracted from burnt buildings....Careful supervision of the clearance failed to establish that any crania were in articulation with vertebrae, and the jaws were rarely attached: in fact, no order was seen in the jumbled mass."

It is suggested that the ossuary was probably connected with the salvage of Lachish after its partial destruction by Sennacherib, King of Assyria, in 701 B.C.

"When floor level was reached it became clear that the tomb had been previously used as a dwelling, a door had been cut at the N.E. [given erroneously as N.W. in the published account] corner, connecting the smaller circular tomb with it, which contained the 500 bodies discovered in 1934. From the style of both chambers it is certain that they were originally excavated to contain early fifteenth-century burials."

Excavation of the area was extended and two other tombs were found, of which no details have yet been published. Fig. 1 is a plan of the area in question, showing all four tombs from which the human remains treated in this study were obtained.

The archaeologists report that:

"All the tomb chambers were adjoining and interconnected, and the deposits in Nos. 107, 108, and 120 were identical and as described by Mr Starkey.

Tomb No. 116 is in a slightly different category; it is an artificial cavern, with a small entrance cut in the south wall of No. 108 near the bottom, containing normal burials comparatively undisturbed. The period however is the same."

It is said that the later discoveries do not provide any evidence which makes closer dating possible. They include a Cypriote juglet, of a type common in Palestine in the eighth century B.C., scarabs and scaraboids, bone pendants, and blue glaze amulets. The last include representations of Egyptian deities

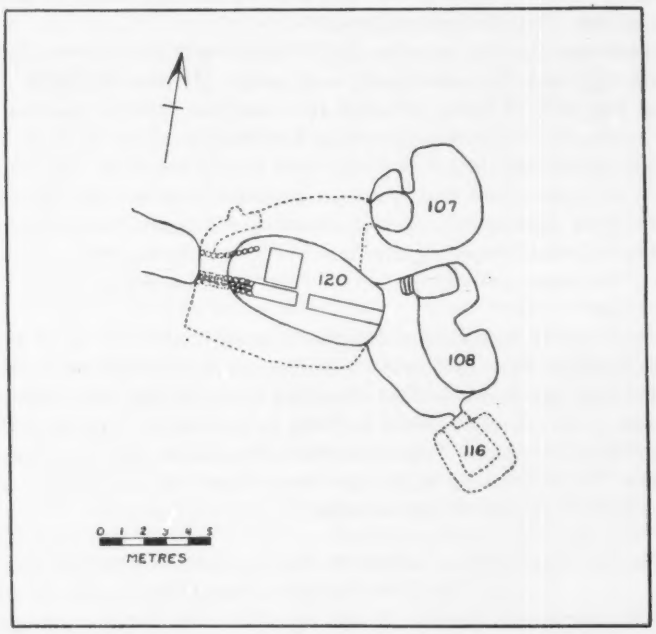


Fig. 1. Plan of the tombs at Lachish from which the human remains were recovered. The broken lines indicate approximately the floor area of Tomb 120, and the outline of the benches (inner line) and the maximum perimeter of Tomb 116. The approximate heights of the tombs were: 107, 2 m.; 108, 2·2 m.; 116, 1·6 m.; and 120, 3·6 m.

which were long popular in Palestine. There is nothing in the objects that one would be surprised to find in normal burials at Lachish in the middle of the Jewish period.

Further particulars regarding the state in which the human remains were found are available. In all the tombs, except No. 116, there were distinct layers of animal bones above the human deposits, and in the shallower part of Tomb 108 they came up to the roof level, the roof itself having been broken away. In Tomb 107 the apex of the pile of human bones was about 1 m. above the floor

level, and the same height was about 1·3 m. in Tomb 120. There were fewer human remains in Tomb 108: these were more scattered, and there were many animal bones above them.

Tomb 116 is in a different category from the others, for it contained several formal burials laid out on the benches, with the usual funerary equipment. These burials were partially disturbed, and the skulls could not be distinguished from others which may have rolled in at the time of the filling of Tomb 108. The roof of Tomb 116 was intact, and most of the long bones collected come from it as they were the best preserved.

The estimate that the remains of 1500 individuals were represented applies to Tomb 120, and it is admittedly very rough. It was impossible to judge whether the piles of bones included the complete skeletal remains of 1500 people or not, but the impression was gained that the skeleton of the head was far better represented than that of any other part of the body. A few mandibles attached to crania were found, but there appeared to be no other cases of parts of the skeleton existing in proper articulation. A suggested explanation of the circumstances which led to the existence of the ossuary is given in the following section of this paper, as evidence derived from the bones preserved has a bearing on this topic.

Plates I and II reproduce photographs taken in the interior of Tomb 120 after the clearance down to floor level. The piles of skulls which had accumulated round the walls are shown *in situ*. Plate II B is a close view of a small group of skulls seen to the left of the beam in Plate I A and above the pick in Plate I B.

Other human remains from a number of tombs at Lachish, representing fewer than 100 individuals in all, have been preserved, and the writer is preparing a separate report on this material.

2. The general nature of the remains and remarks on the origin of the ossuaries

The human remains described in this report comprise all that were preserved from the four tombs referred to in the foregoing section. They are made up principally of crania, which appeared to be more abundant in the deposits than any other parts of the skeleton, but there are also series of mandibles and long bones of the limbs, and a few other bones. A selection was made of the better preserved remains, and preference was given to crania. It should be realized that the relative numbers of different parts in the collection preserved cannot be supposed to be proportional to the frequencies with which they actually occurred in the ossuaries.

The specimens were dipped in paraffin wax in the field, and the cleaning of them in the laboratory was a laborious task. While it was being carried out, four crania distorted by earth pressure were found, as well as a certain number of fragments of other parts of the skeleton which may have belonged to individuals

included in the series, or to individuals not otherwise represented. All this material was discarded, and it is not counted in the totals. The numbers of specimens, most of which are more or less incomplete, from each tomb are given in the table below, no distinctions of sex or age being made:

	Tomb				
	107	108	116	120	Totals
Crania	74	9	45	567	695
Mandibles	12	1	7	56	76
Femora	2	—	50	28	80
Tibiae	2	1	27	15	45
Fibulae	—	—	1	—	1
Humeri	2	—	23	23	48
Radii	1	—	5	8	14
Ulnae	3	3	3	6	15
Sacra	—	—	—	2	2
Clavicles	4	—	2	1	7

There are also a few vertebrae associated with skulls.

The absence of other bones of the skeleton which would normally be well preserved, such as those of the wrist and ankle, should be noted. In this connexion Mr Inge remarks: "Less attention was paid to the skulls of children owing to their supposed smaller anthropometrical value, and the same applies to parts of the skeleton other than skulls and long bones."

None of the bones are known to be associated, except that ten mandibles and crania were preserved together, and are undoubtedly paired. It is probable that a considerable proportion of the other mandibles do belong to the crania, but it is not possible to associate them. Experience has shown that any attempt to pair parallel series of these parts is unprofitable. It was also impossible to associate the other bones with the crania or with one another.

After separating the immature specimens, the crania and mandibles were sexed anatomically, giving the totals shown in the table below. Estimates of

		Tomb				Totals
		107	108	116	120	
Crania	Adult ♂	33	5	19	303	360 (51·8%)
	Adult ♀	31	3	25	215	274 (39·4%)
	Immature	10	1	1	49	61 (8·8%)
Mandibles	Adult ♂	5	0	3	26	34 (44·7%)
	Adult ♀	7	1	4	24	36 (47·4%)
	Immature	0	0	0	6	6 (7·9%)

this kind cannot be exact in all cases, of course, but experience suggests that 80–90 % are likely to be correct. The inclusion of a certain proportion of male-like, but actually female, specimens in the male series, or vice versa, is not likely to affect statistical constants appreciably. It has been shown by Martin (1936) and Cleaver (1937), that isolated mandibles can be sexed anatomically as accurately as crania. Particulars regarding the other bones of the skeleton are given in § 10 below.

The proportions of the sexes and of immature individuals are seen to be very much the same in all the tombs. The corresponding male and female percentages in the table are not particularly close, but the series of mandibles is obviously too small to give reliable proportions. It is safe to assume that there are rather more males than females in the collection preserved. This does not necessarily imply that the same was true for the total number of individuals deposited in the tombs, as the male skulls, being stronger on the average, would stand a slightly better chance of being preserved than female specimens. It is probable, for a similar reason, that the proportion of immature individuals was very considerably higher in the tombs. The proportions of men, women and children there appear to have been very similar to those which would be expected in a normal cemetery population.

Estimates of the ages at death of the adults are discussed in § 3 below. It is shown there that the population was considerably younger than such as is normally found in ancient or recent cemeteries. There are very few aged specimens in the collection.

The condition of the bones throws little further light on the question of how and why they were deposited in the tombs, but the following observations may be noted in this connexion. There is only one skull (No. 108, ♂, Tomb 120) with wounds which are likely to have been the cause of death. This has pieces of bone removed from both parietals (see Plate III B), and a long cut extending across the left temporal line. The other lines on the left parietal and extending across the sagittal suture, along which the bones are broken, may have been made at any time after death. As far as can be told the damage shown in the case of all the other specimens occurred in the tombs, and after the deaths of the individuals. The greater part of it may be attributed to the crushing weight of the piles of bones.

Among the totals for all tombs combined, there are twenty-nine (8·1 %) adult male, eight (2·9 %) adult female, and two (3·3 %) immature skulls with wounds, mostly slight, which had healed completely during life. These frequencies are not higher than those commonly found for excavated series of crania.

In his account of the discovery of the remains, Starkey notes that those in Tomb 107 were much damaged by fire, and that some bones in Tomb 120 were "...partially calcined, suggesting that they were abstracted from burnt

buildings". It is probable that the great majority of the burnt specimens were too damaged to be worth preserving. In the series preserved there are only three skulls with burnt patches.* No. 485 (♀, Tomb 107) has an extensive area on the occipital and right parietal bones affected (see Plate IIIA), No. 537 (♀, Tomb 120) shows a smaller area on the occipital bone, and No. 681 (immature, Tomb 120) shows a small burnt patch above the right orbit.

It may be noted, too, that there are three trepanned skulls (all males from Tomb 120), and a few artificially deformed skulls from the same tomb. There are two male skulls and one female skull which show definite signs of diseased conditions affecting the bone. Again, they all come from Tomb 120 but it must be remembered that the series from this tomb is far longer than that from any of the other tombs.

The evidence relevant to the provenance of the human remains from the four tombs at Lachish consists partly of archaeological observations, and partly of deductions from the remains preserved. It must be remembered that the series available were selected from a larger bulk of material. Taking the four tombs together, the total remains in hand must represent rather more than 700 individuals. It was estimated that those of about 1500 were present in Tomb 120 when it was opened. As the dimensions of the piles of bones are known approximately, it may be asked whether they could possibly have been made up by bodies, or skeletons, which were complete when deposited. Taking Tomb 120 alone, the dimensions of the pile were roughly: height $1{\cdot}3 \times 7{\cdot}5 \times 8{\cdot}5$ m. (see Fig. 1), giving a volume of about 83 cu.m. It is possible that at least 1500 *complete* skeletons, including some of children, were interred in a space of this size. The tomb was nearly three times as high as the pile of bones, and hence it is also possible that it could have contained a heap of 1500 complete bodies, which might be supposed to have subsided on disintegration. The animal bones, which formed a distinct layer over the human remains, may have been thrown in after such a subsidence. So far there is nothing to preclude the possibility of the interment having been either of bodies, or of complete skeletons, such as might have occurred in clearing a neighbouring cemetery.

The hypothesis that the remains were obtained from a cemetery—so that Tomb 120, and possibly Nos. 107 and 108 as well, were really clearance pits—seems to be unacceptable for several reasons. In the first place, it is almost invariably found that among bones cleared from cemeteries, crania and femora are far better represented than any other parts of the skeleton, and in such a case a pile as large as that in Tomb 120, say, would have been likely to contain far more than 1500 skulls. After clearance, too, it would have been unlikely that any skulls and mandibles would have been associated, though a few pairs were

* It was found that the paraffin wax in which the bones had been dipped could be removed most effectively with the aid of a flame, and although great care was exercised, a few specimens now show signs of slight burning, chiefly at the base of the skull, acquired in the laboratory.

found. Another argument against cemetery clearance is that few skeletons of children are likely to have been thrown in the tombs, on the hypothesis considered, and practically all these would probably have been too crushed in the pile to be worth preserving. Of the total number of skulls in the series, 8·8% are immature, including a fair proportion of children under 10 years of age, and it is unlikely that this frequency would have been so high after two stringent selections of the material. The scarcity of aged specimens also tells against the hypothesis of clearance from a cemetery.

In view of the fact that the skeletons of children are far more easily damaged than those of adults, and that bones of women are more easily crushed than those of men, it is not at all unlikely that the proportions of immature to mature, and of female to male, skeletons in the tombs, were very similar to those of the *living* population of Lachish at a particular time. The rarity of aged specimens must be remembered. These facts seem to tell against the theory that the individuals in the tombs were massacred, but a stronger argument against it is the fact that only one skull showing injury which might have been the cause of death is recorded.

The most plausible explanation, which appears to be in accordance with all the evidence, is the following. It may be supposed that some catastrophe, such as pestilence or earthquake, overtook the population of Lachish about the year 700 B.C., and that a large proportion of the inhabitants were victims. Ordinary burial at such a crisis would have been impossible, and in clearing the town, some time after it, the underground chambers in question would have been convenient depositories into which bodies could have been thrown. The fact that some were burnt is not surprising, as the catastrophe may have been accompanied, or followed, by a fire in part of the town. The fact that the age distribution is appreciably younger than would be anticipated for any cemetery population of the period is accounted for on the hypothesis considered.

3. Remarks on the condition and anomalies (other than dental) of the Lachish crania

Remarks on the Lachish skulls are given in the appended tables of individual measurements. Owing to the incomplete nature of many of the specimens, the totals observable in the case of particular anomalies are generally considerably less than the totals for the whole collection, and hence they have to be given separately in considering the frequencies of different conditions. The total numbers of the skulls from different tombs divided into sexes for the adults, and the numbers of immature specimens, are given on p. 103 above. In general, the frequencies are considered separately in the case of the long series from Tomb 120, and of the series made up by combining the skulls from all the other tombs. Subdivision of the latter is inadvisable, as the numbers from all the tombs other than 120 are small.

Sutures. Remarks on the coronal, sagittal, and lambdoid sutures are given in the tables of individual measurements. If no mention of one of these is made there, this indicates that it shows no sign of closing. Every specimen is complete enough to observe the three principal sutures. The approximate estimate of the age constitution of the series, which can be obtained from these observations, will be considered first. Frequencies are given in Table I. If a suture showed any sign of the beginning of closure, the skull was counted in the second category (sutures beginning to close or partly closed). A suture was counted as closed if synostosis was apparent for at least the greater part of its length.

TABLE I

*The age constitution of the adult Lachish and other series estimated from the state of closure of the principal calvarial sutures (coronal, sagittal, and lambdoid)**

	Series	All sutures open	Sutures beginning to close or partly closed	All sutures closed	Totals
♂	Lachish, Tomb 120	123 (43·3%)	153 (53·9%)	8 (2·8%)	284
	Other tombs†	26 (45·6%)	28 (49·1%)	3 (5·2%)	57
	Total‡	149 (43·7%)	181 (53·1%)	11 (3·2%)	341
	Kerma, 12th and 13th dyn.	16 (11·3%)	83 (58·9%)	42 (29·8%)	141
	Gizeh, 26th–30th dyn.	74 (37·0%)	103 (51·5%)	23 (11·5%)	200
	Farringdon St, Londoners	19 (12·2%)	100 (64·1%)	37 (23·7%)	156
	Whitechapel, Londoners	21 (15·4%)	85 (62·5%)	30 (22·1%)	136
	Spitalfields, Londoners	103 (19·3%)	301 (56·3%)	131 (24·5%)	535
	Hythe	23 (20·9%)	62 (56·4%)	25 (22·7%)	110
♀	Lachish, Tomb 120	154 (74·0%)	54 (26·0%)	0 —	208
	Other tombs	45 (76·3%)	14 (23·7%)	0 —	59
	Total	199 (74·5%)	68 (25·5%)	0 —	267
	Kerma, 12th and 13th dyn.	40 (35·1%)	51 (44·7%)	23 (20·2%)	114
	Farringdon St, Londoners	94 (46·3%)	80 (39·4%)	29 (14·3%)	203
	Whitechapel, Londoners	66 (46·8%)	49 (34·8%)	26 (18·4%)	141
	Spitalfields, Londoners	121 (49·4%)	94 (38·4%)	30 (12·2%)	245
	Hythe	51 (59·3%)	26 (30·2%)	9 (10·5%)	86

* The data in this table for all the series other than the Lachish are taken from Stoessiger & Morant (1932, p. 170) and Collett (1933, p. 259).

† Comprising Tombs 107, 108, 116.

‡ The posthumously and artificially deformed Lachish skulls, and those not included in the pooled series on account of the fact that they show premature closing of the sagittal suture, as well as No. 380 (premature closing of the coronal suture) and No. 382 (of unusual form), are not included in the totals given in this table.

It should be noted that all the data relating to sutures refer to their ectocranial aspects. The endocranial surface of the brain-box could only be observed

in a few specimens, as cleaning was impossible, and hence no remarks on the inside of the skull were recorded.

The division of the adult skulls into the three categories shown in Table I was made with the primary purpose of obtaining a rough estimate of the distribution of age of death for the individuals whose skulls are preserved. It is known that there is considerable variation in the ages at which the different sutures close; a skull with the sutures partly closed, for example, may well have belonged to an older individual than one with all sutures open. The actual age distributions for the separate groups, if they could be known, would doubtless show very considerable overlap. In spite of this it is safe to assume that the percentages may fairly be compared with those given for another series observed in precisely the same way, in order to determine differences in the age constitutions of the samples. The data for the other series given in the table were actually collected by Dr Morant, who confirmed the fact that my records had been obtained in a comparable way.

There are very clear differences between the percentages for the Lachish skulls and those for the other series, while the corresponding values for Tomb 120 and the combined series from the other tombs are remarkably close. The usual

TABLE II

*The frequencies of different orders of closing of the principal calvarial sutures for the Lachish skulls (all tombs)**

	♂	♀
Sagittal closing first, coronal and lambdoid open or closing together	74†	23‡
Sagittal closing first, coronal second and lambdoid last	29	3§
Sagittal closing first, lambdoid second and coronal last	17‖	2
Coronal closing first, sagittal and lambdoid open or closing together	21¶	24
Coronal closing first, sagittal second and lambdoid last	3	6
Lambdoid closing first, sagittal and coronal open or closing together	2	1
Lambdoid closing first, sagittal second and coronal last	2	0
Sagittal and coronal closing together before lambdoid	37	12
Sagittal and lambdoid closing together before coronal	11	1
All three sutures closing together	7	2
Totals	203	74

* Excluding the posthumously and artificially deformed, but including the eleven male and six female skulls showing premature closing of the sagittal suture, and No. 380, showing premature closing of the coronal suture.

† Including eight showing premature closing of the sagittal suture.

‡ Including five showing premature closing of the sagittal suture.

§ Including one showing premature closing of the sagittal suture.

‖ Including three showing premature closing of the sagittal suture.

¶ Including No. 380.

sexual difference, due to the fact that the sutures close at a later age for females than for males, is found. It is evident that the average age at death must have been considerably less for the Lachish adults than for any other of the groups of people. All the other series, except the Whitechapel (obtained from a burial pit, probably used in time of plague), are believed to represent ordinary cemetery populations.

Statistics relating to the orders in which the three principal calvarial sutures were closing are given in Table II. It was possible to observe these orders in the case of all the adult skulls for which closure had commenced, except one aged male (No. 357) in which all three sutures are completely obliterated. The special series of adult specimens presumed to have been affected by premature closing of the sagittal suture, and the one (No. 380) distorted owing to premature closing of the coronal suture, are included in Table II. They were omitted from Table I, as their anomalous conditions might give fallacious estimates of age at death.

The figures show that for the majority of skulls the sagittal suture began to close before the other two, and that the coronal normally closed before the lambdoid. The order sagittal-coronal-lambdoid is also found with the greatest frequency for the Kerma Egyptian series (Collett, 1933). The frequencies for the two series may be summarized in another way, as follows, only skulls for which the first suture to close can be observed being included:

	♂		♀	
	Lachish	Kerma	Lachish	Kerma
Sagittal closing before other two	120 (81·1%)	75 (79·8%)	28 (47·5%)	34 (68·0%)
Coronal closing before other two	24 (16·2%)	18 (19·1%)	30 (50·8%)	14 (28·0%)
Lambdoid closing before other two	4 (2·7%)	1 (1·1%)	1 (1·7%)	2 (4·0%)
Totals	148	94	59	50

The values of the corresponding percentages for the male series are quite insignificant, but differences which must be considered significant are found for the first two pairs of percentages in the case of the females. It should be pointed out that the question which suture shows a more advanced stage of closure than the other two may depend on the age of the individual, and if the rates at which the different sutures close are markedly different comparisons between the frequencies considered for different series may be misleading.

For the English series, examined in the same way, the sagittal suture was found to be the first to close, almost invariably followed by the coronal and lambdoid, which appeared to close together. For a Negro series the coronal

was found to show a slight tendency to close before the sagittal. There are certainly racial differences with regard to the order in which the calvarial sutures close, and, as far as can be seen, the Lachish series agrees with Ancient Egyptian in this respect.

It can be seen from Table II that there are only four male and one female skull showing the lambdoid suture closing before either of the others. An examination of these specimens shows that their sutures were definitely closing in an anomalous way in the majority of cases. Of the four males, Nos. 184, 265 and 330 show the occipito-mastoid suture obliterated on both sides; No. 184 also shows the parieto-mastoid suture obliterated on the left. No. 265 shows the temporal squama completely fused to the parietal on the right, and partly fused on the left; and No. 330 shows both parieto-mastoid sutures obliterated and the temporal squamae largely fused to the parietals. The other male skull with the lambdoid suture closing first (No. 268) has all the sutures between the temporal bones on the one hand, and the occipital and parietal, on the other, open. The female specimen (No. 619) has the occipito-mastoid sutures obliterated, and the posterior parts of the temporal squamae fused to the parietals.

There are five other male skulls, and two other female, showing obliteration of the parieto-mastoid suture, and/or partial or complete fusion of the temporal squamae to the parietal bones. In these cases the principal calvarial sutures are open or closing in a normal order, as far as can be seen, and one (No. 357) have them obliterated. The specimens in question are:

No. 90, ♂. Occipito-mastoid suture *L* obliterated, and posterior half of temporal squama *L* fused to parietal.

No. 171, ♂. Occipito-mastoid suture *L* obliterated, and posterior half of temporal squama *L* fused to parietal.

No. 230, ♂. Temporal squama *R* largely fused to parietal.

No. 262, ♂. Occipito-mastoid suture *L* and parieto-mastoid suture *L* obliterated.

No. 357, ♂. Temporals completely fused to occipital and parietal bones.

No. 383, ♀. Occipito-mastoid sutures, *R* and *L*, and parieto-mastoid suture *L* obliterated.

No. 478, ♀. Occipito-mastoid sutures closed, *R* and *L*, temporal squamae fused to parietals, *R* and *L*.

There is one juvenile specimen (No. 680) showing the right parieto-mastoid suture obliterated, but all others open.

In all there are six male skulls showing the temporal squamae completely, or partially, fused to the parietal bones, three on both sides, three on the left only, and one on the right only; and there are two female skulls showing the

same condition on both sides. A count was made of the number of cases showing the occipito-mastoid suture completely obliterated on one or both sides; the totals are:

	Right and left	Left only	Right only	Total no. of adult skulls examined
♂	6	8	1	351
♀	6	2	1	272

When the condition is unilateral, it appears to be shown more frequently on the left than on the right side, but the numbers are too small to warrant a generalization.

Premature closing of the sagittal suture was noted in the case of eleven male and six female adult skulls, all coming from Tomb 120. These were not included in the series used for comparative purposes, but individual measurements for them are given in a separate section of the appended tables. In the case of six male and four female specimens the sagittal suture is completely obliterated, and the coronal and lambdoid completely open. In the case of the remaining five male and two female specimens the sagittal is completely obliterated and one or both of the other two sutures are closing or closed. Few of these skulls show any clear sign of distortion owing to the abnormal closure of the sagittal suture, the only exceptions being No. 364 (male) and Nos. 667 and 672 (female), these having the lowest cephalic indices. The three can safely be called scaphocephalic, but the forms of the others are not exceptional, and hence it may be presumed that their sagittal sutures did not become synostosed until growth of the brain-box was nearly completed. The cephalic indices of all the specimens showing premature obliteration of the sagittal suture compared with those of the normal series (Tombs 107, 108, 116 and 120 combined) are as under:

		60·05–65·05	65·05–70·05	70·05–75·05	75·05–80·05	80·05–85·05	85·05–90·05	Totals
♂	Normal series	1	26	160	112	11	0	310
	Premature closing of sagittal suture	1*	1	7	2	0	0	11
♀	Normal series	0	7	112	117	15	1	252
	Premature closing of sagittal suture	1*	1*	3	1	0	0	6

* Accepted as scaphocephalic.

There are no brachycephalic skulls among those in the special series, but the indices for the majority of them are clearly not abnormal. The fact that all the

adult specimens showing premature closing of the sagittal suture came from the same tomb (No. 120) is suggestive. It may well be that the condition is of genetical origin, and that the affected individuals were related. This explanation would account for the high proportion of specimens exhibiting the anomaly.

One other female skull (No. 577) has the posterior half of the sagittal suture completely obliterated, but the anterior half, and the coronal and lambdoid sutures, completely open. The specimen also shows a distinct post-coronal depression, though this is not apparent in the median sagittal plane.

There is only one other skull (not included in the normal series) which apparently shows deformation resulting from premature closing of a suture. This is No. 380 (Plate VII F): its coronal suture is almost obliterated, while the sagittal and lambdoid are completely open. This is probably a case of oxycephaly, and, though the deformation is not unlike that which was clearly produced artificially in the case of certain specimens, yet all those showing artificial deformation most clearly have the coronal suture completely open. No. 380 has a height-length ($100\ H'/L$) index of 83·3, whereas the highest for a normal adult male skull of the series is 80·8.

No. 59 (male) shows the superior half of the coronal suture on the right side obliterated, but other parts of this suture open, while the sagittal is beginning to close and the lambdoid is open. It does not appear to be deformed, the height-length index of 75·1 being rather high but by no means extreme. No. 584 (female) shows the right side of the coronal suture closed, while the left side and the sagittal and lambdoid sutures are open.

The complete metopic suture was found for twenty-six of the 341 male adult skulls included in Table I, and slight traces of it were observed in a few other cases: it was also found for twenty-two of the corresponding total of 267 female specimens. There are nineteen of the male, and twenty-one of the female affected individuals showing the metopic and sagittal sutures either both open, or closing together. In the remaining female, and six of the remaining male specimens, either one or the other suture is closing first, but the difference between the state of closure of the two is only slight. The greatest difference in this respect is shown by the last male specimen, which has the sagittal suture closed and the metopic open. The conclusion derived from earlier material that the metopic suture normally closes about the same time as the sagittal, if it persists to an adult stage, is thus confirmed.

The male percentage for the condition is 7·6, and the female 8·2. These values are higher than those for the Kerma Egyptian (4·5 male, and 6·2 female), but rather lower than the percentages usually found for European series. Of the sixty-one immature skulls from Lachish, there are eleven (18·0 %) showing the complete metopic suture, an appreciably higher proportion. Of the eleven male adult skulls not included in the normal series on account of premature closing of the sagittal suture, one is metopic, and of the six female in the same group

two show the condition. Of the seven male skulls which show patent artificial deformation, or a suspicion of this condition, two are metopic, and of the two female, one is metopic. The two specimens which are most clearly deformed (Nos. 381 male, and 673 female) are both metopic, which is curious.

A few other sutural anomalies, besides those treated under *supernumerary bones* below, were noted. Among 267 female skulls, twelve were found with traces of the suture between the ex- and supra-occipital bones, but no trace was found in any male specimen. Only one example of a complete suture across a malar bone was found (No. 154 male), the right malar bone being divided and the left normal. No. 216 (male) shows incomplete fusion of the basi-occipital and basi-sphenoid, although it is almost certainly adult. No. 491 (female) shows the suture between the frontal and sphenoid bones obliterated on both sides, although the coronal suture is open. Several examples of fused nasal bones are noted in the remarks.

The region of the pterion and supernumerary bones. The region of the pterion was examined on all the skulls, and the totals for which the sutures there are visible are given in the table below. Epipteric bones are recorded in the remarks on individual specimens, and, as usual, they are found to be diversified in number and size. The numbers of specimens showing contact between the temporal and frontal bones are given in the following table, the right and left sides being considered separately:

	♂		♀		Immature	
	R	*L*	*R*	*L*	*R*	*L*
Total no. of skulls with sutures at pterion visible on the side in question*	257	258	136	138	61	61
No. of skulls with fronto-temporal articulation	2	3	3	3	1	2
No. of skulls with pterion in *K*	—	—	1	2	—	—

* For the normal series (all tombs), excluding the posthumously and artificially deformed skulls, those showing premature closing of the sagittal or coronal suture, and No. 382. Among all these specimens excluded there is one case of contact between the temporal and frontal bones.

Of the four male skulls with fronto-temporal articulation, one shows the condition on the right side only, two show it on the left side only, and the last shows it on both sides. Of the five female skulls, two show the condition on the right side only, two on the left side only, and for the last it is bilateral. Of the two juvenile specimens, one has fronto-temporal articulation on the left side only, and for the other the condition is bilateral. Of the two female specimens with the pterion in *K*, one case is on the left side only, and for the other the condition is bilateral. The low frequency of cases showing contact between the temporal and frontal bones is comparable with those recorded for European

series. One of the male skulls suspected to be artificially deformed shows fronto-temporal articulation on both sides, but all the others have normal pteria.

Examples of one or more wormian bones between the temporal squamae and parietals were observed with a frequency which appears to be unusually high. There are ten cases of the condition exhibited unilaterally, and five bilaterally, among the male skulls; four unilaterally and four bilaterally for the female; and one unilaterally and one bilaterally among the immature specimens.

The normal series of 341 male skulls (all tombs) includes the following cases of complete or partial division of the occipital bone: *os épactal* (preinterparietal) 2, interparietal bones 8 (only *os pentagonale* and *os triangulare R* separate 2; only *os pent.* separate 2; only, *os tri. R* separate 2; only *os tri. L* separate 1; two large bones meeting below lambda 1—No. 60, Plate XIV B), traces of horizontal suture of interparietal bones near asteria 11. The normal series of 267 female skulls includes the following cases: *os épactal* 3, interparietal bones 2 (only *os tri.* separate 1; only *os pent.* separate 1), traces of horizontal suture of interparietal bones 9. The series of 61 immature specimens include the following cases: *os épactal* 1, only *os pent.* separate 1. All the skulls not included in the normal series, on account of deformation or for other reasons, have normal occipital bones. The percentages of the occurrence of true interparietal bones, of one form or another, are 2·3 for the male adults, and 0·7 for the female adults. These values are close to those given for the Kerma Egyptian series (Collett, 1933, p. 265), viz. 3·6 and 0·9 %, respectively.

One male and one female skull with an ossicle of bregma were noted in the total series. Ossicles of lambda of varying sizes were far more frequent. In general the sutures appeared to be moderately complicated and wormian bones in the lambdoid suture are by no means rare. A few cases of supernumerary bones in the sagittal and coronal sutures were noted, but these are all small, with one exception. This is a male specimen (No. 299), the sagittal suture being closed and the coronal closing. There is a large supernumerary bone in the right side of the coronal suture above the pterion. It is roughly triangular in shape (see Plate XIV A), with its maximum length over 56 mm.—the inferior margin being broken—and its maximum breadth 24 mm. The suture bounding this line anteriorly is symmetrically disposed with regard to the left half of the coronal suture, which is normal, so the additional element appears to occupy part of the area normally occupied by the right parietal bone.

Other anatomical anomalies. The rarest anatomical anomaly noted in the whole series is a case of complete absence of the right auricular passage. This specimen (No. 324) is male and Plate XV A shows the condition of the affected region. There is no opening in the bone taking the place of the right auditory meatus. The corresponding region on the left side is normally formed. Apart from asymmetry in general form—the auricular region of the temporal and

adjoining parts of the base of the skull being more protruding on the left than on the right—there are no clear differences between the two sides of the base of the skull. Hrdlička (1932-3) has discussed similar cases of complete congenital absence of the external auditory meatus and tympanic bone, with reference to seven American specimens, all of which are affected on the right side only.

An orifice in the right temporal squama of a male skull (No. 266) was noted. This is the end of a canal leading through the bone to the interior of the brain cavity. Another male specimen (No. 301) exhibits complete blockage of the left jugular foramen (see Plate XV B), the right foramen being normal.

Wounds. Small healed wounds were found on several of the skulls, and they are noted in the remarks given in the appended tables of individual measurements. One male skull (No. 47) has a healed wound on the left malar bone, but all other injuries noted are on the cranial vault. The most severe examples of completely healed injury are on three male—No. 5 (frontal bone, Plate XVIII A), No. 190 (frontal bone), No. 301 (left parietal)—and four female—No. 419 (frontal, Plate XVIII C), No. 454 (frontal), No. 544 (frontal), and No. 670 (left parietal)—skulls. Only one specimen (No. 108, male, Plate III B), shows wounds which were probably inflicted not long before death. The outer table was completely removed in a region on the left, and another on the right, parietal: the edges of the affected area apparently show signs of healing, but the diploe is still exposed. There are long cracks in the vault of the same skull, but these are probably due to post-mortem injury.

Diseased and other conditions. I am indebted to Dr L. W. Proger and Dr A. M. El Batrawi for commenting on the following and some of the exceptional specimens previously described.

A female cranium (No. 662, Plate XVII) has what appears to be a diseased area extending over the greater part of the right side of the frontal bone. All the bones of the vault are softened and in a bad condition, with numerous small cracks, and the area in question is raised above the general level of the outer table. The roof of the right orbit is also affected and the endocranial surface of the right side of the frontal bone is slightly roughened. The inflammatory condition may possibly be due to osteomyelitis. Another male (No. 1, Plate XVIII B) has a roughened area on the parietal above the mid-point of the right side of the lambdoid suture. There seems to be no doubt that this is a pathological, and not a traumatic, lesion.

A cranium, presumed to be male (No. 382, Plate XVI) is peculiar on account of the "swollen" appearance of its brain-box. This suggests hydrocephaly, but the bones are not exceptionally thin. The basi-occipital is unusually short and broad, so the condition may be due to achondroplasia.

Artificially deformed skulls. There are eight of the Lachish skulls, all from Tomb 120, which are clearly artificially deformed, or which suggest this condition. The most marked cases are Nos. 381 (male) and 673 (female), both of

which happen to be metopic, and they are both young adult (see Plate VI). They are typical examples of fronto-occipital deformation, the two bones being flattened and a post-coronal depression being apparent in each case. Remarks on these two, and on the other six specimens, are in the appended tables of individual measurements. Of the latter, two male specimens (Nos. 376 and 378) show a clear suggestion of both frontal and occipital flattening, while the former shows slight post-coronal depression, but the latter does not. Nos. 377, 379, 380 (male) and 451 (female), show some degree of frontal flattening only, but the occipital bones appear to be perfectly normal: of these No. 379 is the only one showing any suggestion of a post-coronal depression. No. 375 (male) shows some degree of both frontal and occipital flattening but no post-coronal depression. Photographs of all these specimens, except No. 451, are reproduced in Plate VII. It is not possible to assert definitely that any of them were intentionally deformed in childhood. No. 380 has the coronal suture nearly obliterated and the sagittal and lambdoid open, and it is possible that its peculiar form is due solely to premature closing of the anterior suture. Artificial deformation was extremely rare in ancient Egypt, if it was ever practised there at all. There are records of affected specimens of earlier date than the Lachish from other parts of Western Asia, Crete, Cyprus, and some countries of Eastern Europe (see Dingwall, 1931).

Trepanned skulls. The three trepanned skulls have been described by Dr T. Wilson Parry (1936), and new photographs of them are given in Plates IV and V. He says that they are the first specimens exhibiting evidence of this surgical operation to have been found in Asia, and no others have been discovered since in the continent. On two of the skulls (No. 114, an ageing male, and No. 115, a young adult male) a quadrilateral of bone has been removed by sawing, and it is said that there is no evidence that the primitive operation was performed in the same way in any part of the world except Peru. The third skull (No. 340, a young adult male) shows the results of an operation of a different type. It is suggested that the individual had a depressed fracture, and that following this a piece of bone, which had become partly free as a result of the accident, was separated by sawing and removed. He survived long enough to enable the edges of the cavity to become completely healed, while the other two men must have died shortly after the operation. There appears to be no recorded case of a trepanned skull from ancient Egypt, Dr Batrawi (1935, Plate XV) has given a photograph of a young adult female specimen of Meroitic age from Lower Nubia with a circular opening on the right side of the frontal bone, supposed due to a trepan.

4. REMARKS ON THE CONDITION AND ANOMALIES OF THE JAWS OF THE LACHISH SKULLS

Considerably fewer than half of the skulls in the Lachish series have the upper dental arch complete or nearly complete. Table III gives statistics regarding the loss of teeth before death for the adult specimens having complete upper jaws, and for the smaller number of adult mandibles with the dental arch complete. The percentages of cases with no teeth lost before death are high, but it must be remembered that the age constitution, judging from the state of the sutures, indicates a younger group, on the average, than that expected in a cemetery population. The frequencies of adult upper jaws with one or both third molars absent are not unusual, but it is customary to find the female percentage greater than the male. The samples are too small, however, to give a reliable sexual comparison in this respect. Remarks on the condition and anomalies of the upper jaws, and of the few mandibles associated with crania, are given in the appended tables of individual measurements.

TABLE III

The frequencies of different conditions of the teeth in adult jaws having complete dental arcades

	Upper jaw		Lower jaw	
	♂	♀	♂	♀
(i) All teeth including third molars present at death	61	70	13	13
(ii) Third molars apparently absent and no teeth lost before death	6	8	1	4
(iii) One third molar apparently absent and no teeth lost before death	5	0	0	2
(iv) Third molars erupted, or believed erupted, and one or more teeth lost before death	55	29	12	12
(v) Third molars apparently absent, and one or more teeth lost before death	4	2	3	0
(vi) One third molar only apparently absent, and one or more teeth lost before death	1	1	1	1
Total no. of complete arches	132	110	30	32
Total no. with no teeth lost before death: (i), (ii) and (iii)	72 (54·5%)	78 (70·9%)	14 (46·7%)	19 (59·4%)
Total no. with one or both third molars believed unerupted: (ii), (iii), (v) and (vi)	16 (12·1%)	11 (10·0%)	5 (16·7%)	7 (21·9%)

Any dental anomalies of special interest were examined by Mr C. Bowdler Henry, M.R.C.S., and he has kindly allowed me to incorporate his notes in the following descriptions. The skiagrams reproduced in Plates XIX, XXII and XXIII were provided by him.

Deflected canines. There are four adult upper jaws with a canine on one side grossly deflected and not completely erupted.

No. 111, male. The unerupted right canine is misplaced and buried obliquely, so that the tip of the crown is situated in the palatal alveolus, between the sockets of the first and second incisors, and the apex of the root is seen (uncovered by bone) in the external surface of the maxilla over the apex of the socket of the first premolar. The third molars were congenitally absent, and all the other teeth were present and in good condition.

No. 142, male. The unerupted right canine is placed with the tip of its crown close to the anterior palatine fossa, and its root directed superiorly and posteriorly on the palatal side of the roots of the two premolars. It is possible that the left third molar was either abnormally small or else congenitally absent. The region of the right third molar is defective.

No. 401, female: Plate XXI D. The anterior part of the dental arch is damaged, and it is not possible to judge how many teeth were present. The unerupted left canine is placed with the tip of its crown in the anterior palatine fossa, and its root directed externally, posteriorly, and superiorly, towards the malar process of the maxilla, and placed above the apices of the premolars. The right third molar is normal, but it is probable that the left third molar was reduced in size.

No. 659, female. The left side of the palate and the part of the right side anterior to the canine are missing. The unerupted canine is placed with the tip of its crown near the normal position of the alveolus of the lateral incisor, and its root directed externally, posteriorly, and superiorly towards the malar process of the maxilla. The right third molar was probably erupted and lost before death.

Supernumerary denticles. There are two crania and one mandible exhibiting this anomaly.

No. 705, immature. The dental arch is symmetrical, with the left third molar erupting and the right apparently congenitally absent. There is a supernumerary denticle buried in the palate behind the right central incisor (see Plate XXI C). It is lying obliquely, with its crown in the anterior palatine fossa, and its tip impinging against the socket of the left central incisor. The condition of the residual root of the second right incisor suggests that this tooth had been broken during life.

No. 383, female. All the teeth appear to have been present and in good condition at death. There is a small supernumerary denticle placed unusually far back in the middle of the palate, to the left side of the suture (see Plate

XXII C, D). The skiagram shows that the apex of the crown is directed posteriorly.

No. 437, female. In the mandible all teeth were present at death, and there are five dental rudiments embedded in the outer alveolar margin. The positions of these can be seen from Plate XX. In the upper jaw all teeth were present at death, except the left third molar which had been lost. There are diastemae between the central incisors and between the lateral incisors and canines on both sides. There was post-normal occlusion of the jaws, and the teeth were markedly and irregularly worn.

Diastemae. There is one cranium showing diastemae between teeth, in addition to No. 437 described above.

No. 132, male. The third molars appear to have been congenitally absent, and all other teeth were present at death in the upper jaw. There is a diastema between the canine and first premolar on both sides (see Plate XXI B).

Misplaced and missing teeth, and retained milk teeth. In addition to the four examples of deflected canines described above, there are four crania and two mandibles falling in this category.

No. 72, male. The dental arch appears to be sufficiently roomy for the normal dentition, but crowding and some irregularity are present, due to the abnormal persistence of the right milk canine, and to the rotation of the left second premolar (see Plate XXIII C, D). The right third molar appears to have been congenitally absent, and the left is unerupted with the occlusal surface of its crown facing posteriorly and slightly laterally in the posterior wall of the antrum, and in the developmental position of the tooth. The deciduous canine is a well-developed tooth retained in its functional position. Limitation of space has caused the permanent right canine to be slightly rotated. The left second premolar is rotated so that its external, or buccal, cusp is antero-internal, and its internal, or palatal, cusp is postero-external.

No. 445, female. No teeth had been lost before death, but there are only sockets for three in the right side anterior to the second premolar (see Plate XXI A). It is probable that the lateral incisor was missing. The antero-inferior part of the inter-maxillary region is deflected to the right, and the anterior palatine foramen appears to be completely absent.

No. 496, female. The upper left third molar is below the alveolar margin and apparently impacted (see Plate XXII E).

No. 506, female. The dental arch is well formed, and all teeth except three molars are normally erupted. The left third molar was erupting at the time of death. The second and third right molars exhibit an unusual form of delayed eruption (see Plate XXII A, B). Neither tooth had emerged from the gum, although their direction and root formation appear to be normal, judging from the skiagram. The second molar is not impacted against the first, which might have accounted for the delayed eruption, but in fact they are clearly separated.

No. 1039, female. The left third molar of this mandible appears to be slightly "over-erupted", and it has a backward tilt. In the region of the second and third right molars, which are missing, there is a deep pathological excavation, suggestive of an abscess cavity or cyst. The place of the second premolar on the left side, which was congenitally absent, is taken by the retained deciduous second molar. The third molar on the left has an abnormally deep pit where the limbs of the cruciform fissures intersect: the aperture is 1 mm. in diameter. There are two smaller cavities in the occlusal surface of the second molar on the same side.

No. 1068, immature. This mandible had lost no teeth before death. The canine and first premolar on the right are unerupted and completely buried, the premolar lying obliquely towards the canine so that their crowns are in contact (see Plate XXIII A, B). A skiagram shows that the same teeth, unerupted on the other side, are in a similar condition.

Diseased conditions of the jaws. Only the more marked cases of diseases of the jaws were noted. Notes on a mandible (No. 1039), with a large abscess cavity or cyst, are given above, and there are three skulls and two mandibles exhibiting similar conditions.

No. 14, male. All upper teeth were present at death except the lateral incisor and canine on the right side. The shrunken and eburnated condition of the alveolus suggests that these two teeth might have been lost before death through traumatic injury.

No. 467, female. Nearly all teeth had been lost before death, and there is a large cyst in the anterior part of the palate, penetrating to the nasal aperture (see Plate XXIII F).

No. 469, female. All the upper molars had been lost before death, and there is a large abscess cavity or cyst in the molar region on the right side (see Plate XXIII E).

No. 1017, male. No teeth had been lost from the right side of this mandible before death, the left side being defective. There is a large carious cavity in the posterior half of the first molar, which evidently involved the dental pulp. In consequence of this, an abscess formed which discharged through a sinus in the external alveolar plate, over the apex of the posterior root of the tooth.

No. 1051, female. This mandible had lost no teeth before death, the third molars being congenitally absent. The bone of the external surface immediately above the left mental foramen is diseased.

Apparent adventitious filling of a tooth. A female cranium (No. 518) is remarkable on account of the fact that a piece of metal was found firmly embedded in, and level with, the occlusal surface of the second right molar near its centre. The fact that the surface of the metal that could be seen was flat, and that its edges conformed to the surface of the tooth, shows conclusively that it must have been where it was found during the life of the individual. Its appearance

was precisely similar to that of an artificial stopping. Plate XIX reproduces photographs of the whole palate and of the right molar region, before and after the removal of the metal, and a skiagram showing its depth. The filling was removed and its dimensions were found to be 1 × 3 mm. (exposed surface) by 1 mm. (height). The cavity in the tooth, which shows no sign of disease, is almost circular in form, and its position, where the fissures between the cusps join, is one in which a small pit is sometimes found. The description of mandible No. 1039, on p. 120 above, refers to pits of this kind in the occlusal surfaces of a second and third molar. There is a small pit in the surface of the second right molar of skull No. 518. These circumstances suggest that the person accidentally bit the piece of metal, which became lodged in the natural pit of the second molar and worn down as teeth are normally. Such an explanation appears much more plausible than an alternative one which might attribute the filling to surgical interference.

5. THE STATISTICAL NATURE OF THE MATERIAL

The statistical nature of the Lachish series is discussed in this section, topics considered being the question whether the male and female adult and immature samples can be supposed to represent the same population or not, sexual comparisons of average types and variabilities, and allied matters. As the series came from four tombs, which were adjoining, it is advisable to ask first whether there are any significant differences between the series from each, and whether it is legitimate to pool all the material for statistical purposes. In making all metrical comparisons, the specimens patently, or apparently, artificially deformed, those showing premature closing of the sagittal or coronal suture, and the one of unusual (? hydrocephalic) form were excluded. The numbers in these exceptional groups are in the table below, and measurements of the specimens, which are all adult, are given separately in the appended tables of individual measurements.

The numbers of anomalous crania

Tomb		♂	♀
120	Artificially deformed	6	1
	Premature closing of sagittal suture	11	6
	Premature closing of coronal suture	1	—
	Hydrocephalic?	1	—

When seen all together, the series from each tomb showed a variety of "types", but no specimens which clearly stood out from the others on account of their characters were noted, with the exception of two males, viz. No. 156, Tomb 120, and No. 179, Tomb 116. *Norma facialis* views of these two are

given in Plate III C, D, and it will be seen that they are of a very similar type, which is distinctly different from that of the male specimen (Plate X, right), which was selected because none of its measurements are at all peculiar (see p. 163 below). The cephalic index of No. 156 (81·7) is high, but not extreme, and that of No. 179 (78·4) is less exceptional. In spite of these two, it was thought best not to exclude any specimens from the series on account of their appearance.

In making male comparisons between tombs, it was necessary to pool data from Tombs 108 and 116, as the series from them are too small to stand alone. Male and female means are given in Table IV for various series from single tombs, or groups of tombs, in the case of those coefficient of racial likeness characters for which the pooled means for Tombs 107, 108 and 116 are based on more than ten skulls. It will be seen that the series from Tomb 120 is the only one long enough to provide adequate statistical constants. The others are just long

TABLE IV

Mean measurements of series of Lachish crania from different tombs

Character	Male series from tombs				Female series from tombs	
	120	107	108, 116	107, 108, 116	120	107, 108, 116
L	184·3 (266)	185·0 (32)	185·6 (24)	185·2 (56)	176·6 (200)	177·6 (59)
B	137·1 (276)	136·2 (28)	134·6 (23)	135·5 (51)	133·1 (204)	133·8 (57)
B′	95·4 (263)	96·5 (32)	95·1 (24)	95·9 (56)	92·2 (190)	92·1 (55)
H′	133·8 (226)	133·2 (24)	134·3 (18)	133·7 (42)	128·3 (166)	128·6 (47)
S	375·9 (221)	372·7 (20)	372·5 (14)	372·6 (34)	362·9 (164)	363·5 (48)
βQ′	308·7 (255)	307·7 (29)	308·3 (22)	308·0 (51)	297·6 (183)	298·8 (51)
U	518·1 (255)	519·0 (27)	517·1 (22)	518·2 (49)	500·1 (170)	501·4 (44)
fml	37·0 (211)	36·4 (21)	38·0 (15)	37·1 (36)	35·8 (151)	35·9 (42)
fmb	30·5 (213)	29·7 (20)	30·4 (11)	30·0 (31)	29·0 (154)	28·8 (38)
LB	100·6 (205)	101·3 (21)	101·1 (17)	101·2 (38)	96·6 (159)	95·7 (47)
G′H	70·1 (76)	68·8 (14)	72·5 (8)	70·2 (22)	67·2 (59)	66·4 (28)
NB	25·2 (96)	25·0 (18)	24·9 (9)	25·0 (27)	24·6 (62)	24·3 (26)
NH, L	51·3 (103)	51·7 (20)	52·1 (13)	51·8 (33)	49·1 (78)	48·6 (38)
O_1L	41·4 (117)	41·8 (21)	42·0 (10)	41·9 (31)	40·6 (78)	40·4 (33)
O_2L	32·8 (120)	33·6 (19)	33·3 (13)	33·5 (32)	33·1 (78)	33·4 (34)
G_1'	47·0 (72)	45·3 (16)	46·4 (12)	45·7 (28)	45·0 (63)	44·4 (27)
G_2	40·4 (43)	39·8 (8)	39·5 (6)	39·7 (14)	39·2 (48)	39·4 (20)
100 *B/L*	74·5 (259)	73·6 (28)	72·6 (23)	73·1 (51)	75·5 (196)	75·4 (56)
100 *H′/L*	72·7 (215)	72·5 (24)	72·0 (18)	72·3 (42)	72·7 (162)	72·8 (47)
100 *B/H′*	102·5 (219)	103·2 (20)	100·8 (17)	102·1 (37)	103·7 (162)	104·3 (44)
Oc.I.	59·5 (238)	60·0 (25)	59·4 (15)	59·8 (40)	60·1 (166)	59·7 (49)
100 *fmb/fml*	82·9 (195)	81·8 (16)	79·7 (11)	80·9 (27)	81·5 (134)	80·8 (33)
100 *NB/NH*	49·6 (88)	48·7 (17)	47·9 (9)	48·4 (26)	50·3 (59)	49·7 (25)
100 O_2/O_1, *L*	79·3 (113)	80·4 (18)	78·3 (10)	79·7 (28)	81·7 (69)	82·8 (31)
100 G_2/G_1'	85·7 (33)	86·9 (6)	86·0 (5)	86·5 (11)	87·0 (39)	86·5 (16)
N∠	64°·0 (70)	64°·9 (12)	62°·2 (7)	63°·9 (19)	64°·4 (54)	64°·7 (21)
A∠	73°·8 (70)	74°·0 (12)	74°·5 (7)	74°·2 (19)	73°·9 (54)	73°·3 (21)

enough, however, to make comparison of interest. The following crude coefficients of racial likeness,* using standard deviations for the Tomb 120 series alone, were found:

Male, Tomb 120 (177·7†) and Tomb 107 (21·4) ... $-0{\cdot}45 \pm 0{\cdot}19$ (25‡).

Male, Tomb 120 (202·1) and Tombs 108, 116 (16·0) ... $0{\cdot}44 \pm 0{\cdot}21$ (20).

Male, Tomb 107 (23·9) and Tombs 108, 116 (16·6) ... $-0{\cdot}18 \pm 0{\cdot}21$ (20).

Male, Tomb 120 (167·3) and Tombs 107, 108, 116 (34·5) ... $0{\cdot}41 \pm 0{\cdot}18$ (27).

Female, Tomb 120 (116·3) and Tombs 107, 108, 116 (36·8) ... $-0{\cdot}20 \pm 0{\cdot}18$ (29).

It is known from experience that no reliance can be based on comparisons of measurements for short series of skulls. Two series, each being made up by fewer than twenty specimens, and actually representing two distinct races, are quite likely to indicate an insignificant difference, and series of fifty or more are usually required before any reliance can be placed on comparisons such as those given by coefficients of racial likeness. All of the five values above may be considered to differ insignificantly from zero, so the evidence, as far as it goes, indicates that the series from all the tombs may have been random samples from the same population. Larger samples would be required to justify this hypothesis in an adequate way, but for practical purposes there can be no objection to pooling all the material, on the supposition that it represents a single population. This conclusion is in conformity with the archaeological evidence.

It appeared worth while to compare the variabilities of two sub-groups of the total material. All the standard deviations given in Table V are for forty or more skulls. Two of the differences for corresponding constants might be considered just significant if considered by themselves (δ *L*, Δ/P.E. $\Delta = 3{\cdot}4$, and δ S_2, 3·2, the Tomb 120 constant being the greater in both these cases), but the conclusion for all characters must be that the two groups are almost identical in variability.

* With the usual notation, the form of the crude coefficient used is:

$$\frac{1}{M}\Sigma\left(\frac{n_s n_{s'}}{n_s+n_{s'}} \times \frac{(m_s-m_{s'})^2}{\sigma_s^2}\right) - 1 \pm 0{\cdot}67449\sqrt{\frac{2}{M}} = \frac{1}{M}\Sigma(\alpha) - 1 \pm 0{\cdot}67449\sqrt{\frac{2}{M}}.$$

If $\overline{n}_s$ is the mean number of skulls available for the characters used in the case of the first series, and $\overline{n}_{s'}$ is the same for the second series, then the "reduced" coefficient is defined to be:

$$50 \times \frac{\overline{n}_s+\overline{n}_{s'}}{\overline{n}_s\overline{n}_{s'}}\left\{\frac{1}{M}\Sigma(\alpha) - 1\right\} \pm 0{\cdot}67449\sqrt{\frac{2}{M}} \times 50 \times \frac{\overline{n}_s+\overline{n}_{s'}}{\overline{n}_s\overline{n}_{s'}}.$$

† This figure is the average number of skulls for the series ($\overline{n}$), in the case of the comparison in question, no means based on fewer than ten specimens being used.

‡ The number in brackets following a coefficient is the number of characters on which it is based.

TABLE V

Standard deviations for series from Tombs 107, 108 *and* 116 *combined, and from Tomb* 120

Character	Male		Female	
	Tomb 120	Tombs 107, 108, 116	Tomb 120	Tombs 107, 108, 116
L	6·03 ± 0·18*	4·82 ± 0·31	5·03 ± 0·17	5·19 ± 0·32
B	4·95 ± 0·14	5·65 ± 0·38	4·68 ± 0·16	3·95 ± 0·25
B'	4·16 ± 0·12	4·19 ± 0·27	4·53 ± 0·16	3·89 ± 0·25
H'	4·85 ± 0·15	5·73 ± 0·42	4·18 ± 0·15	4·32 ± 0·30
S_1	5·97 ± 0·18	5·84 ± 0·40	5·87 ± 0·20	5·13 ± 0·32
S_2	7·44 ± 0·21	5·96 ± 0·41	7·18 ± 0·25	7·02 ± 0·45
S_3	7·04 ± 0·22	5·89 ± 0·44	6·56 ± 0·24	6·84 ± 0·46
$\beta Q'$	9·54 ± 0·29	9·78 ± 0·65	9·47 ± 0·33	8·54 ± 0·57
U	13·86 ± 0·41	11·55 ± 0·79	12·54 ± 0·46	12·52 ± 0·90
100 B/L	3·02 ± 0·09	3·29 ± 0·22	3·01 ± 0·10	2·91 ± 0·18
100 H'/L	2·88 ± 0·09	3·18 ± 0·24	3·02 ± 0·10	2·51 ± 0·17
Oc.I.	2·62 ± 0·09	2·36 ± 0·18	2·51 ± 0·09	2·30 ± 0·16

* The symbol ± indicates probable errors throughout this paper. They were used instead of standard errors as probable errors have been given far more frequently than standard errors in craniometric studies.

The data for material from all four tombs were pooled, so that a single Lachish series, subdivided according to sex and age, is considered in all comparisons made below. It may be asked, next, whether the total adult male and female series can be supposed to represent the same population or not. The constants for them are given in Table VI.

The corresponding mean measurements of shape—indices and angles—for the two sexes are obviously very close. The differences only exceed three times their probable errors in the case of:

100 B/L (Δ/P.E. $\Delta = 6{\cdot}7$), 100 B/H' (4·5), 100 $(B-H')/L$ (5·0), *Oc.I.* (3·3),

100 O_2/O_1, L (6·5), 100 SS/SC (5·7), 100 fmb/fml (3·4), 100 S/C, ml (5·3).

In the case of the first six of these characters, it is normally found that the male and female means for long series show small differences of the same signs as those observed for the Lachish series. For the last character there is no good comparative material, but the longest series available, described by Woo (1937), shows a sex difference of the same sign. As far as can be told from these comparisons, the male and female Lachish series represent precisely the same population.

TABLE VI

Constants for the male and female series of adult crania from Lachish (all tombs combined)

Character	Means (± P.E.)		Standard deviations		Coefficients of variation	
	Male	Female	Male	Female	Male	Female
C*	1425·1 (108)	1286·5 (89)	—	—	—	—
L	184·5 ± 0·22 (322)	176·8 ± 0·21 (259)	5·88 ± 0·16	5·09 ± 0·15	3·19 ± 0·08	2·88 ± 0·09
B	136·8 ± 0·19 (327)	133·3 ± 0·19 (261)	5·10 ± 0·13	4·55 ± 0·13	3·73 ± 0·10	3·41 ± 0·10
B'	95·5 ± 0·16 (319)	92·2 ± 0·19 (245)	4·26 ± 0·11	4·37 ± 0·13	4·46 ± 0·12	4·74 ± 0·14
H'	133·8 ± 0·20 (268)	128·4 ± 0·23 (213)	5·00 ± 0·15	5·05 ± 0·16	3·74 ± 0·11	3·93 ± 0·13
OH	115·1 ± 0·24 (108)	109·4 ± 0·30 (89)	3·69 ± 0·17	4·22 ± 0·21	3·20 ± 0·15	3·86 ± 0·20
S_1'	112·9 ± 0·18 (299)	108·7 ± 0·18 (248)	4·51 ± 0·12	4·11 ± 0·12	3·99 ± 0·11	3·78 ± 0·11
S_2'	116·0 ± 0·22 (323)	112·1 ± 0·23 (251)	5·88 ± 0·16	5·48 ± 0·16	5·07 ± 0·13	4·89 ± 0·15
S_3'	96·3 ± 0·19 (280)	94·0 ± 0·21 (216)	4·73 ± 0·13	4·52 ± 0·15	4·91 ± 0·14	4·81 ± 0·16
S_1	129·2 ± 0·24 (296)	124·7 ± 0·25 (248)	6·01 ± 0·17	5·75 ± 0·17	4·65 ± 0·13	4·61 ± 0·20
S_2	129·9 ± 0·27 (321)	124·9 ± 0·31 (249)	7·25 ± 0·19	7·16 ± 0·22	5·58 ± 0·15	5·73 ± 0·17
S_3	116·9 ± 0·28 (279)	113·3 ± 0·31 (215)	6·98 ± 0·20	6·68 ± 0·22	5·97 ± 0·17	5·89 ± 0·19
S	375·5 ± 0·54 (255)	363·0 ± 0·53 (212)	12·73 ± 0·38	11·48 ± 0·38	3·39 ± 0·10	3·16 ± 0·10
$\beta Q'$	308·6 ± 0·37 (306)	297·8 ± 0·41 (234)	9·60 ± 0·26	9·31 ± 0·29	3·11 ± 0·08	3·13 ± 0·10
U	518·1 ± 0·52 (304)	500·4 ± 0·58 (214)	13·51 ± 0·37	12·48 ± 0·41	2·61 ± 0·07	2·49 ± 0·08
fml	37·0 ± 0·11 (247)	35·8 ± 0·10 (193)	2·52 ± 0·08	2·14 ± 0·07	6·81 ± 0·21	5·98 ± 0·21
fmb	30·4 ± 0·09 (244)	28·9 ± 0·10 (192)	2·16 ± 0·07	1·99 ± 0·07	7·09 ± 0·22	6·87 ± 0·24
LB	100·7 ± 0·17 (243)	96·4 ± 0·20 (206)	3·82 ± 0·12	4·24 ± 0·14	3·79 ± 0·12	4·40 ± 0·15
GL	94·3 ± 0·34 (89)	90·6 ± 0·36 (76)	4·71 ± 0·24	4·65 ± 0·25	5·00 ± 0·25	5·13 ± 0·28
$G'H$	70·1 ± 0·30 (98)	66·9 ± 0·28 (87)	4·36 ± 0·21	3·94 ± 0·20	6·22 ± 0·30	5·88 ± 0·30
J	128·4 ± 0·48 (49)	121·3 ± 0·46 (40)	4·97 ± 0·34	4·28 ± 0·32	3·87 ± 0·26	3·53 ± 0·27
GB	94·4 ± 0·31 (107)	91·8 ± 0·30 (81)	4·81 ± 0·22	4·04 ± 0·21	5·09 ± 0·24	4·40 ± 0·23
NB	25·2 ± 0·11 (123)	24·5 ± 0·12 (88)	1·74 ± 0·07	1·73 ± 0·09	6·89 ± 0·30	7·49 ± 0·38
NH, L	51·4 ± 0·15 (136)	48·9 ± 0·17 (116)	2·64 ± 0·11	2·70 ± 0·12	5·14 ± 0·21	5·52 ± 0·25
SC	10·5 ± 0·09 (188)	10·0 ± 0·10 (146)	1·80 ± 0·06	1·87 ± 0·07	17·14 ± 0·61	18·63 ± 0·76
SS	5·0 ± 0·06 (186)	4·5 ± 0·07 (134)	1·23 ± 0·04	1·21 ± 0·05	24·40 ± 0·90	27·07 ± 1·19
O_2L	32·9 ± 0·11 (152)	33·2 ± 0·13 (112)	2·05 ± 0·08	2·01 ± 0·09	6·22 ± 0·24	6·05 ± 0·27
O_1L	41·5 ± 0·09 (148)	40·6 ± 0·11 (111)	1·64 ± 0·06	1·65 ± 0·07	3·95 ± 0·15	4·07 ± 0·19
G_1'	46·7 ± 0·19 (100)	44·8 ± 0·21 (90)	2·79 ± 0·13	3·00 ± 0·15	5·98 ± 0·29	6·69 ± 0·34
G_2	40·3 ± 0·20 (57)	39·2 ± 0·19 (68)	2·29 ± 0·14	2·28 ± 0·13	5·69 ± 0·36	5·81 ± 0·34
Ml_1	59·8 ± 0·33 (81)	56·3 ± 0·36 (58)	4·44 ± 0·24	4·11 ± 0·26	7·43 ± 0·40	7·29 ± 0·46
Ml_2	49·8 ± 0·20 (134)	47·2 ± 0·19 (101)	3·35 ± 0·14	2·83 ± 0·13	6·73 ± 0·28	5·99 ± 0·29
$C(ml)$	52·4 ± 0·25 (83)	49·9 ± 0·32 (58)	3·32 ± 0·17	3·56 ± 0·22	6·33 ± 0·33	7·13 ± 0·45
$S(ml)$	11·9 ± 0·12 (83)	10·6 ± 0·13 (58)	1·63 ± 0·09	1·47 ± 0·09	13·72 ± 0·73	13·85 ± 0·88
100 B/L	74·3 ± 0·12 (310)	75·5 ± 0·13 (252)	3·08 ± 0·08	2·99 ± 0·09	—	—
100 H'/L	72·7 ± 0·12 (257)	72·7 ± 0·14 (209)	2·93 ± 0·08	2·92 ± 0·10	—	—
100 B/H'	102·4 ± 0·20 (256)	103·8 ± 0·24 (206)	4·72 ± 0·14	5·11 ± 0·17	—	—
100 $(B-H')/L$	1·7 ± 0·14 (246)	2·8 ± 0·17 (202)	3·25 ± 0·10	3·52 ± 0·12	—	—
$Oc.I.$	59·5 ± 0·10 (278)	60·0 ± 0·11 (215)	2·59 ± 0·07	2·48 ± 0·08	—	—
100 fmb/fml	82·7 ± 0·26 (222)	81·4 ± 0·28 (167)	5·82 ± 0·19	5·31 ± 0·20	—	—
100 $G'H/GB$	74·6 ± 0·39 (63)	72·9 ± 0·45 (59)	4·64 ± 0·28	5·14 ± 0·32	—	—
100 NB/NH, L	49·4 ± 0·25 (114)	50·2 ± 0·28 (84)	4·00 ± 0·18	3·81 ± 0·20	—	—
100 O_2/O_1, L	79·4 ± 0·28 (141)	82·0 ± 0·30 (100)	4·96 ± 0·20	4·50 ± 0·21	—	—
100 G_2/G_1'	85·9 ± 0·50 (44)	86·9 ± 0·52 (55)	4·95 ± 0·36	5·73 ± 0·37	—	—
100 SS/SC	48·6 ± 0·48 (186)	44·2 ± 0·59 (134)	9·71 ± 0·34	10·17 ± 0·42	—	—
100 Ml_2/Ml_1	84·0 ± 0·55 (78)	84·5 ± 0·64 (57)	7·16 ± 0·39	7·19 ± 0·45	—	—
100 $S/C(ml)$	22·8 ± 0·20 (83)	21·2 ± 0·22 (58)	2·72 ± 0·14	2·49 ± 0·16	—	—
$P\angle$	86°·0 ± 0·24 (81)	84°·9 ± 0·28 (62)	3°·14 ± 0·17	3°·23 ± 0·20	—	—
$N\angle$	64°·0 ± 0·24 (89)	64°·5 ± 0·30 (75)	3°·43 ± 0·17	3°·79 ± 0·21	—	—
$A\angle$	73°·9 ± 0·20 (89)	73°·7 ± 0·27 (75)	2°·83 ± 0·14	3°·50 ± 0·19	—	—
$B\angle$	42°·0 ± 0·20 (89)	41°·8 ± 0·23 (75)	2°·77 ± 0·14	2°·90 ± 0·16	—	—

* Determined from reconstruction formulae: see Appendix I.

The few juvenile and more numerous adolescent skulls together, a total of sixty-one unsexed specimens, give the mean indices in the following table, where comparison is made with the male and female adult means.

	Male	Female	Immature
100 *B*/*L*	74·3 ± 0·12 (310)	75·5 ± 0·13 (252)	76·2 (53)
100 *H*′/*L*	72·7 ± 0·12 (257)	72·7 ± 0·14 (209)	72·6 (41)
100 *B*/*H*′	102·4 ± 0·20 (256)	103·8 ± 0·24 (206)	104·0 (39)
100 (*B* − *H*′)/*L*	1·7 ± 0·14 (246)	2·8 ± 0·17 (202)	2·7 (38)
100 *fmb*/*fml*	82·7 ± 0·26 (222)	81·4 ± 0·28 (167)	76·5 (35)
100 *NB*/*NH*	49·4 ± 0·25 (114)	50·2 ± 0·28 (84)	52·0 (22)
100 O_2/O_1, *L*	79·4 ± 0·28 (141)	82·0 ± 0·30 (100)	83·8 (22)

The sequence, male adult—female adult—immature (unsexed) mean, is expected in the case of the last three of these indices. The first four give means for the immature series which differ quite insignificantly from the female adult means, and a close correspondence of this kind is expected. There is thus every reason to believe that the children and adults belonged to the same population.

The sex ratios (male mean/female mean) for a few measurements of size are given in Table VII for the Lachish, three Egyptian, and four English series. All the means involved are based on eighty-six or more crania.

TABLE VII

*Sex ratios for the Lachish and other series**

	Lachish	Egyptian, *E*	Egyptian, Kerma	Egyptian, Denderah	English, Farringdon St	English, Whitechapel	English, Spitalfields	English, Hythe
B	1·027	1·025	1·029	1·031	1·049	1·045	1·050	1·046
S	1·034	1·034	1·039	—	1·046	1·039	1·051	1·040
U	1·035	1·038	1·044	—	1·044	1·041	—	1·038
B′	1·036	1·027	1·040	—	1·038	1·053	1·044	1·041
H′	1·042	1·038†	1·044	1·041	1·059	1·059	1·053	1·054
L	1·043	1·047	1·045	1·044	1·040	1·048	1·052	1·038
LB	1·045	1·053	1·054	1·047	1·045	1·066	1·058	1·053

* The series are: Egyptian *E*, Gizeh, 26th–30th dynasties (Davin & Pearson, 1924); Kerma, 12th–13th dynasties (Collett, 1933); Denderah, 6th–12th dynasties (Thomson & MacIver, 1905); Farringdon St, seventeenth century (Hooke, 1926); Whitechapel, seventeenth century (Macdonell, 1904); Spitalfields, Roman or medieval (Morant & Hoadley, 1931); Hythe, medieval (Stoessiger & Morant, 1932).

† For *H* in place of *H*′. These very similar calvarial heights give almost identical sex ratios in the case of long series for which both are given.

The characters are arranged in the table in order of the sex ratios for the Lachish series. The three Egyptian series give very similar orders, that for the

Kerma being particularly like the Lachish. As is usually found, the preponderance of the average male over the average female skull is clearly different for different measurements, the sex ratios being rather less for cranial diameters than for stature and the lengths of the long bones. It is curious that the orders in which the characters are arranged by the Lachish and Egyptian sex ratios are distinctly different from all those given by the constants for the four English series, while these last show a general, though not very close, correspondence *inter se*. There appears to be a distinct difference between the sex differences of the Lachish and ancient Egyptian races, on the one hand, and the later English ones, on the other.

Measures of variability for the total male and female adult series of crania from Lachish are given in Table VI. Comparison of the relative degrees of variation exhibited by the material for the two sexes may be considered first. For the thirty-three absolute measurements the male standard deviation exceeds the corresponding female value in twenty-four cases, and for the remaining nine the position is reversed. In one or two instances the excess of the male over the female constant is clearly significant, but where the female standard deviation exceeds the male the differences are quite insignificant. In comparing parallel samples representing the same population it is customary to find a slight preponderance of male over female variation for size characters. A closer approach to equality is normally found for these if coefficients of variation are used.

The following comparisons are based on coefficients of variation for absolute (size) characters, and standard deviations for measurements of shape (indices and angles), the total number of characters being fifty. In twenty-four cases the male measure of variation exceeds the female, and for the other twenty-six the position is reversed. The only difference which exceeds three times its probable error is for LB (Δ/P.E. $\Delta = 3{\cdot}2$), but no importance whatever can be attached to a divergence of this order in fifty comparisons. The male and female series thus show a remarkably close approach to equality in variation, and the agreement in this respect again suggests that a single population is represented.

It may be asked next how the variability of this population compares with that of others which are usually accepted as being racially homogeneous. It will be sufficient to make comparisons with the long series of 26th–30th dynasty skulls from Gizeh (Davin & Pearson, 1924), as these have often been used for the purpose. There are thirty-three characters for which the constants required (coefficients of variation for absolute measurements and standard deviations for indices and angles) are available. In the case of the male series the Lachish value exceeds the Egyptian E in fifteen cases, and the Egyptian E exceeds the Lachish in the remaining eighteen. Differences exceeding three times their probable errors are only found in the case of three measurements of shape, viz. 100 B/L (Δ/P.E. $\Delta = 4{\cdot}0$, Lachish σ the greater), $A\angle$(4·1, Egyptian E greater),

and *Oc.I.* (8·3, Egyptian *E* greater). In the case of the female series the Lachish constant exceeds the Egyptian *E* in twenty-three cases, and the Egyptian *E* exceeds the Lachish in the remaining ten. The "significant" differences are for $B\angle$ (3·1), B' (3·8), 100 B/L (4·2) and LB (4·6), the Lachish variability being the greater for these four, and *Oc.I.* (7·2), in which case the Egyptian $E\sigma$ exceeds the Lachish. It is clear that there is a close agreement between the variabilities of the two populations, and, in fact, it is not possible to say that one was more homogeneous than the other. No importance can be attached to any of the differences for single characters, except those for the occipital index, which is decidedly more variable for the Egyptian series in the case of both sexes.

Comparison with data for other series shows that the Lachish standard deviations of the occipital index are quite unexceptional, while those for the Egyptian *E* series are peculiarly large, in view of the fact that all other characters for it indicate a low order of variability. It may be noted that a few of the Lachish crania (excluded from the series) are clearly artificially deformed. If artificial deformation had been generally practised to a slight degree, this would have been expected to increase the variability of the occipital index. Such an effect is not observed, however, and it is very probable that the exceptional cases noted are the only ones exhibiting artificial deformation.

All the statistical evidence thus points to the fact that the male and female adult and immature Lachish series represent random samples from a single population, which showed the same order of variability as ancient Egyptian populations. It is known that these were slightly less variable than Western European populations in later times.

6. The relationships of the Lachish and Ancient Egyptian populations judged from comparisons of mean measurements

The Lachish is the first series of skulls of any period from Palestine which is of an adequate length, and for which adequate measurements are available. A rough comparison of mean measurements for it showed that the type is very similar to that of certain ancient Egyptian series, and this was fully substantiated by statistical comparisons. As far as can be judged from the appearance of the skulls, no clear distinction from dynastic Egyptian material is evident. Comparisons with Egyptian series only are dealt with in this section, and reference is made to other material from Palestine in § 9.

Far more skulls have been preserved and studied from an anthropological point of view from Egypt than from any other country in the world. The vast majority of these relate to Roman and earlier times, and no comparisons with the meagre post-Roman material from the country were made. Most of the

Locality ...	Lachish, Palestine	Badari	Abydos	Naqada *A* and *Q*	El Amrah & Hou	Abyd private
Period ...	700 B.C.	Early predyn.	Early predyn.	Middle predyn.	Late predyn.	1st d
Measured by ...	Risdon	Stoessiger & Derry	Thomson & MacIver	Fawcett	Thomson & MacIver	Thom Mac
$\overline{n}$	184·0	50·3	40·9	66·1	105·6	33
Serial no. ...	1	2	3	4	5	6
1	—	24·12 (29)	18·06 (14)	9·36 (29)	9·62 (14)	8·10
2	24·12 (29)	—	1·72 ± 0·69 (14)	4·02 (31)	7·62 (14)	13·64
3	18·06 (14)	1·72 ± 0·69 (14)	—	0·62 ± 0·52 (14)	3·68 (14)	6·78
4	9·36 (29)	4·02 (31)	0·62 ± 0·52 (14)	—	0·91 ± 0·32 (14)	4·34
5	9·62 (14)	7·62 (14)	3·68 (14)	0·91 ± 0·32 (14)	—	4·26
6	8·10 (14)	13·64 (14)	6·78 (14)	4·34 (14)	4·26 (14)	
7	7·87 (29)	36·24 (31)	33·15 (14)	14·74 (31)	19·44 (14)	16·08
8	2·72 (14)	34·58 (14)	29·36 (14)	21·83 (14)	19·72 (14)	15·32
9	6·57 (14)	7·78 (14)	4·86 (14)	4·93 (14)	6·11 (14)	5·14
10	10·62 (29)	23·89 (29)	30·74 (14)	13·77 (30)	30·69 (14)	17·70
11	13·53 (27)	3·71 (28)	4·45 (14)	3·91 (28)	8·55 (14)	8·17
12	4·62 (14)	10·34 (14)	4·99 (14)	2·38 (14)	2·79 (14)	4·22
13	7·86 (29)	7·77 (31)	5·65 (14)	1·11 ± 0·21 (31)	1·15 ± 0·19 (14)	3·84
14	1·29 ± 0·32 (14)	25·41 (14)	21·24 (14)	13·05 (14)	11·59 (14)	8·44
15	5·06 (14)	7·91 (14)	5·73 (14)	2·50 ± 0·52 (14)	3·19 (14)	9·36
16	7·91 (25)	41·42 (28)	47·51 (14)	22·91 (28)	28·15 (14)	27·37
17	2·13 (25)	15·15 (29)	19·72 (14)	10·26 (28)	10·08 (14)	8·19
18	5·27 (26)	18·19 (30)	20·78 (14)	9·49 (27)	10·85 (14)	12·09
19	5·74 (29)	33·53 (31)	38·32 (14)	14·35 (30)	23·50 (14)	24·79
20	1·24 ± 0·23 (14)	23·78 (14)	20·16 (14)	9·44 (14)	7·85 (14)	8·61
21	3·31 (14)	29·08 (14)	25·35 (14)	17·96 (14)	14·82 (14)	17·37
22	6·89 (26)	12·29 (24)	12·10 (14)	6·16 (28)	6·81 (14)	12·50

* The $\overline{n}$ for a particular series is the average

TABLE VIII

Reduced coefficients of racial likeness between the Lachish, Ancient Egy

u	Abydos, private tombs 1st dyn. Thomson & MacIver 33·6 6	Abydos, royal tombs 1st dyn. Motley 32·0 7	Deshasheh & Medum 4th and 5th dyn. Thomson & MacIver 39·9 8	Denderah 6th–12th dyn. Thomson & MacIver 168·6 9	Sedment 9th dyn. Woo 37·1 10	El Kubanieh, N. Middle dyn. Toldt 32·5 11	Abydos & I 12th–15th d Thomson MacIver 65·9 12
)	8·10 (14)	7·87 (29)	2·72 (14)	6·57 (14)	10·62 (29)	13·53 (27)	4·62
)	13·64 (14)	36·24 (31)	34·58 (14)	7·78 (14)	23·89 (29)	3·71 (28)	10·34
)	6·78 (14)	33·15 (14)	29·36 (14)	4·86 (14)	30·74 (14)	4·45 (14)	4·99
)	4·34 (14)	14·74 (31)	21·83 (14)	4·93 (14)	13·77 (30)	3·91 (28)	2·38
	4·26 (14)	19·44 (14)	19·72 (14)	6·11 (14)	30·69 (14)	8·55 (14)	2·79
)	—	16·08 (14)	15·32 (14)	5·14 (14)	17·70 (14)	8·17 (14)	4·22
)	16·08 (14)	—	7·73 (14)	21·34 (14)	17·03 (31)	21·93 (28)	11·75
)	15·32 (14)	7·73 (14)	—	12·42 (14)	5·07 (14)	13·60 (14)	10·50
)	5·14 (14)	21·34 (14)	12·42 (14)	—	12·25 (14)	1·46 ± 0·46 (14)	2·68
)	17·70 (14)	17·03 (31)	5·07 (14)	12·25 (14)	—	7·83 (28)	13·25
)	8·17 (14)	21·93 (28)	13·60 (14)	1·46 ± 0·46 (14)	7·83 (28)	—	3·20 ± 0·56
)	4·22 (14)	11·75 (14)	10·50 (14)	2·68 (14)	13·25 (14)	3·20 ± 0·56 (14)	—
)	3·84 (14)	15·52 (31)	19·56 (14)	5·85 (14)	16·14 (31)	6·27 (28)	5·78
)	8·44 (14)	2·08 ± 0·64 (14)	1·96 ± 0·58 (14)	11·08 (14)	13·12 (14)	14·84 (14)	7·08
)	9·36 (14)	18·48 (14)	11·36 (14)	4·44 (14)	15·70 (14)	4·67 (14)	0·22 ± 0·41
)	27·37 (14)	8·82 (29)	8·21 (14)	29·63 (14)	10·39 (28)	28·59 (26)	19·45
)	8·19 (14)	12·03 (29)	2·39 ± 0·59 (14)	7·66 (14)	8·38 (29)	7·44 (28)	6·56
)	12·09 (14)	14·00 (30)	7·29 (14)	9·41 (14)	9·18 (30)	8·31 (27)	8·03
)	24·79 (14)	15·80 (31)	4·50 (14)	22·64 (14)	14·20 (31)	18·71 (28)	14·68
)	8·61 (14)	4·31 (14)	8·12 (14)	9·36 (14)	18·12 (14)	13·90 (14)	3·81
)	17·37 (14)	8·87 (14)	8·05 (14)	12·96 (14)	15·55 (14)	16·55 (14)	6·81
)	12·50 (14)	17·83 (28)	10·21 (14)	5·60 (14)	8·99 (28)	8·08 (25)	5·79

the average number of skulls for the characters used in computing the coefficient for the series which is based on the largest nu

III

, *Ancient Egyptia*

N.	Abydos & Hou 12th–15th dyn. Thomson & MacIver 65·9 12	
7)	4·62	(14)
8)	10·34	(14)
4)	4·99	(14)
8)	2·38	(14)
4)	2·79	(14)
4)	4·22	(14)
8)	11·75	(14)
4)	10·50	(14)
4)	2·68	(14)
8)	13·25	(14)
	3·20 ± 0·56	(14)
4)	—	
8)	5·78	(14)
4)	7·08	(14)
4)	0·22 ± 0·41	(14)
6)	19·45	(14)
8)	6·56	(14)
7)	8·03	(14)
8)	14·68	(14)
4)	3·81	(14)
4)	6·81	(14)
5)	5·79	(14)

n the largest number

TABLE VIII

Reduced coefficients of racial likeness between the Lachish, Ancient Egyptian and allied ser

ydos, e tombs dyn. nson & clver 3·6 6	Abydos, royal tombs 1st dyn. Motley 32·0 7	Deshasheh & Medum 4th and 5th dyn. Thomson & MacIver 39·9 8	Denderah 6th–12th dyn. Thomson & MacIver 168·6 9	Sedment 9th dyn. Woo 37·1 10	El Kubanieh, N. Middle dyn. Toldt 32·5 11	Abydos & Hou 12th–15th dyn. Thomson & MacIver 65·9 12	Kerma 12th–13th dyn. Collett 107·3 13
(14)	7·87 (29)	2·72 (14)	6·57 (14)	10·62 (29)	13·53 (27)	4·62 (14)	7·86 (29)
(14)	36·24 (31)	34·58 (14)	7·78 (14)	23·89 (29)	3·71 (28)	10·34 (14)	7·77 (31)
(14)	33·15 (14)	29·36 (14)	4·86 (14)	30·74 (14)	4·45 (14)	4·99 (14)	5·65 (14)
(14)	14·74 (31)	21·83 (14)	4·93 (14)	13·77 (30)	3·91 (28)	2·38 (14)	1·11 ± 0·21 (31)
(14)	19·44 (14)	19·72 (14)	6·11 (14)	30·69 (14)	8·55 (14)	2·79 (14)	1·15 ± 0·19 (14)
—	16·08 (14)	15·32 (14)	5·14 (14)	17·70 (14)	8·17 (14)	4·22 (14)	3·84 (14)
(14)	—	7·73 (14)	21·34 (14)	17·03 (31)	21·93 (28)	11·75 (14)	15·52 (31)
(14)	7·73 (14)	—	12·42 (14)	5·07 (14)	13·60 (14)	10·50 (14)	19·56 (14)
(14)	21·34 (14)	12·42 (14)	—	12·25 (14)	1·46 ± 0·46 (14)	2·68 (14)	5·85 (14)
(14)	17·03 (31)	5·07 (14)	12·25 (14)	—	7·83 (28)	13·25 (14)	16·14 (31)
(14)	21·93 (28)	13·60 (14)	1·46 ± 0·46 (14)	7·83 (28)	—	3·20 ± 0·56 (14)	6·27 (28)
(14)	11·75 (14)	10·50 (14)	2·68 (14)	13·25 (14)	3·20 ± 0·56 (14)	—	5·78 (14)
(14)	15·52 (31)	19·56 (14)	5·85 (14)	16·14 (31)	6·27 (28)	5·78 (14)	—
(14)	2·08 ± 0·64 (14)	1·96 ± 0·58 (14)	11·08 (14)	13·12 (14)	14·84 (14)	7·08 (14)	10·17 (14)
(14)	18·48 (14)	11·36 (14)	4·44 (14)	15·70 (14)	4·67 (14)	0·22 ± 0·41 (14)	7·33 (14)
(14)	8·82 (29)	8·21 (14)	29·63 (14)	10·39 (28)	28·59 (26)	19·45 (14)	18·66 (29)
(14)	12·03 (29)	2·39 ± 0·59 (14)	7·66 (14)	8·38 (29)	7·44 (28)	6·56 (14)	8·09 (29)
(14)	14·00 (30)	7·29 (14)	9·41 (14)	9·18 (30)	8·31 (27)	8·03 (14)	7·77 (30)
(14)	15·80 (31)	4·50 (14)	22·64 (14)	14·20 (31)	18·71 (28)	14·68 (14)	15·25 (31)
(14)	4·31 (14)	8·12 (14)	9·36 (14)	18·12 (14)	13·90 (14)	3·81 (14)	7·09 (14)
(14)	8·87 (14)	8·05 (14)	12·96 (14)	15·55 (14)	16·55 (14)	6·81 (14)	13·82 (14)
(14)	17·83 (28)	10·21 (14)	5·60 (14)	8·99 (28)	8·08 (25)	5·79 (14)	5·01 (28)

number of skulls for the characters used in computing the coefficient for the series which is based on the largest number of characters. A

*and allied series of male skulls**

Kerma	Abydos	Sheikh Ali	Abydos	Thebes	Thebes	
12th–13th dyn. Collett	18th dyn. Thomson & MacIver	18th dyn. Thomson & MacIver	18th–19th dyn. Schmidt	18th–20th dyn. Stahr	18th–21st dyn. Schmidt	26t
107·3	49·9	40·0	29·9	54·0	161·9	
13	14	15	16	17	18	
7·86 (29)	1·29 ± 0·32 (14)	5·06 (14)	7·91 (25)	2·13 (25)	5·27 (26)	5·7
7·77 (31)	25·41 (14)	7·91 (14)	41·42 (28)	15·15 (29)	18·19 (30)	33·5
5·65 (14)	21·24 (14)	5·73 (14)	47·51 (14)	19·72 (14)	20·78 (14)	38·3
1·11 ± 0·21 (31)	13·05 (14)	2·50 ± 0·52 (14)	22·91 (28)	10·26 (28)	9·49 (27)	14·3
1·15 ± 0·19 (14)	11·59 (14)	3·19 (14)	28·15 (14)	10·08 (14)	10·85 (14)	23·5
3·84 (14)	8·44 (14)	9·36 (14)	27·37 (14)	8·19 (14)	12·09 (14)	24·7
15·52 (31)	2·08 ± 0·64 (14)	18·48 (14)	8·82 (29)	12·03 (29)	14·00 (30)	15·8
19·56 (14)	1·96 ± 0·58 (14)	11·36 (14)	8·21 (14)	2·39 ± 0·59 (14)	7·29 (14)	4·5
5·85 (14)	11·08 (14)	4·44 (14)	29·63 (14)	7·66 (14)	9·41 (14)	22·6
16·14 (31)	13·12 (14)	15·70 (14)	10·39 (28)	8·38 (29)	9·18 (30)	14·2
6·27 (28)	14·84 (14)	4·67 (14)	28·59 (26)	7·44 (28)	8·31 (27)	18·7
5·78 (14)	7·08 (14)	0·22 ± 0·41 (14)	19·45 (14)	6·56 (14)	8·03 (14)	14·6
—	10·17 (14)	7·33 (14)	18·66 (29)	8·09 (29)	7·77 (30)	15·2
10·17 (14)	—	10·97 (14)	11·97 (14)	2·87 ± 0·49 (14)	8·92 (14)	6·2
7·33 (14)	10·97 (14)	—	21·08 (14)	8·40 (14)	6·62 (14)	13·0
18·66 (29)	11·97 (14)	21·08 (14)	—	5·13 (26)	4·91 (29)	3·4
8·09 (29)	2·87 ± 0·49 (14)	8·40 (14)	5·13 (26)	—	2·91 (25)	3·8
7·77 (30)	8·92 (14)	6·62 (14)	4·91 (29)	2·91 (25)	—	7·5
15·25 (31)	6·23 (14)	13·02 (14)	3·48 (27)	3·81 (27)	7·56 (28)	
7·09 (14)	1·47 ± 0·42 (14)	6·05 (14)	8·19 (14)	2·70 (14)	4·34 (14)	7·1
13·82 (14)	5·89 (14)	7·70 (14)	4·53 (14)	2·75 ± 0·49 (14)	1·61 ± 0·33 (14)	5·7
5·01 (28)	12·95 (14)	4·30 (14)	13·04 (25)	5·25 (30)	3·83 (24)	7·1

f characters. A probable error is only given if a coefficient differs from zero by less than six times its probable error.

Thebes 18th–21st dyn. Schmidt 161·9 18	Gizeh 26th–30th dyn. Davin & Pearson 855·4 19	Denderah Ptolemaic Thomson & MacIver 76·6 20	Denderah Roman Thomson & MacIver 49·3 21	Tigré, Abyssinia Modern Sergi 61·8 22	
5·27 (26)	5·74 (29)	1·24 ± 0·23 (14)	3·31 (14)	6·89 (26)	1
18·19 (30)	33·53 (31)	23·78 (14)	29·08 (14)	12·29 (24)	2
20·78 (14)	38·32 (14)	20·16 (14)	25·35 (14)	12·10 (14)	3
9·49 (27)	14·35 (30)	9·44 (14)	17·96 (14)	6·16 (28)	4
10·85 (14)	23·50 (14)	7·85 (14)	14·82 (14)	6·81 (14)	5
12·09 (14)	24·79 (14)	8·61 (14)	17·37 (14)	12·50 (14)	6
14·00 (30)	15·80 (31)	4·31 (14)	8·87 (14)	17·83 (28)	7
7·29 (14)	4·50 (14)	8·12 (14)	8·05 (14)	10·21 (14)	8
9·41 (14)	22·64 (14)	9·36 (14)	12·96 (14)	5·60 (14)	9
9·18 (30)	14·20 (31)	18·12 (14)	15·55 (14)	8·99 (28)	10
8·31 (27)	18·71 (28)	13·90 (14)	16·55 (14)	8·08 (25)	11
8·03 (14)	14·68 (14)	3·81 (14)	6·81 (14)	5·79 (14)	12
7·77 (30)	15·25 (31)	7·09 (14)	13·82 (14)	5·01 (28)	13
8·92 (14)	6·23 (14)	1·47 ± 0·42 (14)	5·89 (14)	12·95 (14)	14
6·62 (14)	13·02 (14)	6·05 (14)	7·70 (14)	4·30 (14)	15
4·91 (29)	3·48 (27)	8·19 (14)	4·53 (14)	13·04 (25)	16
2·91 (25)	3·81 (27)	2·70 (14)	2·75 ± 0·49 (14)	5·25 (30)	17
—	7·56 (28)	4·34 (14)	1·61 ± 0·33 (14)	3·83 (24)	18
7·56 (28)	—	7·14 (14)	5·73 (14)	7·12 (26)	19
4·34 (14)	7·14 (14)	—	1·20 ± 0·43 (14)	8·74 (14)	20
1·61 ± 0·33 (14)	5·73 (14)	1·20 ± 0·43 (14)	—	7·29 (14)	21
3·83 (24)	7·12 (26)	8·74 (14)	7·29 (14)	—	22

imes its probable error.

longer series of male Egyptian skulls representing populations of the period considered have been compared statistically by Morant (1925). A selection of the material with which he dealt was made, and certain series were excluded for various reasons, viz.:

(i) Fouquet's predynastic—the so-called Aeneolithic—series from Middle Egypt. The male means of size characters for this series are decidedly larger than those for any other predynastic series dealt with in Morant's paper, or described since, while its mean indices are very close to those for some contemporary series. It appears to be very probable that the distinction was due, either to a process of selection which favoured the larger specimens, or to inaccurate sexing. While the series is suspect it appears better not to use it for comparative purposes.

(ii) The series measured by Broca and Chantre, for which the means are unsatisfactory as they are not recorded in fine enough units.

(iii) Four other series, each comprising fewer than thirty skulls.

(iv) The El Kubanieh South series, measured by Toldt, which relates to a period covering Early and Middle Dynastic times, while all the other series relate to shorter intervals.

The modern Abyssinian series measured by Sergi, and reduced by Morant, is included in our list. This accounts for eighteen of the series used. The three other ancient Egyptian series included are the 9th dynasty from Sedment (Woo, 1930); the Kerma (Collett, 1933)—from a Nubian site, although the population was unquestionably of Egyptian type—and the pooled early predynastic Badari series (Morant, 1935), measured by Stoessiger (1927) and Derry. Including the Lachish, there is thus a total of twenty-two male series, made up by about 3000 skulls. The localities from which the collections were obtained are shown in the sketch-map, Fig. 2.

Reduced coefficients of racial likeness between all pairs of the twenty-two series are given in Table VIII. Many of these were obtained from the crude coefficients given by Morant (1925), and others were copied from tables given by Woo (1930), and Collett (1933). In addition to these I was able to use a number of unpublished values calculated by Morant, and to complete the table I calculated all the coefficients with the Lachish series and several others which were wanting. The $\overline{n}$ for a particular series in the table gives the average number of skulls on which the means are based, in the case of all the characters used in computing the coefficients which are available for the series. The $\overline{n}$'s range from 29·9 to 855·4, and there are only six, including the Lachish, greater than 100. Experience has suggested that a series of fifty complete skulls of one sex, or a larger number of incomplete specimens providing an equivalent amount of evidence, is required in order to give reliable racial comparisons with

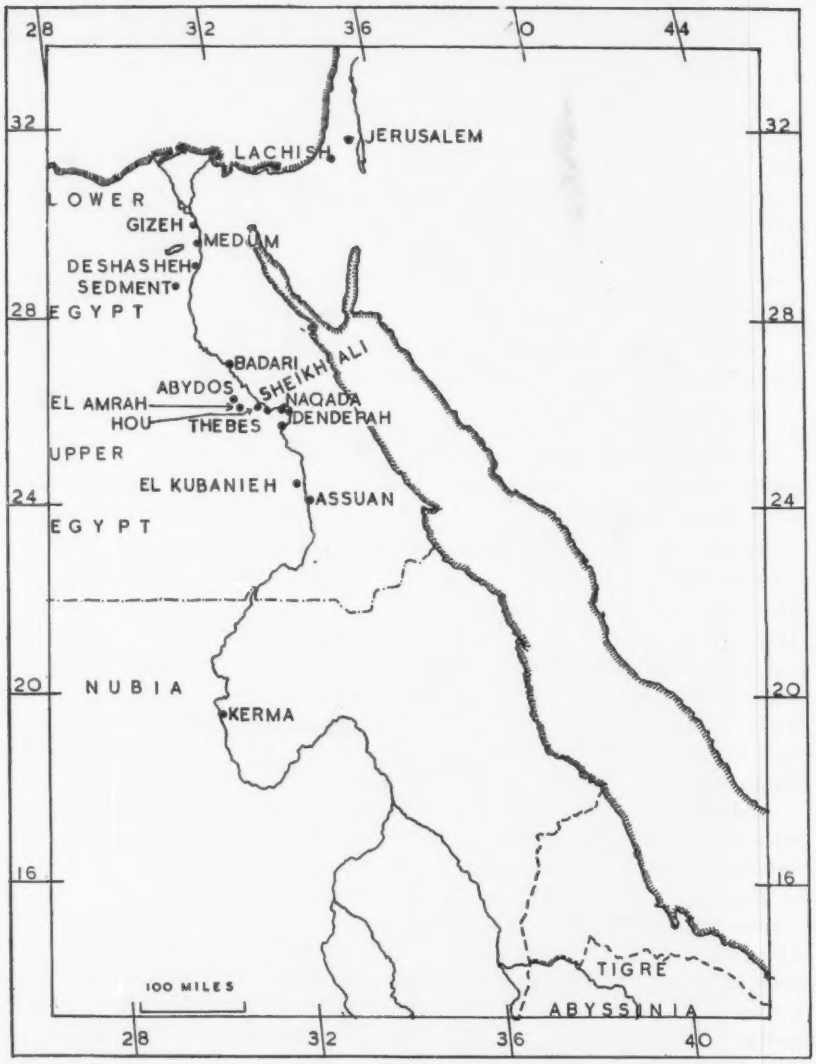


Fig. 2. Sketch-map of Egypt and neighbouring countries showing localities from which cranial series were obtained.

a particular population. Of the twenty-two series, there are eight which fail to satisfy this ideal requirement, but these may be supposed large enough to give fairly reliable comparisons.

When possible coefficients of racial likeness between cranial series have been based on a standard set of thirty-one characters, nineteen of these being absolute measurements (chords and arcs), seven indices, and three angles. For the Egyptian material considered it was only possible to use the complete set in a few cases, owing to the fact that one or several of the measurements are not available for the majority of the series. In all cases as many as possible of the selected characters were used in computing the coefficients. The distribution of the numbers of characters on which the 231 coefficients could be based is:

No. of characters	14	24	25	26	27	28	29	30	31
No. of comparisons	165	2	5	5	5	16	14	7	12

More than half of the coefficients are thus based on only fourteen characters, all these being comparisons of the ten series measured by Thomson and MacIver with one another, and with the remaining twelve series. The coefficients between these twelve are all based on a number of characters which can be supposed adequate, but it may be suggested that comparisons between them and those for which only fourteen characters are involved are likely to be unreliable. The question of how far the generalized estimate of resemblance for fourteen characters is likely to diverge from that obtained from about twice as many was investigated by computing some of the more reliable coefficients for the incomplete set of fourteen characters available for Thomson and MacIver's series. Comparisons of the corresponding reduced coefficients for (*a*) all of the characters available, and (*b*) for the fourteen characters only, are made in Table IX, in the case of fifteen pairs of series selected at random.

For seven of the fifteen comparisons the reduced coefficient for fourteen characters is greater than the corresponding value for the larger number, and markedly greater in four cases; for the remaining eight comparisons the position is reversed, but there are no marked differences between the pairs. It appears that the generalized estimate of divergence derived from the fourteen characters can usually be expected to give a fairly close approximation to that which would be obtained from about twice as many characters, but occasionally it will suggest a rather misleading conclusion. In the case of the material considered, the fourteen characters show a tendency to indicate a wider separation of the types than that likely to be found if more evidence were available, and this might have been anticipated from the fact that they include the characters which normally show the most significant differences in the comparisons of Ancient Egyptian material, viz. B, $100\,B/L$, J and $N\angle$.

TABLE IX

Comparisons of corresponding pairs of reduced coefficients of racial likeness (male series) for all coefficient characters available and for a set of fourteen characters

	Reduced coefficients of racial likeness	
	All characters	14 characters
Lachish (191·5*) with Thebes, 18th–20th dyn. (54·1)	2·13 ± 0·23 (25)	0·01 ± 0·30
Naqada *A* and *Q* (64·9) with El Kubanieh North (33·1)	3·91 ± 0·41 (28)	6·41 ± 0·58
Abydos, 18th–19th dyn. (31·5) with Thebes, 18th–20th dyn. (54·1)	5·13 ± 0·48 (26)	8·76 ± 0·64
Lachish (191·5) with Thebes, 18th–21st dyn. (167·9)	5·27 ± 0·11 (26)	3·23 ± 0·14
Lachish (191·5) with Gizeh, 26th–30th dyn. (869·5)	5·77 ± 0·06 (29)	5·37 ± 0·08
Lachish (191·5) with Abyssinian (62·6)	6·89 ± 0·20 (26)	4·25 ± 0·27
Lachish (191·5) with Kerma (107·6)	7·86 ± 0·13 (29)	7·78 ± 0·19
Abyssinian (62·6) with El Kubanieh North (33·1)	8·08 ± 0·44 (25)	5·56 ± 0·59
Sedment (37·5) with Thebes, 18th–21st dyn. (167·9)	9·18 ± 0·29 (30)	11·09 ± 0·42
Thebes, 18th–20th dyn. (54·1) with Abydos, 1st dyn. Royal (33·2)	12·03 ± 0·44 (29)	10·63 ± 0·62
Abyssinian (62·6) with Abydos, 18th–19th dyn. (31·5)	13·04 ± 0·46 (25)	19·37 ± 0·61
Sedment (37·5) with Abydos, 18th–19th dyn. (31·5)	13·12 ± 0·60 (28)	13·97 ± 0·73
Naqada *A* and *Q* (64·9) with Gizeh, 26th–30th dyn. (869·5)	14·35 ± 0·14 (30)	19·03 ± 0·21
Lachish (191·5) with Badari (61·1)	24·12 ± 0·22 (29)	21·07 ± 0·27
Badari (61·1) with Abydos, 1st dyn. Royal (33·2)	36·24 ± 0·44 (31)	45·33 ± 0·59

* The numbers in brackets following the titles of the series give the average numbers of skulls ($\bar{n}$'s) in the case of the fourteen coefficient of racial likeness character available for all the series. In the case of the comparisons for "all characters" the $\bar{n}$'s are changed, but they are invariably of the same order. Fuller particulars regarding the series are given in Table VIII.

The point in question can be examined in an indirect way by comparing the distributions of reduced coefficients in Table VIII based (*a*) on fourteen characters, and (*b*) on twenty-four to thirty-one characters. These are:

	Insignificant*	Significant and <3·5	3·5–5	5–13	13–20	20–30	30–40	40–50	Totals
14 characters	9	16	18	72	27	17	5	1	165
24–31 characters	0	4	6	31	17	5	2	1	66
All comparisons	9	20	24	103	44	22	7	2	231

* A coefficient is counted here as insignificant if it differs from zero by less than 3·5 times its probable error.

Allowing for the differences in their sizes, these two distributions appear to be very similar, and it is clear that the coefficients for fourteen characters only do not show any marked tendency to diverge from the others on the average.

It is curious that there are no insignificant coefficients based on twenty-four or more characters, but little importance need be attached to this fact.

In computing all the reduced coefficients of racial likeness given in Table VIII, the Egyptian *E* standard deviations (Davin & Pearson, 1924) were used. It is thus assumed that the variabilities of all the populations are so closely similar that any one of them can be adequately represented by the constants for the longest series of the group available. It has been shown above that the Lachish standard deviations are very similar to the corresponding values for the *E* series. The constants for three of the other series—viz. the Badari, Sedment, and Kerma—have been compared with the Egyptian *E* in the same way, and it was found that the variabilities indicated by them are also of almost precisely the same order. At the same time it is known that the Ancient Egyptian series show a clear tendency to be rather less variable than Western European series of later times.

In the comparison of any two series the full coefficient of racial likeness involves both sets of standard deviations. The question of the extent to which the assumption that the population variabilities were precisely the same is likely to affect reduced coefficients was examined in an indirect way in the case of a few comparisons. For twenty-nine of the comparisons reduced coefficients happened to be available computed by using both (*a*) the Egyptian *E* σ's, and (*b*) the Farringdon St (seventeenth-century London, Hooke, 1926) σ's. In every case the former value is greater than the latter, as would have been anticipated, and the ratio of the reduced coefficients found with Egyptian *E* σ's to the corresponding values found with Farringdon St σ's, range from 1·05 to 1·79, while for 21 of the 29 ratios, the range is between 1·3 and 1·6. The Farringdon St variablities are obviously inappropriate for use in comparisons of Ancient Egyptian material.

For the ten cases giving the largest ratios the reduced coefficient was also computed using the Lachish σ's, and the values for the three sets are given in Table X. The ratios of coefficients found with the Egyptian *E* σ's to the corresponding values found with the Lachish σ's range from 0·91 to 1·17. For seven of the ten comparisons the use of the Lachish σ's gives the lower value, but all the differences here are so small that they may be considered of no importance.

The reduced coefficients of racial likeness which were used in this paper to provide a classification of a number of Ancient Egyptian and related populations are clearly not statistical estimates of an ideal kind. Owing to the nature of the material, certain devices have to be used in practice which are likely to have an appreciable affect on the coefficients obtained. It has been shown above that there is good reason to believe that the use of a constant set of standard deviations does not distort the value to any appreciable extent. The use of different sets of the complete group of thirty-one characters appears to be of far more practical importance. More than half of the 231 coefficients computed are

actually based on fourteen characters only, the numbers for the remainder ranging from twenty-four to thirty-one. Comparisons with the shorter series might have been omitted, but this would have reduced the evidence to such an extent that it appeared better to include them, while remembering their imperfection. There are other objections to the constants. It is known that the theoretical requirement that all the characters used should be uncorrelated with one another

TABLE X

Corresponding reduced coefficients of racial likeness for male cranial series computed by using (1) *the Egyptian E*, (2) *the Lachish, and* (3) *the Farringdon St standard deviations*

	No. of characters	With Egyptian E σ's	With Lachish σ's	With Farringdon St σ's
Abyssinians with Abydos (Early predyn.) ($\bar{n}$ 62·6) ($\bar{n}$ 40·9)	14	12·60 ± 0·52	10·76 ± 0·52	7·64 ± 0·52
Abyssinians with El Amrah and Hou (Late predyn.) ($\bar{n}$ 62·6) ($\bar{n}$ 105·6)	14	7·04 ± 0·32	6·33 ± 0·32	4·61 ± 0·32
El Amrah and Hou (Late predyn.) with Thebes (18–20 dyn.) ($\bar{n}$ 105·6) ($\bar{n}$ 54·1)	14	10·08 ± 0·36	8·86 ± 0·36	6·61 ± 0·36
Abydos (Early predyn.) with Abydos (12–15 dyn.) ($\bar{n}$ 40·9) ($\bar{n}$ 65·9)	14	4·99 ± 0·51	4·34 ± 0·51	3·13 ± 0·51
Naqada A and Q with Abydos (12–15 dyn.) ($\bar{n}$ 64·9) ($\bar{n}$ 65·9)	14	2·38 ± 0·39	2·39 ± 0·39	1·33 ± 0·39
Thebes (18–20 dyn.) with Thebes (18–21 dyn.) ($\bar{n}$ 53·1) ($\bar{n}$ 166·4)	23	1·43 ± 0·24	1·57 ± 0·24	0·90 ± 0·24
Naqada A and Q with Thebes (18–20 dyn.) ($\bar{n}$ 66·9) ($\bar{n}$ 53·4)	27	8·27 ± 0·30	7·08 ± 0·30	5·18 ± 0·30
Thebes (18–20 dyn.) with Egyptian E ($\bar{n}$ 53·2) ($\bar{n}$ 866·0)	25	3·37 ± 0·18	3·45 ± 0·18	2·35 ± 0·18
Denderah (Roman) with Abydos (12–15 dyn.) ($\bar{n}$ 49·3) ($\bar{n}$ 65·9)	14	6·81 ± 0·45	5·88 ± 0·45	4·34 ± 0·45
Thebes (18–20 dyn.) with Deshasheh (4–5 dyn.) ($\bar{n}$ 54·1) ($\bar{n}$ 39·9)	14	2·39 ± 0·56	2·25 ± 0·56	1·42 ± 0·56

is not fulfilled, some pairs of the characters being almost certainly quite highly correlated in all the series, but it is probable that the relative values of the estimates of resemblance are not more disturbed by this than by some of the other factors. In the case of a particular measurement used, comparisons are only made between mean readings said to have been obtained by following a particular definition of the measurement. Even so, the personal equations of different measurers may have been large enough, in some cases, to disturb the results quite appreciably. It is not possible to investigate this matter in the case of all the material used, but it must be recognized that the disturbance due to it is probably quite sufficient to invalidate any rigid application of statistical rules to the material. Mistakes in sexing the crania provide another possible source of error, though probably this is one of minor importance.

It is not at all likely, of course, that the extent to which a particular coefficient is distorted is determined by the accumulation of the various kinds of disturbing factors mentioned. One may hope that they will usually counteract one another. In view of all the circumstances, it is obvious that no great reliance can be placed on the numerical accuracy of any particular reduced coefficient of racial likeness, or on the comparison of any pair of the constants. It would be most inadvisable, for example, to compare the difference between any pair with regard to the probable error of this difference. It is not unreasonable, however, to accept the coefficients as indicating different, rather broad, grades of resemblance, and it is in this sense that they are used for purposes of classification here, in conformity with previous biometric practice.

The classification of the series treated in this section is based on reduced coefficients of racial likeness for pairs of the male series: the female series are in most cases shorter, but they may be expected to lead to very similar conclusions. Corresponding male and female values for the Lachish compared with six other series are given in Table XI. For the first of these comparisons the difference

TABLE XI

*Corresponding male and female reduced coefficients of racial likeness for comparisons of the Lachish with other series**

		Abydos 18th dyn.	Thebes 18th–20th dyn.	Abydos and Hou 12th–15th dyn.
		♂ (40·0) ♀ (66·1)	♂ (53·8) ♀ (43·5)	♂ (65·9) ♀ (87·4)
Lachish	♂ (184·0) ♀ (147·1)	1·29 ± 0·32 (14) 1·05 ± 0·28 (14)	2·13 ± 0·23 (24) 6·37 ± 0·29 (24)	4·62 ± 0·26 (14) 2·51 ± 0·23 (14)

		Egyptian *E* 26th–30th dyn.	Denderah 6th–12th dyn.	Kerma 12th–13th dyn.
		♂ (859·5) ♀ (569·7)	♂ (168·6) ♀ (140·4)	♂ (107·5) ♀ (84·2)
Lachish	♂ (184·0) ♀ (147·1)	5·74 ± 0·06 (29) 6·87 ± 0·08 (29)	6·57 ± 0·14 (14) 8·67 ± 0·17 (14)	7·86 ± 0·13 (29) 12·66 ± 0·17 (29)

* The series compared in this table with the Lachish, are ones for which fuller particulars are given in Table VIII. The numbers in brackets after the sex signs are the $\bar{n}$'s for the numbers of skulls on which the means used in calculating the coefficients are based. The $\bar{n}$'s given for the Lachish series are for the twenty-nine characters on which two of the comparisons are based, and its values for the other groups of characters are very close to these.

between the male and female coefficient is of no account, but for some of the others the difference is appreciable, even in the case of the Lachish compared with the Egyptian *E* series, for which all the means involved are based on an adequate number of skulls. The estimates of resemblance are in fact rather different for the male and female comparisons, though there is a fairly close agreement in their orders, except in the case of the comparison with the 18th–20th dynasty series from Thebes. The means for this series suggest that the groups of male and female skulls composing it may not have represented precisely the same population, and any slight discordance of this kind is likely to have an appreciable effect on the comparisons considered. Errors in sexing are also likely to affect them to some extent. We are again led to the conclusion that there is no justification for attaching any importance to small differences between coefficients of racial likeness.

The conclusions regarding the racial affinities of the populations represented by the twenty-two Ancient Egyptian and allied series may now be considered. Reduced coefficients of racial likeness between all pairs of them are given in Table VIII, and the arrangement suggested by the lowest orders of these constants is shown in Fig. 3. Three sets of series, representing races in other parts of the world, have been treated in the same way: the results for a North American set have been published by von Bonin & Morant (1938), for an Asiatic set by Woo & Morant (1932), and for a European set by Morant (1928). For this last material crude coefficients of racial likeness only are given for forty-one series, four of which relate to North African peoples. I am indebted to Morant for the corresponding reduced coefficients, which he has calculated but which have not been published. The four North African series were excluded, so the numerical results to be considered refer to thirty-seven European series.

It is of interest to compare the distributions of the reduced coefficients for the new Egyptian and the three other sets. These are given in Table XII. There are marked differences between the ranges of the constants for the four sets of material. On the average the Egyptian shows much lower values than the other three sets, the highest reduced coefficient for it being 47·5, while for each of the others more than one quarter of the coefficients exceed 30 and several values greater than 100 are found.

The relationships of the modern races of man are of such a kind that they can be roughly divided into groups, so that all the members of a particular group show close resemblances, while there are links between the different groups. This conception of constellations of races, as it were, which are joined to one another is undoubtedly useful, though it is often difficult to give precision to it, owing to the existence of subgroups within the major groups. The distributions in Table XII suggest that the selected set of twenty-two Ancient Egyptian and allied series may be considered to represent a single constellation of populations which had very similar types, while each of the other three sets

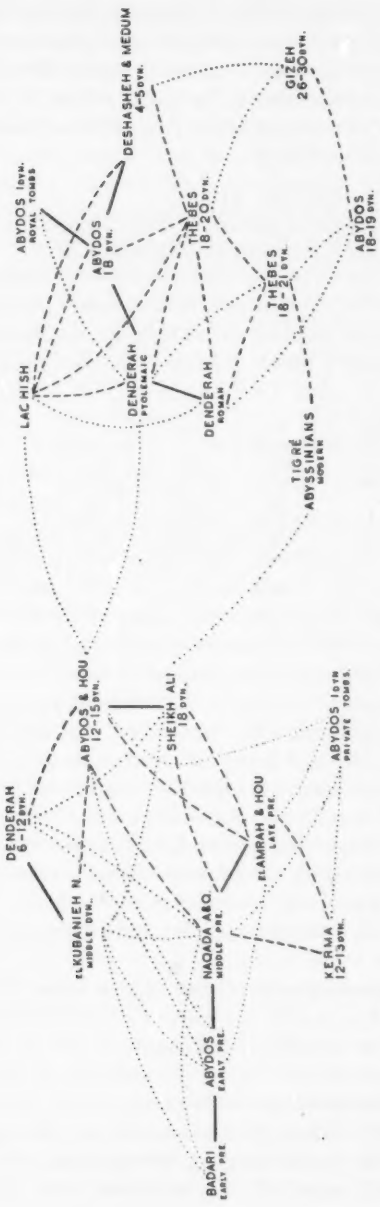


Fig. 3. Reduced coefficients of racial likeness between Ancient Egyptian and allied series of male skulls.

represents two or more constellations. The Egyptian series actually relate to peoples who lived from early predynastic to modern times, a span of about 7000 years, but there is no suggestion that the group changed radically within this period either as the results of evolutionary change, or owing to infiltration by alien peoples. The area represented extends from Abyssinia to Palestine, though nearly all the material comes from Egypt.

TABLE XII

Distributions of reduced coefficients of racial likeness for various groups of male cranial series

Groups of series	No. of series	Coefficients of racial likeness							Total comparisons
		<5	5–13	13–50	50–100	100–200	200–400	400–430	
Ancient Egyptian and allied	22	53 (22·9%)	103 (44·6%)	75 (32·5%)	—	—	—	—	231
European	37	33 (5·0%)	102 (15·3%)	362 (54·4%)	135 (20·3%)	34 (5·1%)	—	—	666
North American Indian	16	2 (1·7%)	9 (7·5%)	59 (49·2%)	45 (37·5%)	5 (4·2%)	—	—	120
Asiatic	26	7 (2·2%)	23 (7·1%)	102 (31·4%)	110 (33·8%)	56 (17·2%)	26 (8·0%)	1 (0·3%)	325

It is known from experience that in the cross comparisons of two groups of cranial series, representing two distinct families of races (A and B), some lower reduced coefficients will be found between them than *some* for pairs of series belonging to group A, or than *some* for pairs of series belonging to group B. In comparisons of the Ancient Egyptian with North American Indian series, for example, it would be anticipated that a certain proportion of the coefficients would be below 50, though the majority would doubtless exceed this limit. The peculiarity of the generalized measure of resemblance employed must be supposed due to the peculiar nature of the material, and it need not be taken to indicate any defect in the method. A classification based on such generalized estimates of resemblance must evidently proceed on certain lines, and the way in which the constants can most usefully be employed for the purpose has to be determined empirically.

Some of the cross comparisons $A \times B$ referred to give lower reduced coefficients than some for pairs of A's, or some for pairs of B's, but at the same time they have always been found to exceed a certain limit, so that the lowest order of coefficients are not represented by any $A \times B$ comparisons. This suggests that any classification of the material ought to be derived solely from coefficients less than the limit in question, i.e. from the evidence of close resemblance only, such as is never found, as far as experience goes, between any two populations belonging to distinct families of races.

The earlier studies of groups of cranial series treated by the method of the coefficient of racial likeness suggested that the limit in question might be taken as a reduced coefficient of about 20, so that the classification would be based on all values less than 20, while all greater than this limit would be neglected entirely. Recent experience of the cross comparisons between North American Indian and Asiatic series (von Bonin & Morant, 1938) has shown that it is safer to employ a considerably lower limiting value, which was provisionally taken at 13. If the particular set of material treated is made up by a considerable number of very closely related series, a clearer picture of the interrelationships is obtained, however, by taking a still lower limiting value.

In the present case the limit was chosen arbitrarily at 5·0, that is to say the evidence presented by all reduced coefficients less than 5·0 is taken into account —bringing in 23 % of the total of 231 comparisons—while all values greater than 5·0 are neglected. It should be realized that a reduced coefficient less than 5·0 represents very close similarity of the cranial types compared, such as has hitherto only been found in cases where close relationship between the populations represented was anticipated, from the cultural (historical or archaeological) evidence.

The grades of resemblance indicated by reduced coefficients of racial likeness less than 5·0 are shown in Fig. 3. The full lines represent "insignificant" coefficients—i.e. values which differ from zero by less than 3·5 times their probable errors. In these cases the two series might have represented precisely the same population, as far as can be seen from the direct comparison, though, owing to the imperfection of the material, no stress can be laid on the difference between an "insignificant" coefficient and one of the same order which indicates clear differentiation judged by the ratio of the constant to its probable error. The second grade of resemblance is represented by coefficients which differ significantly from zero, but which are less than 3·5, and the third by values between 3·5 and 5·0.

It should be stressed that any coefficients of racial likeness less than 5·0 indicate a very close resemblance of the cranial types compared. There are two adequately long male series of seventeenth-century London skulls, from Whitechapel and Farringdon St, and the coefficient between them is 3·8 (see Morant & Hoadley, 1931, p. 233), so a value of this order can only be taken to show the degree of divergence which may be found between parochial subgroups of the same population.

The arrangement shown in Fig. 3 was arrived at by placing the series, as far as possible, in position relative to one another so that their distances apart are proportional to the estimates of resemblance considered. One of the series treated—viz. that of the 9th dynasty skulls from Sedment—is not shown, because the lowest reduced coefficient found with it is 5·1 (with the 4th–5th dynasty series from Deshasheh and Medum).

There is a clear suggestion of a division into two groups.

Group A includes eight series from sites in Upper Egypt (covering a stretch of the Nile, about 100 miles in length, from Badari in the north to Thebes in the south—see Fig. 2), a series from El Kubanieh (100 miles south of Thebes), and a Nubian series from Kerma. The period represented by these ten series extends from early predynastic times to the 18th dynasty.

Group B includes seven series from the same area in Upper Egypt, two from Lower Egypt, one from Abyssinia, and the Lachish from Palestine. The Upper Egyptian series here range in time from the 1st dynasty—only represented by a single series of skulls from Royal Tombs—to Roman times. The two Lower Egyptian series and the Lachish (of about 25th dynasty date) fall within the same period, and the Abyssinian series is of modern date.

The population of Upper Egypt is represented by fifteen of the twenty-two series, representing a time sequence from early predynastic to Roman times. All the series of earlier date than the 18th dynasty except one fall in group *A*, and show very close interconnexions. At the same time there is a clear suggestion that the type was changing gradually with time, the earliest predynastic (Badari) series being at one extreme, and the middle dynastic series at the other. The only aberrant Upper Egyptian series of a date prior to the 18th dynasty is one from Royal Tombs at Abydos, and it may be supposed that these represent an intrusive group coming from Upper Egypt. The fact that the early Upper Egyptian type persisted almost unchanged until the 18th dynasty is evidenced by the series of this date from Sheikh Ali.

There are four Upper Egyptian series of the 18th–21st dynasties—two from Abydos and two from Thebes—which are quite distinct from the Upper Egyptian (*A*) group of series, and which must hence be supposed to represent mainly an intrusive population in the region. Immediately before the 18th dynasty the whole of Egypt was under the Hyksos dominion, and when this lapsed at the end of the 17th dynasty the country was in an unsettled state. It may be noted that if there were immigrants into Upper Egypt at this time, they were most likely to have been of the ruling classes, who would have settled and been buried in the principal towns in Upper Egypt, viz. Abydos and Thebes. The population of Upper Egypt in later times is only represented by two series from Denderah, one of Ptolemaic and the other of Roman date. The types of these two are very similar to those of the presumed intrusive groups of the 18th–21st dynasties, though they stand somewhat closer to the indigenous Upper Egyptian types. This may obviously be explained as being due to some slight degree of intermixture between the intrusive group and the settled population of Upper Egypt between the 18th dynasty and Ptolemaic times.

If an explanation of this kind is on the right lines, then it becomes necessary to discover the source of the people who are presumed to have migrated to Upper Egypt about the time of the 18th dynasty. The relationships of the

cranial series in Fig. 3 provide a clear answer to this question, as they show intimate connexions between two late Upper Egyptian series and two of the three Lower Egyptian series available, viz. the 4th–5th dynasty series from Deshasheh and Medum, and the 26th–30th dynasty from Gizeh, forming with them what is here called the *B* group.

Unfortunately, the evidence available is quite inadequate to provide an outline of the racial history of Lower Egypt. A third series from that region, viz. one from Sedment of 9th dynasty skulls, is not shown in the diagram, because its lowest reduced coefficient exceeds 5·0; this is 5·1 with the 4th and 5th dynasty series from Deshasheh and Medum, sites close to Sedment. The three Lower Egyptian series are very similar to one another, and they stand on the same side, as it were, of all the Upper Egyptian material. Another factor which has to be taken into account in interpreting the evidence is that the Upper Egyptian type was apparently becoming gradually modified, *prior* to the 18th dynasty, in the direction of the Lower Egyptian type. The evidence suggests the following general conclusions:

(1) It may be supposed that originally the populations of Upper and Lower Egypt formed two distinct groups, the purest representatives of these known being the Early predynastic from Badari, and the 4th–5th dynasty series from Deshasheh and Medum. These are the earliest series available from Upper and Lower Egypt, respectively. For convenience, the groups may be called, following Morant, the Upper (corresponding to our Group *A*), and the Lower (Group *B*) Egyptian types. From the earliest times until about the end of the 17th dynasty, the type of the Upper Egyptian population became *gradually* modified in the direction of that of the Lower Egyptian. This may be supposed due to a gradual infiltration of Lower Egyptians into Upper Egypt. The relationships of the Kerma series from Nubia are of particular interest in this connexion. It is of 12th–13th dynasty date, but its closest connexions are with two predynastic series from Upper Egypt. Hence it may be supposed that the Kerma people represent the descendants of colonists who left Upper Egypt in predynastic times, and that this stock remained stable, and was not modified by intermixture, though the parent group itself was changing owing to contacts with the North.

There is only one Upper Egyptian series of earlier date than the 18th dynasty which stands apart from the constellation formed by all the others. This is of 1st dynasty skulls from Royal Tombs at Abydos, and as their type is very similar to the Lower Egyptian, it may be supposed that they represent an intrusive group from that region. This group, which was probably small, may have been absorbed in the Upper Egyptian population without affecting the type of the latter appreciably.

(2) The situation in Upper Egypt appears to have changed radically in the 18th dynasty. There is one Upper Egyptian series of this date (from Sheikh Ali)

which is very similar to the earlier series from that region, but four others (ranging from the 18th to the 21st dynasty) stand quite apart and show close relationships with the Lower Egyptian series. The movement from Lower to Upper Egypt appears to have been greatly accelerated about the time of the 18th dynasty. The evidence suggests that at this time it virtually became a peaceful invasion of Upper Egypt, which resulted in the almost complete displacement of the earlier population there. This view is supported by the fact that there are no series whatever later than the 18th dynasty of the early Upper Egyptian type, while two late series from Denderah are still of Lower Egyptian type, though somewhat closer to the early Upper Egyptian than are the 18th–21st dynasty series from Thebes and Abydos. These relationships suggest that the prevailing population in Upper Egypt after the 18th dynasty was of Lower Egyptian origin, and that it became mixed to some extent with descendants of the earlier population of the region.

The fact that a modern population from the north of Abyssinia stands between the two Ancient Egyptian groups is interesting, but far more evidence would obviously be required to elucidate the significance of this relationship.

The relationships of the Lachish series may now be considered. All its closest connexions are with series of the Lower Egyptian type, and these are close enough to suggest that the population in the Palestinian town was entirely, or almost completely, of Egyptian origin. The skeletal material from Lachish is believed to represent people who died about 700 B.C., which is the time of the 25th dynasty in Egypt. The series shows closest resemblance, however, not to the 26th–30th dynasty series from Gizeh, but to the early dynastic series from Lower Egypt (Deshasheh and Medum), and to three Upper Egyptian series of 18th dynasty or later dates which are assumed to represent populations of Lower Egyptian origin. There is also a rather less close connexion between the Lachish and one of the Upper Egyptian types (Abydos and Hou, 12th–15th dynasty). These relationships suggest that the Lachish population represents descendants of a colonizing group of men and women, which was derived primarily from Upper Egypt, at some time later than the 18th dynasty, and which maintained its type unchanged—free from intermixture—until 700 B.C. or later It is not at all unlikely, of course, that a colonial group in Palestine, such as the one which settled in Lachish, originally included people from several parts of Egypt, and not from a single restricted locality. The characteristics of the Lachish sample give no hint of heterogeneity, however, which may be due to the fact that any diversity which originally existed in the group was obscured by intermarriage within the community for several generations. The evidence suggests quite clearly that the Lachish people were derived principally from a population of Upper Egypt which was itself derived principally from emigrants who left Lower Egypt about the time of the 18th dynasty.

7. COMPARISONS OF THE LACHISH AND ANCIENT EGYPTIAN SERIES FOR CHARACTERS CONSIDERED SINGLY

Comparison of the Lachish and a number of Ancient Egyptian and allied cranial series, based on the averages for a number of characters considered in conjunction, have been made in the preceding section. The means for the same material will now be compared for characters considered singly.

A convenient way of summarizing the statistical comparisons of means for groups having a number of features recorded, is provided by considering the α's calculated in computing the coefficients of racial likeness. An α is an approximation to the square of a quantity which is the difference of the means divided by the standard error of this difference. If an α is greater than 10, it may be supposed that the difference between the two means is clearly significant. The percentages of α's greater than 10 may be used to distinguish characters which are practically constant for all the groups from those which frequently denote differentiation.

The twenty-one comparisons between the Lachish and each of the other series may be considered first. The numbers of α's available for this set range for the different characters from seven to twenty-one, only fourteen of the thirty-one coefficient of racial likeness characters being available for all the series.* These fourteen may be considered first, and they give the following grouping:

(*a*) characters showing a high proportion of significant differences:

B (percentage of α's $> 10 = 47{\cdot}6$), $100\,B/L$ (42·9), and $100\,B/H'$ (42·9);

(*b*) characters showing a lesser proportion of significant differences:

$N\angle$ (28·6), H' (23·8), NH (23·8), J (19·0), and $100\,H'/L$ (19·0);

(*c*) characters showing few significant differences:

L (14·3), $100\,NB/NH$ (9·5), $A\angle$ (9·5), $G'H$ (4·8), NB (4·8), and LB (0·0).

It is clear that some characters can be supposed practically constant for all the series, while others show many significant differences.

The remaining characters which can be treated in the same way are only available for numbers varying from eight to twelve of the twenty-two series. The numbers of comparisons are very restricted for these, but they suggest that S, O_2, G'_1, $100\,G'H/GB$, $100\,G_2/G'_1$, and $P\angle$ are practically constant for all the series, while the Lachish is most frequently differentiated by B', *Oc.I.*, *fml*, *fmb*, U, and rather less frequently by G_2, O_1, $100\,O_2/O_1$ and $100\,fmb/fml$.

Table XIII gives the ranges of certain male means for the Upper Egyptian group of series (i.e. the ten on the left-hand side of Fig. 3), the means for the Lachish

* The transverse arc ($\beta Q'$) for the Lachish series is not available for any of the others, and the estimated cranial capacities were not included, so the maximum number of characters used in computing coefficients with the Lachish series is 29.

alone, and the ranges for the remaining nine comprising the Lower Egyptian group (i.e. all on the right-hand side of Fig. 3 except the Lachish and Abyssinian). There is seen to be complete separation of the ranges for the two sets of series, with the Lachish falling within the Lower Egyptian limits, in the case of B, B', U, and 100 B/H', and a close approach to the same condition in the case of J and 100 B/L. All these measurements are transverse breadths, or indices including a breadth, and there is no doubt that the values of the reduced coefficients of racial likeness are largely determined by differences between them. Owing to the use of correlated measurements, the differences in cranial breadth affect the generalized measure of resemblance unduly. This factor was apparently showing far more significant variation in Ancient Egyptian populations than any other relating to the cranium.

TABLE XIII

Ranges of mean measurements for two groups of Ancient Egyptian male skulls and the Lachish means

Series	Period	*B*	*J*	*B'*
Upper Egyptian type	Early predyn.–18th dyn.	131·4–134·3 (10)*	123·6–127·5 (8)	90·4–92·8 (4)
Lachish	*ca.* 25th dyn.	136·8	128·4	95·5
Lower Egyptian type	1st dyn.–Roman	135·3–139·3 (9)	127·5–131·3 (8)	93·0–96·2 (5)

Series	Period	*U*	100 *B/L*	100 *B/H'*
Upper Egyptian type	Early predyn.–18th dyn.	500·0–510·4 (4)	71·7–73·7 (10)	98·1–101·1 (10)
Lachish	*ca.* 25th dyn.	518·1	74·3	102·4
Lower Egyptian type	1st dyn.–Roman	510·8–518·7 (5)	73·7–76·0 (9)	102·3–106·4 (9)

Series	Period	*L*	*H'*	*N*∠
Upper Egyptian type	Early predyn.–18th dyn.	182·2–185·2 (10)	132·4–135·9 (10)	64°·9–67°·0 (8)
Lachish	*ca.* 25th dyn.	184·5	133·8	64°·0
Lower Egyptian type	1st dyn.–Roman	181·4–185·8 (9)	130·7–136·0 (9)	63°·4–66°·1 (8)

Series	Period	*fml*	*fmb*	*Oc.I.*
Upper Egyptian type	Early predyn.–18th dyn.	34·9–36·4 (4)	28·7–30·1 (4)	60·2–62·3 (4)
Lachish	*ca.* 25th dyn.	37·0	30·5	59·5
Lower Egyptian type	1st dyn.–Roman	35·1–37·0 (5)	29·7–30·2 (5)	59·9–61·5 (3)

* The numbers in brackets indicate the number of series to which the ranges relate.

The remaining characters treated in the table make no clear distinctions between the two sets of series, and the same situation is observed in the case of all the other coefficient of racial likeness characters. The Upper Egyptian series show a slight tendency to be more prognathous than the Lower Egyptian (judging by $N\angle$), but the two ranges overlap appreciably. It may be noted that the foraminal length and breadth of the Lachish type are extremely large, and its occipital index is extremely low compared with those for the other series.

The majority of the measurements not used in computing the coefficients appear to be fairly constant for all the Ancient Egyptian series, and the Lachish means fall within their ranges. This is so far the bimaxillary breadth (GB), in spite of the clear differences found between the bizygomatic and calvarial breadths. The index 100 $(B\text{-}H')/L$ does make a clear distinction between the Upper and Lower Egyptian sets of series, but this again appears to be due to the fact that a breadth measurement is involved.

The simotic measurements, giving estimates of the "flattening" of the nasal bridge, are only available for a few series. Comparison with the data for these given by Woo & Morant (1934), shows that the breadth of the nasal bones (SC) is unexceptional for the Lachish type, but the subtense (SS) and index (100 SS/SC) are decidedly larger for it than for the Badari, Kerma, Sedment, and an Ancient Nubian type. The greater curvature of the nasal bones in the Lachish skulls places them within the range found for European populations.

Comparative material for the malar bone measurements is still more restricted. Comparison with the data given by Woo (1937) suggests that the Lachish means are unexceptional, but for the index measuring the curvature of a horizontal section of the bone (100 S/C, *ml*). For this the Palestine series has a mean which is decidedly greater than those for the two Egyptian series, and this mean places it close to the extreme for the types from all parts of the world hitherto described. For the vast majority of measurements the Lachish skulls are not distinguishable as a group from series representing Ancient Egyptian populations. The most distinguishing features of the type appear to be the exceptional curvature of the nasal bridge and horizontal sections of the malar bones. This divergence is suggestive, but more abundant comparative material would be needed to assess its significance.

8. THE CONTOURS OF THE LACHISH SERIES

Contours of adult skulls in the Lachish series were drawn in accordance with the methods used in a number of earlier craniometric studies published in *Biometrika*, and a selection of the total group had to be made for this purpose. Types were constructed from the measurements of these contours in the usual way (Figs. 4–9), and they relate to material from the four tombs combined (see p. 123). The selection was made by first excluding all specimens too incomplete to give the Frankfurt orientation, and then excluding from the

remainder those for which the horizontal and transverse sections are defective to more than a slight extent. The totals included are 108 male and 89 female crania, and almost all the measurements used in constructing the types (Tables XV–XVII) are available for every one of these specimens, except in the case of facial measurements of the sagittal contours for which the numbers vary appreciably. The type contours of the Lachish series are based on samples which are almost as large as any previously used for the purpose.

A comparison is made in Table XIV between certain measurements of the sagittal type contours and corresponding caliper measurements for the total

TABLE XIV

A comparison of mean caliper and type contour measurements

Character	Male		Female	
	Contour	Caliper	Contour	Caliper
L	185·3 (108)	184·5 (322)	178·8 (89)	176·8 (259)
H'	135·0 (96)	133·8 (268)	127·6 (74)	128·4 (213)
S_1'	113·6 (108)	112·9 (299)	110·2 (89)	108·7 (248)
S_2'	116·1 (108)	116·0 (323)	111·8 (89)	112·1 (251)
S_3'	96·7 (97)	96·3 (280)	94·5 (79)	94·0 (216)
fml	36·7 (96)	37·0 (247)	35·1 (74)	35·8 (193)
G'H	69·7 (66)	70·1 (98)	66·2 (44)	66·8 (87)
GL	94·2 (62)	94·3 (89)	91·1 (38)	90·6 (76)
LB	101·8 (96)	100·7 (243)	97·1 (74)	96·4 (206)
P∠	87°·0 (66)	86°·0 (81)	86°·5 (44)	84°·9 (62)
N∠	63°·4 (62)	64°·0 (89)	65°·1 (38)	64°·5 (75)
A∠	75°·2 (62)	73°·9 (89)	73°·6 (38)	73°·7 (75)
B∠	41°·4 (62)	42°·0 (89)	41°·3 (38)	41°·8 (75)

series. The former are either used in the construction of the type (*G'H*, *GL*, *LB*), or calculated from measurements used in its construction (*H'*, S_1', S_2', S_3', *fml*, *P*∠, *N*∠, *A*∠ and *B*∠), or measured on the figure (*L*). In comparing the means of the two kinds, it must not be forgotten that one series represented is a selected group of the other. It might be anticipated that the process of selection described would favour the larger and stronger skulls, more likely to be well preserved, and hence rather larger means would be expected for the contour series. The divergences between corresponding values are actually found to be very small. Of the nine absolute measurements the male contour mean exceeds the caliper in six cases, the largest difference being 1·2 mm., and the position is reversed for the other three, which show a maximum difference of 0·4 mm. For the same measurements the female contour mean exceeds the caliper value in five instances, the maximum difference being 2·0 mm., and the maximum difference for chords showing the caliper greater than the contour mean is

0·8 mm. The angles show a close correspondence. On the whole there is a good agreement between the measurements obtained in the two ways, suggesting that the contours were drawn with sufficient accuracy, and that the sub-series gives a fair representation of the type for the total series.

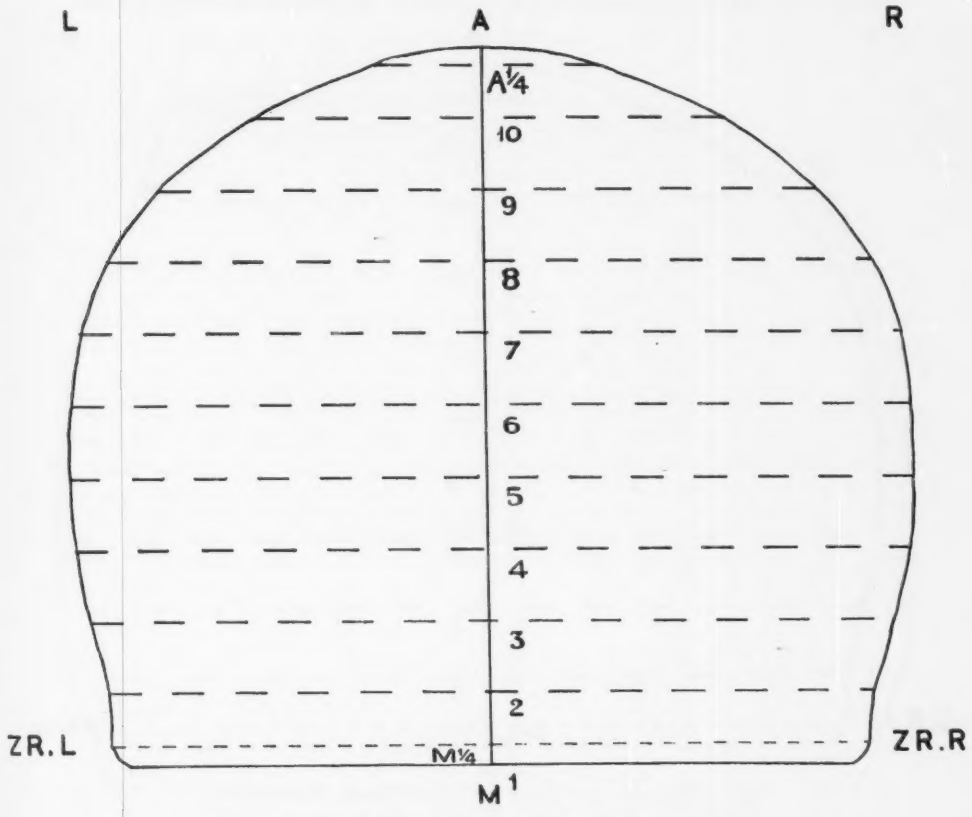


Fig. 4. Transverse type contour based on 108 male Lachish skulls.

The type contours (Figs. 4–9) have no striking peculiarities, and superficially, at any rate, they appear to be very similar to those given for Ancient Egyptian and even some European series. They are average in size, show an orthognathous facial skeleton, and only moderate muscular development. The male contours are larger than the female in all respects to the same extent as is usually found.

Fig. 10 shows the three male Lachish types and those given by Miss Collett (1933) for the Kerma, and by Miss Stoessiger (1927) for the Badari series, superposed. There is clearly a close agreement between the average contours for these two Ancient Egyptian and the Palestinian series, though the differences are probably almost as great as those which would be found between any pairs of the types of Ancient Egyptian series. The Kerma and Badari series are assigned

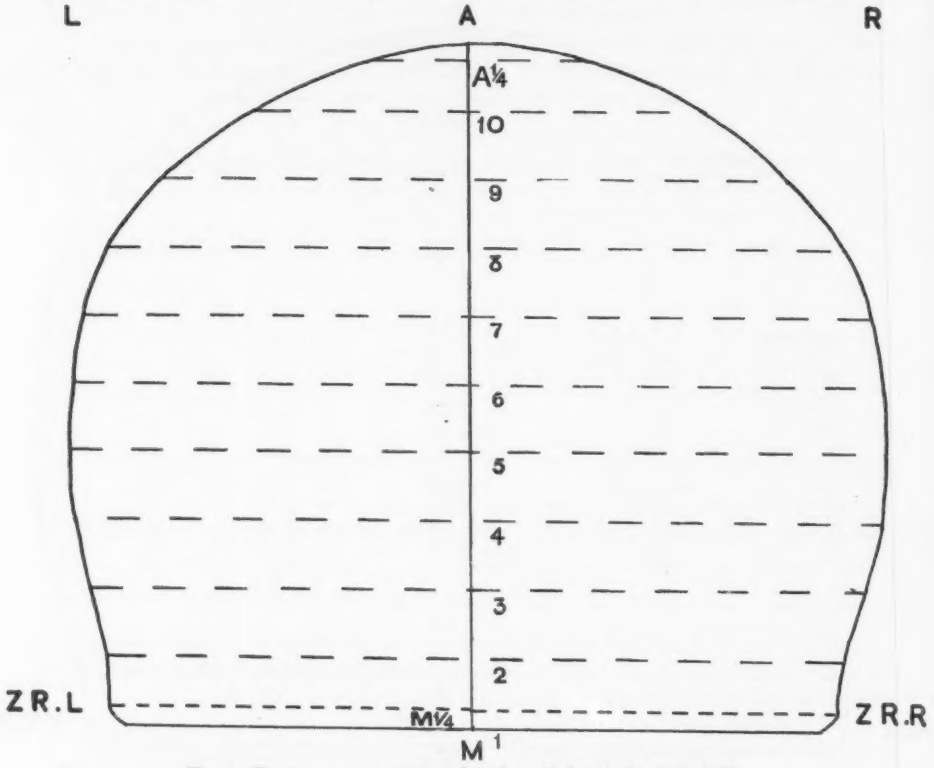


Fig. 5. Transverse type contour based on 89 female Lachish skulls.

by their mean measurements to the Upper Egyptian group, and the Lachish belongs to the Lower Egyptian (see Fig. 3). The superposed types show the greatest differences in calvarial breadth (transverse and horizontal sections), and decidedly smaller ones in calvarial lengths (horizontal and sagittal), as would have been anticipated. The Lachish facial skeleton is seen to be rather less prognathous than the Kerma or Badari, but the section of its nasal bones is the most projecting (cf. p. 145). The type for the predynastic series is the smallest in nearly all respects and particularly in facial height.

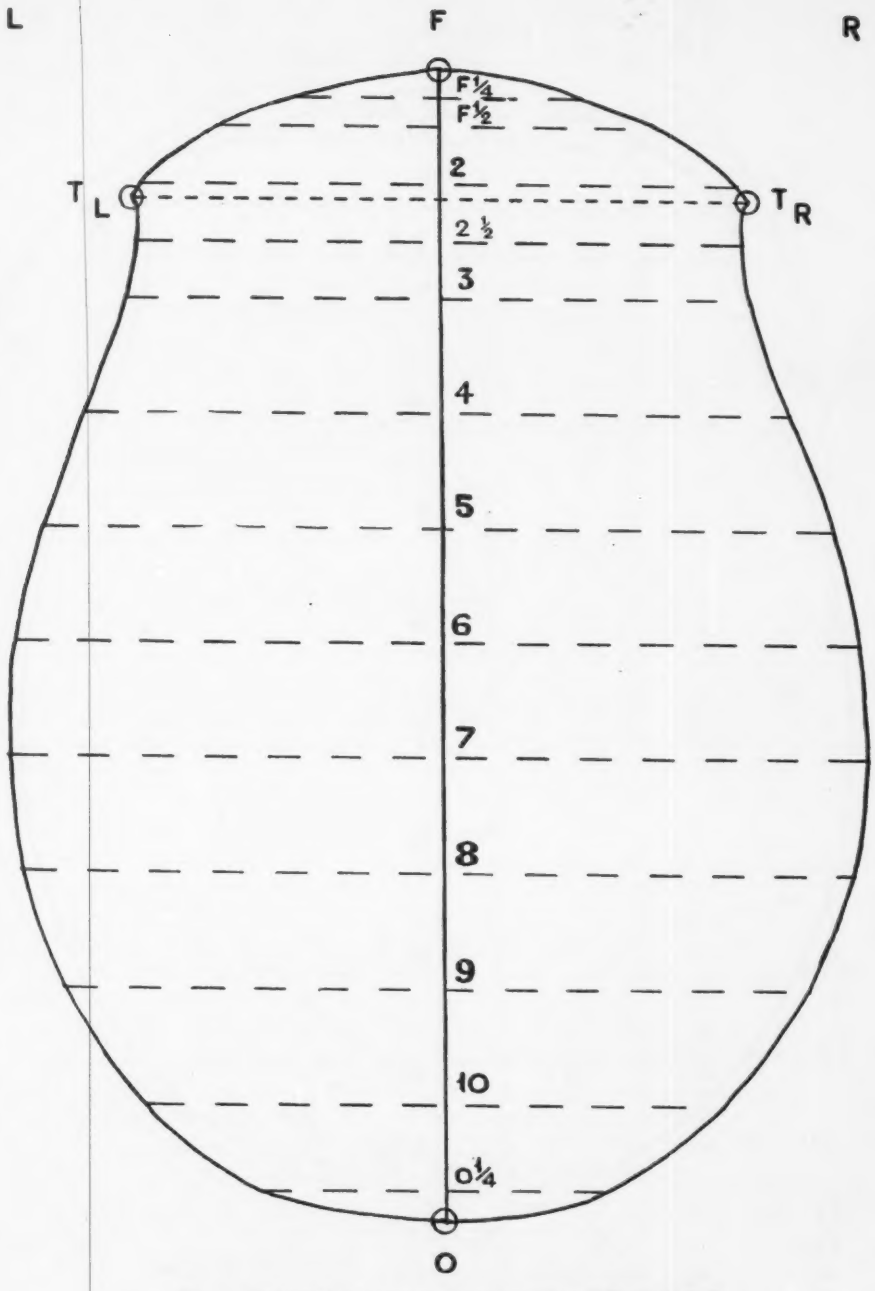


Fig. 6. Horizontal type contour based on 108 male Lachish skulls.

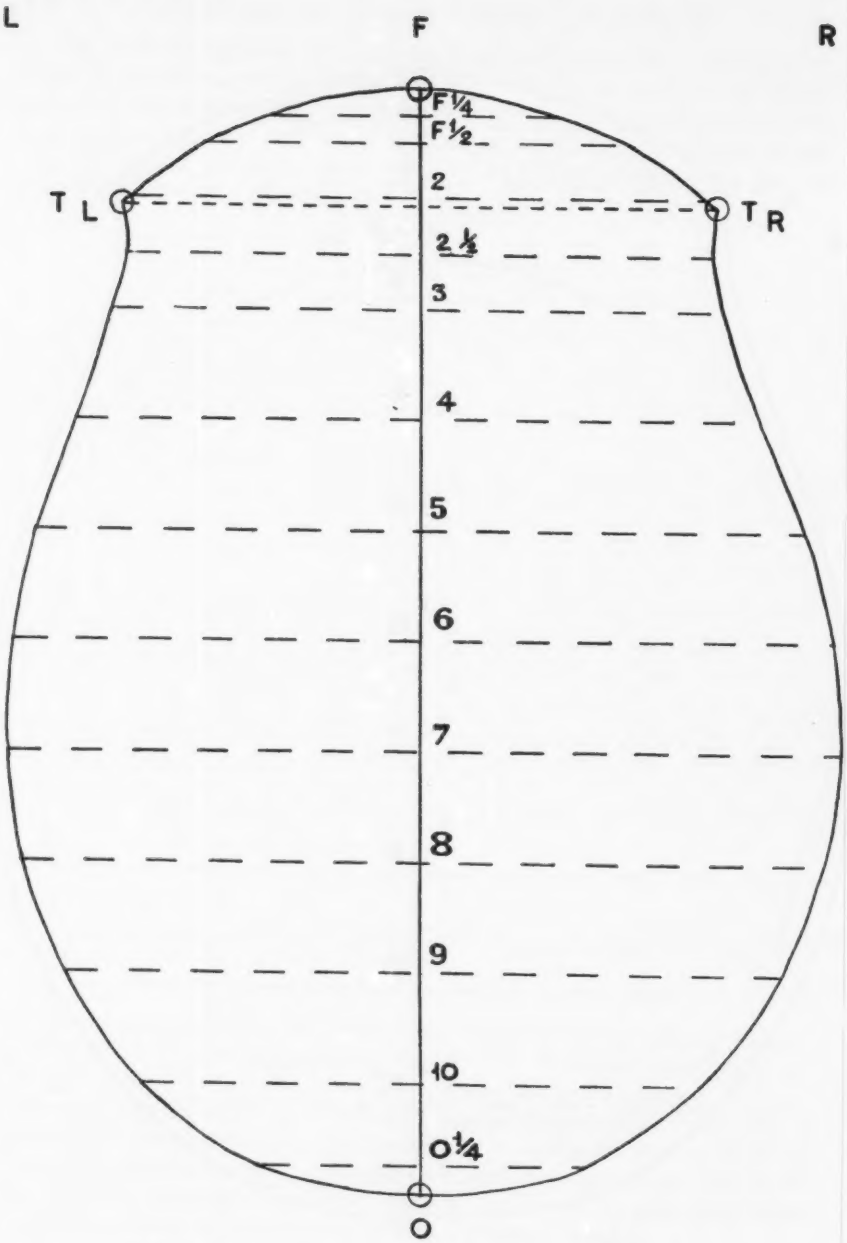


Fig. 7. Horizontal type contour based on 89 female Lachish skulls.

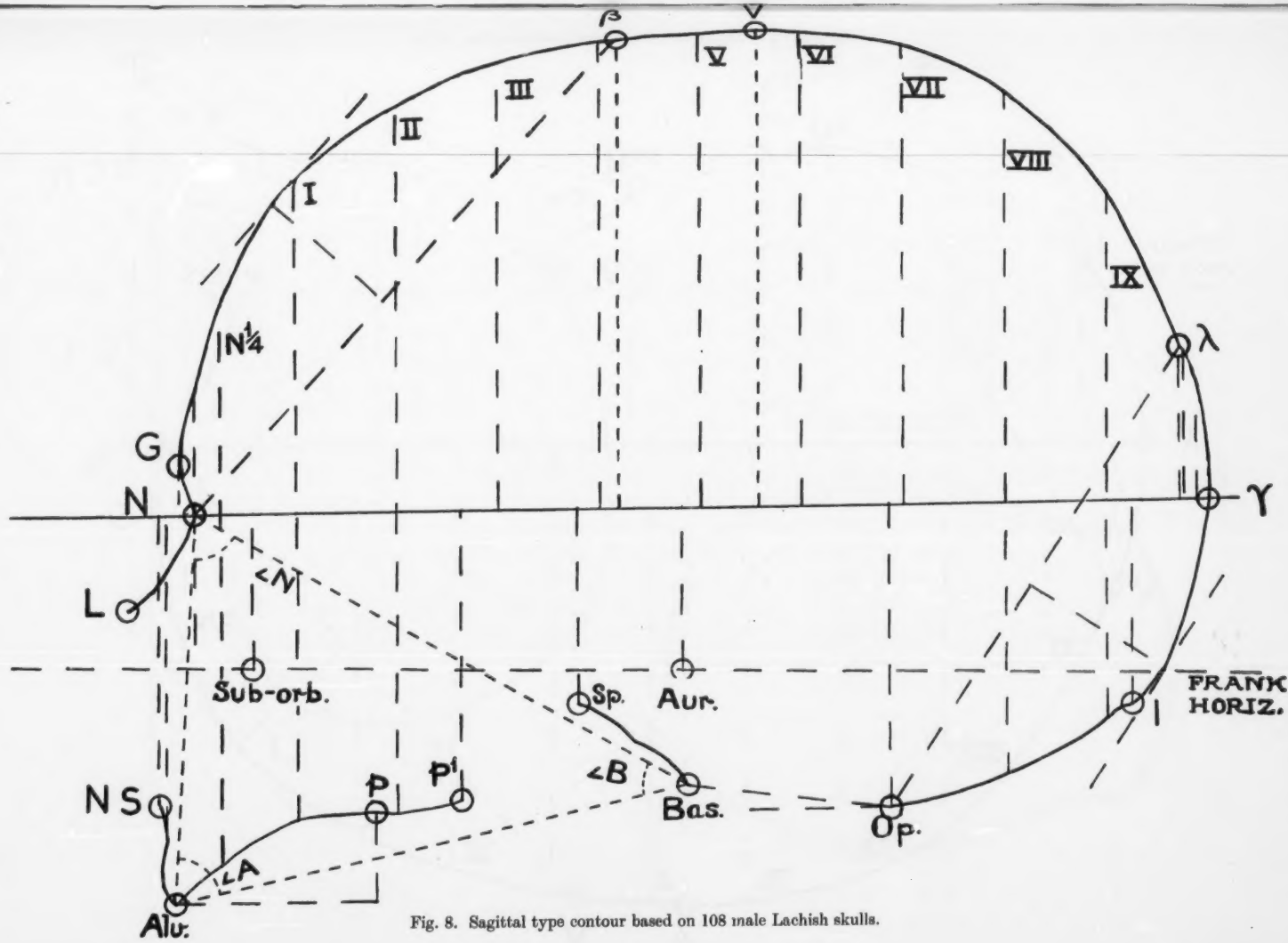


Fig. 8. Sagittal type contour based on 108 male Lachish skulls.

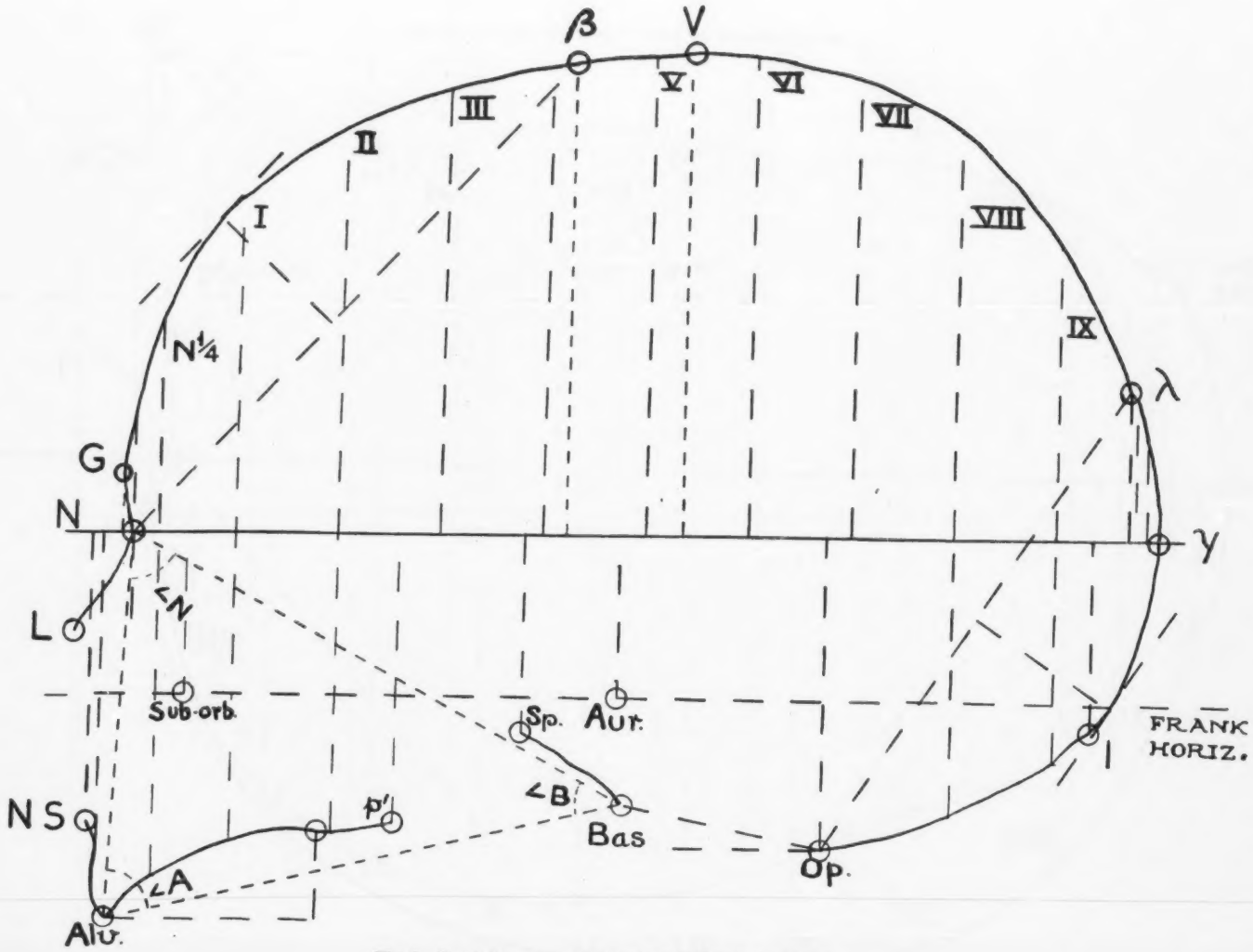


Fig. 9. Sagittal type contour based on 89 female Lachish skulls.

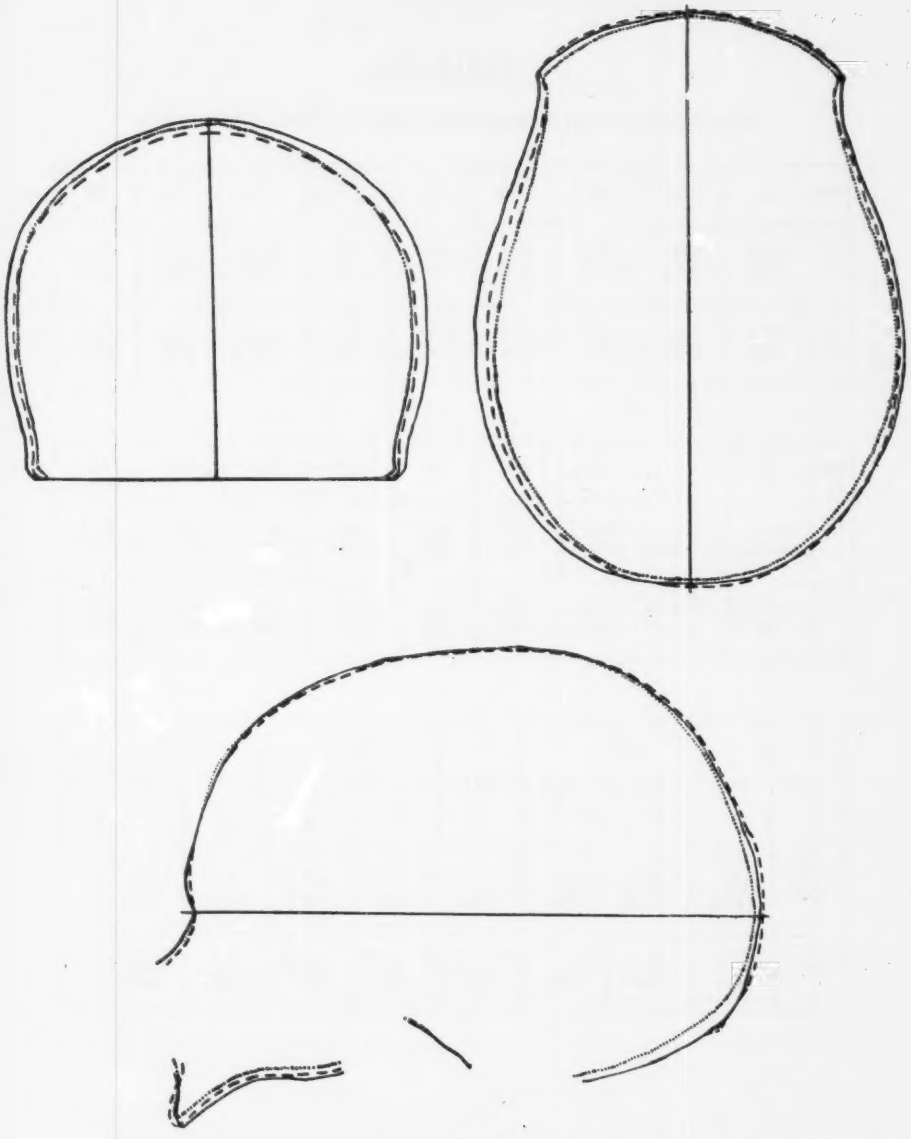


Fig. 9. Sagittal type contour based on 89 female Lachish skulls.

Fig. 10. Superposed male type contours of the Lachish (full line), Kerma (broken line) and Badari (dotted line) series. Scale half linear.

TABLE XV

Mean measurements of transverse contours of the Lachish skulls

Sex	*MA*	1*R*=1*L*	2*R*	2*L*	3*R*	3*L*	4*R*	4*L*	5*R*
♂	114·9 (108)	57·6 (108)	61·0 (108)	60·9 (108)	64·5 (108)	63·6 (108)	67·3 (108)	65·8 (108)	68·0 (108)
♀	109·1 (89)	55·4 (89)	59·0 (89)	58·4 (89)	62·3 (89)	61·1 (89)	65·3 (89)	63·3 (89)	65·8 (89)

Sex	5*L*	6*R*	6*L*	7*R*	7*L*	8*R*	8*L*	9*R*	9*L*
♂	66·5 (108)	67·9 (108)	66·0 (108)	66·3 (108)	64·3 (108)	62·0 (108)	60·1 (108)	53·1 (108)	51·4 (108)
♀	63·9 (89)	65·4 (89)	63·3 (89)	63·9 (89)	61·6 (89)	59·9 (89)	57·6 (89)	51·6 (89)	48·9 (89)

Sex	10*R*	10*L*	*A*¼*R*	*A*¼*L*	*ZR*, *R*		*ZR*, *L*	
					x	*y*	*x*	*y*
♂	38·4 (108)	36·5 (108)	18·5 (108)	17·0 (108)	3·1 (108)	60·2 (108)	3·1 (108)	60·7 (108)
♀	37·5 (89)	34·0 (89)	18·6 (89)	14·9 (89)	2·6 (89)	58·0 (89)	2·9 (89)	58·1 (89)

TABLE XVI

Mean measurements of horizontal contours of the Lachish skulls

Sex	*FO*	*F¼R*	*F¼L*	*F½R*	*F½L*	*2R*	*2L*	*2½R*	*2½L*	*3R*	*3L*
♂	184·3 (108)	23·0 (108)	22·6 (108)	35·0 (108)	35·4 (108)	47·6 (108)	47·9 (107)	48·5 (108)	48·5 (108)	49·6 (108)	50·1 (108)
♀	177·2 (89)	22·5 (89)	22·4 (89)	34·2 (89)	34·1 (89)	45·9 (89)	45·9 (89)	46·7 (89)	46·6 (89)	48·6 (89)	48·9 (89)

Sex	*4R*	*4L*	*5R*	*5L*	*6R*	*6L*	*7R*	*7L*	*8R*	*8L*
♂	55·8 (108)	56·8 (108)	62·9 (108)	63·6 (108)	66·9 (108)	68·0 (108)	68·0 (107)	69·2 (108)	65·4 (107)	67·1 (108)
♀	54·1 (89)	54·4 (89)	61·7 (88)	60·9 (89)	65·8 (88)	64·9 (89)	66·9 (89)	65·8 (89)	64·3 (89)	63·6 (89)

Sex	*9R*	*9L*	*10R*	*10L*	*O¼R*	*O¼L*	*TR*		*TL*	
							x	*y*	*x*	*y*
♂	58·1 (108)	60·4 (108)	44·4 (108)	47·7 (108)	25·1 (108)	29·4 (108)	20·9 (108)	49·3 (108)	20·9 (108)	49·3 (108)
♀	57·1 (89)	57·0 (89)	44·2 (89)	44·7 (89)	25·4 (89)	26·6 (89)	18·5 (89)	47·2 (89)	18·5 (89)	47·2 (89)

TABLE XVII

Mean measurements of sagittal contours of the Lachish skulls

Sex	$N\gamma$	Ordinates above $N\gamma$						
		$O=N$	$N\frac{1}{4}$	1	2	3	4	5
♂	182·3 (108)	21·6 (108)	37·6 (107)	59·4 (107)	72·5 (108)	80·1 (108)	84·2 (108)	85·7 (108)
♀	177·0 (89)	19·7 (89)	37·6 (39)	57·8 (89)	69·8 (89)	76·7 (89)	80·5 (89)	82·1 (89)

Sex	Ordinates above $N\gamma$						
	6	7	8	9	$\gamma\frac{1}{4}$	$\gamma\frac{1}{2}$	γ
♂	85·9 (107)	82·8 (107)	73·7 (108)	55·0 (108)	26·6 (108)	19·3 (108)	6·24 (108)
♀	82·2 (89)	78·6 (88)	69·9 (88)	50·6 (89)	24·4 (89)	17·1 (89)	5·0 (89)

Sex	Ordinates below $N\gamma$						Vertex	
	$O=N$	$N\frac{1}{4}$	1	2	8	9	x from N	y
♂	65·9 (87)	62·6 (95)	55·0 (105)	53·7 (97)	48·5 (106)	38·7 (108)	101·3 (108)	86·4 (108)
♀	62·1 (66)	58·5 (79)	52·2 (89)	51·2 (82)	47·3 (89)	37·7 (89)	94·5 (89)	82·9 (89)

Sex	Bregma		Glabella		λ		Suborb.	
	x from N	y	x from N	y	x from γ	y	x from N	y
♂	76·2 (108)	84·3 (108)	2·7 (108)	8·9 (108)	5·1 (108)	27·1 (108)	10·2 (108)	27·8 (108)
♀	74·7 (89)	81·0 (89)	1·4 (89)	9·4 (89)	5·3 (89)	25·4 (89)	9·6 (89)	27·2 (89)

TABLE XVII (*cont.*)

Mean measurements of sagittal contours of the Lachish skulls (cont.)

Sex	Aur. Pt.		Opisthion		Inion		Basion	
	x from N	y	x from γ	y	x from γ	y	From γ	From N
♂	87·7 (108)	29·3 (108)	57·3 (97)	54·3 (97)	13·8 (107)	36·5 (107)	106·3 (96)	101·8 (96)
♀	84·0 (89)	27·8 (89)	57·3 (79)	53·5 (79)	11·4 (89)	32·8 (89)	102·6 (74)	97·1 (74)

Sex	Alv. Pt.		Nose				
	From Bas.	From N	(i)	(ii)	(iii)	$\angle\gamma NL$	NL
♂	94·2 (62)	69·7 (66)	1·3 (100)	4·7 (70)	7·0 (33)	125°·3 (13)	21·0 (13)
♀	91·1 (38)	66·2 (44)	1·2 (83)	3·5 (67)	7·1 (29)	122°·8 (11)	19·4 (11)

Sex	Palate				Frontal		Occipital	
	P'		P		Max. Sub. to $N\beta$		Max. Sub. to λ Op.	
	x from N	y	x from Alv.	y	x from N	y	x from λ	y
♂	47·9 (76)	51·6 (76)	36·1 (65)	16·0 (65)	50·9 (107)	26·6 (107)	50·1 (97)	28·6 (97)
♀	45·7 (56)	49·6 (56)	36·1 (44)	15·0 (44)	49·0 (89)	26·7 (89)	46·8 (79)	27·5 (79)

Sex	Sp.		NS		Vert. Tan. below NS		Sub. from $\frac{1}{2}$ Bas.-Sp. chord above
	x from N	y	x from N	y	x from N	y	y
♂	68·9 (105)	34·9 (105)	6·3 (36)	52·0 (36)	5·3 (67)	56·9 (67)	0·4 (96)
♀	67·7 (83)	33·8 (83)	6·9 (25)	49·4 (25)	5·0 (51)	53·9 (51)	0·4 (74)

9. Comparisons of the Lachish series with series of crania and living people other than Egyptian

The comparisons made above have shown that the Lachish series is very similar in type to several Ancient Egyptian series. In fact it bears as close a resemblance to some of these as they do in general to one another. Hence it may be concluded that the population of Lachish in the year 700 B.C. was primarily, at least, of Egyptian origin. The cranial evidence from other countries of the Near East is very meagre. Comparisons are made with the only other Palestinian series of any length, and with a few series of living people, in this section.

A considerable number of human skeletons was excavated by Prof. R. A. S. Macalister at Gezer, 17 miles south-east of Jaffa, from 1902 to 1905 and 1907 to 1909. These are dealt with in a chapter in his report (1912). Material of the pre-Semitic period was very fragmentary, and no measurements of it could be taken. Five indices for a series of skulls of the Semitic periods are given in the form of frequency distributions only. Division is made between a series for the first and second periods together, and a series for the third and fourth periods together, in the case of the cephalic index. A distribution of reconstructed statures is also provided. The material was apparently unsexed, and it is not clear how the absolute frequencies could be determined from the diagrams. All that can be said is that the average cephalic index for both series of unsexed skulls is about 76. The Lachish male mean is 74·3, and the female 75·5.

There appear to be no published records for any other long series of skulls of any date from Palestine, or any neighbouring country except Egypt. Measurements of small numbers of ancient Jewish and Phoenician and modern Arab specimens have been given, but these are practically worthless for statistical purposes. There are no adequate data for Ancient Jewish skulls from any locality. The longest modern series representing this people is one published by Prof. J. Matiegka (1926) of seventeenth-century Jews buried in Prague. The average cephalic index for the fifty-three male skulls is 82·0, which is sufficient to show that there can be no close connexion with the Lachish people (74·3), or with any of the Ancient Egyptian groups, for which the highest index is 76·0. The coefficient of racial likeness was computed for twenty-two characters between the male Jewish series from Prague ($\overline{n} = 32{\cdot}8$) and the Lachish series ($\overline{n} = 202{\cdot}8$): a reduced value of 52·6 is found. This is greater than the maximum found between any pair of the series of Egyptian type, including the Lachish (see Table XII). It is not suggested, of course, that the Palestinian Jews in 700 B.C. were necessarily of the same type as Jews in Prague in the seventeenth century.

Measurements of a certain number of Jewish people in Palestine have been published, and only the cephalic index for series of men will be considered here. No accurate comparisons with cranial data can be given in the case of any

other characters recorded. Weissenberg (1909) gives a mean value of 79·8 for fourteen men in Galilee. All the remaining series of Jews in Palestine represent the Samaritans, the sources and mean indices being:

Weissenberg (1909)—76·2 (20); Kappers (1934)—77·2 (27);
Szpidbaum (1927)—77·6 (27); Huxley (1906)—78·1 (35);
Genna (1938)—79·1 (39).

In view of the small sizes of the series, these averages accord remarkably well. The pooled mean for the 148 men is 77·9.

Weissenberg (1909) has also given cephalic indices of 76·9 for twenty-five fellahin measured near Jaffa, and 75·7 for thirty in the Safed district, or a pooled mean of 76·2 for the fifty-five men. This value differs markedly from that of 81·6 given by Kappers (1934) for 139 Arabs from "towns north of the line Jaffa–Jericho". There appear to be some well-marked regional differences between Arabs in different localities in Palestine.

The mean cephalic index for the male Lachish series of skulls is 74·3, which can be supposed to correspond to a value of about 76·3 in the living. The latter is decidedly less than the mean for one series of Arabs, rather less than the Jewish means, and practically identical with that for the Arabs measured by Weissenberg. It is quite unsafe to lay stress on comparisons of a single character, but those made show that a population with the same cephalic index as that of the ancient inhabitants of Lachish is living in Palestine to-day. The possibility that the pre-Christian type has persisted until modern times is not precluded, though far more evidence—and particularly that of later series of skulls from the country—will be needed to disclose its racial history in any detail.

10. Mandibles and long bones of the Lachish series

In all there are seventy-six mandibles in the Lachish series—fifty-six from Tomb 120 and the remainder from the three other tombs—most of them being defective to some extent. Of the total, one is associated with an adult male cranium, and nine with adult female crania. The remaining sixty-six are unassociated, and they were sexed by anatomical appreciation. Remarks on a few of the specimens noted as being anomalous are in § 4 above. Measurements were taken of the adult bones in accordance with the biometric technique (Morant, Collett & Adyanthāya, 1936). There are thirty-four male and thirty-five female specimens. Means for these are given in Table XVIII, and comparisons with other material would not be profitable, as it has been shown (Cleaver, 1937) that considerably larger numbers would be required to reveal small racial differences in type. Neither the measurements nor the appearance of the Lachish mandibles suggest any clear divergence from Ancient Egyptian types.

TABLE XVIII

Mean measurements of Lachish mandibles

Character	Male	Female	Character	Male	Female
w_1	118·6 (7)	114·9 (7)	c_1h	64·3 (11)	60·0 (16)
c_1l	20·5 (17)	19·4 (18)	m_2h	26·9 (14)	24·5 (14)
rb'	32·6 (26)	29·9 (28)	$M\angle$	125°·9 (14)	124°·5 (15)
m_2p_1	27·4 (26)	27·2 (23)	$R\angle$	71°·6 (5)	72°·1 (9)
h_1	34·8 (20)	31·8 (19)	$C'\angle$	74°·6 (11)	72°·7 (13)
zz	44·8 (31)	43·2 (33)			
c_2l	73·1 (14)	70·1 (15)	$100\ c_1h/ml$	60·7 (5)	60·5 (6)
rl	59·75 (14)	54·9 (15)	$100\ c_1c_1/ml$	86·6 (4)	90·5 (5)
g_0g_0	97·1 (11)	85·5 (11)	$100\ g_0g_0/c_2l$	133·3 (11)	122·9 (11)
c_1c_1	91·1 (4)	92·1 (8)	$100\ rb'/rl$	54·7 (14)	53·9 (14)
ml	104·8 (8)	100·3 (8)	$100\ g_0g_0/c_1c_1$	101·4 (4)	92·1 (6)

Other parts of the skeleton are only represented in the Lachish series (all tombs) by two sacra and nearly 200 long bones, many of which are incomplete. These are not associated together and no attempt was made to sex them. The maximum lengths of the adult long bones were determined, and means for them are given in Table XIX. As far as can be seen from these constants, the Lachish people were rather short, but no approximation of any value to the average statures of the men and women can be given. One ulna (No. 2) has a healed fracture of the lower shaft, and one femur (No. 39) has condyles affected by arthritis.

TABLE XIX

Means of the maximum lengths of unsexed adult long bones of the Lachish series

	Femora (oblique)	Tibiae (oblique)	Humeri (oblique)	Radii	Ulnae	Clavicles
R	427·9 (18)	347·7 (16)	298·5 (11)	243·7 (6)	264·6 (5)	147·25 (4)
L	430·45 (20)	372·25 (8)	300·1 (15)	239·8 (5)	266·0 (6)	154·0 (1)

11. Summary and conclusions

The skeletal remains reported on in this paper for the Trustees of the late Sir Henry Wellcome were collected at Tell Duweir (Lachish), twenty-five miles south-west of Jerusalem, by the Wellcome-Marston Expedition to the Near East from 1933 to 1936. The bones were found in four adjoining tomb chambers and they are assigned to the seventh and eighth centuries B.C. In all there are 695 crania, the majority of which are more or less imperfect, and much smaller numbers of mandibles and other bones of the skeleton (see table on p. 103). Of the crania 360 were judged to be adult male and 274 adult female, the

remaining sixty-one being immature. The origin of the collection is discussed, and it is concluded that the remains are probably those of people who died as the result of some catastrophe. The frequencies of occurrence of different states of closure of the principal calvarial sutures show, from comparisons with other cranial series, that the adults from Lachish were younger, on the average, than cemetery populations are expected to be. Very few aged individuals were interred in the tombs. The normal order of closing of the sutures was sagittal—coronal—lambdoid. Remarks on unusual conditions and anomalies are given, the most interesting specimens being three trepanned skulls; two showing marked artificial deformation and six suspected to have been deformed artificially; a series of seventeen showing premature closing of the sagittal suture without clear deformation except in three cases; one believed to be distorted owing to premature closing of the coronal suture; one with absence of the right auricular passage; and one with an extensive diseased area on the vault.

Statistics regarding the loss of teeth before death show that they were remarkably well preserved. Remarks on dental anomalies are given. The most interesting skull from this point of view is one which was found to have a metal filling in one of its molars, presumably acquired by accident.

Judging from comparisons of the measurements, there is no reason to doubt that the series from the four tombs represent precisely the same population, and the differences found between the male and female adult and juvenile constants are no greater than those expected in such a case. The variabilities and sex ratios of the total series (combining skulls from all tombs) are quite unexceptional.

Comparisons are made between the Lachish and twenty-one Ancient Egyptian and allied series of skulls by the method of the coefficient of racial likeness, and a classification of the material is presented. The relationships found suggest that the population of the town in 700 B.C. was entirely, or almost entirely, of Egyptian origin, very close connexions with some contemporary Egyptian series being found. They show, further, that the population of Lachish was probably derived principally from Upper Egypt. Comparisons of measurements considered singly indicate that the Lachish cranial type has no features which would be unusual for an Ancient Egyptian type, other than the prominence of its nasal bones and the curvature of its malar bones. Transverse, horizontal, and sagittal type contours based on 108 male and eighty-nine female Lachish skulls are given, and it is shown that they are very similar to some previously provided for Ancient Egyptian series. There are no good records for any series of skulls from Palestine other than the Lachish. Its mean cephalic index accords fairly well with that given for a series of living Palestine Arabs, and it is close to that for Samaritans. The possibility that the Lachish people have persisted until to-day is not precluded, but far more evidence would be required to substantiate such a hypothesis. The series of mandible and long bones from Lachish are too small to be of any value for comparative purposes. As far as can be seen the people were rather short.

APPENDICES

I. *Definitions of skull measurements taken*

Measurements of the Lachish material were taken in accordance with biometric practice. The definitions of cranial points given by Buxton & Morant (1933), and those of mandibular measurements given by Morant, Collett & Adyanthāya (1936), were followed. The contractions below are used to denote measurements in the tables and text, and their numbers in Martin's list are given.

C = capacity in c.c. It was not possible to determine the capacities of any of the Lachish skulls directly, owing to the fact that the interiors of the brain-boxes are coated with mud and wax which cannot be removed. The reconstruction formulae using L, B, and H' given by Pearson & Stoessiger (1927) were applied to give the estimates in Table VI, and these were not used in computing coefficients of racial likeness. L = maximum glabella-occipital length (M. 1). B = maximum horizontal breadth (M. 8). H' = basio-bregmatic height (M. 17). LB = basion to nasion (M. 5). B' = minimum frontal breadth (M. 9). S = arc nasion to opisthion (M. 25). S_1 = arc nasion to bregma (M. 26). S_2 = arc bregma to lambda (M. 27). S_3 = arc lambda to opisthion (M. 28). S_1' = chord nasion to bregma (M. 29). S_2' = chord bregma to lambda (M. 30). S_3' = chord lambda to opisthion (M. 31). U = horizontal circumference measured through the ophyron and directly above the superciliary ridges (M. 23*a*). $\beta Q'$ = transverse circumference from one auricular point to the other, passing through bregma (M. 24). fml = basion to opisthion (M. 7). fmb = maximum breadth of foramen magnum (M. 16). $G'H$ = nasion to alveolar point (M. 48). GL = basion to alveolar point. GB = facial breadth between lowest points on zygomatic-maxillary sutures (M. 46). J = maximum breadth between zygomatic arches (M. 45). NH, L = nasion to lowest edge of pyriform aperture on the left side. NB = maximum breadth of pyriform aperture (M. 54). O_1L = maximum breadth of left orbit (M. 51). O_2L = maximum height of left orbit (M. 52). G_1' = length of palate from orale to staphylion (M. 62). G_2 = breadth of palate between inner alveolar walls of second molars (M. 63). OH = maximum projection from biporial axis in the transverse vertical plane, measured on transverse contour. SC = simotic chord, minimum breadth of nasal bones (M. 57). SS = subtense of simotic chord. Measurements of the left malar bones, taken in accordance with Woo's instructions (1937), are; Ml_1 = minimum horizontal arc. Ml_2 = minimum vertical arc. $C(ml)$ = chord between terminals of horizontal arc. $S(ml)$ = maximum subtense from the chord. The occipital index is the only one which needs definition: it is

$$Oc.I. = 100\,\frac{S_3}{S_3'}\sqrt{\left(\frac{S_3}{24(S_3 - S_3')}\right)}.$$

Values for the individual skulls were found with the aid of Miss Tildesley's table of the function (*Biometrika*, **13** (1921), 261–2). $P\angle$ = profile angle between Frankfurt horizontal plane and the chord joining nasion to alveolar point. $N\angle$, $A\angle$ and $B\angle$ are the angles of the triangle of which the nasion, alveolar point and basion are the apices. w_1 = maximum breadth outside condyles. c_1l = maximum length of the left condyle. rb' = minimum antero-posterior "breadth" of the left ramus. m_2p_1 = chord between the points on the outer left alveolar margin from the middle of the second molar to the middle of the first premolar. h_1 = symphyseal height from intradental to the point farthest removed from it in the symphyseal plane. zz = minimum chord between the anterior margins of the right and left *foramina mentalia*. c_1c_r = coronial breadth from right coronion to left coronion. $M\angle$ = mandibular angle. c_1l = projective length of the corpus. rl = projective length of the left ramus. g_0g_0 = chord from left gonion to right gonion. ml = maximum projective length of the mandible. c_1h = projective height of the left coronoid process. m_2h = projective height of the corpus at the middle point of the outer alveolar margin of the second left molar. $R\angle$ = angle of condylar-coronoidal line with ramus tangent. $C'\angle$ = angle between the standard horizontal plane and the line joining the infradental to the most anterior point in the standard sagittal plane of the symphysis.

No.	L	B	B'	H'	S_1'	S_2'	S_3'	S_1	S_2	S_3	S	$\beta Q'$	U	fml	fmb	LB	GL	$G'H$
1	184	131·5	95·7	133	113·3	120·8	95·0	125·5	136·5	112	374	301	505	36·6	31·3	100·2	95·9	70·1
2	178	144	95·7	135·5	116·8	117·5	94·4	130·5	134	111	376	311	511	35·3	27·3	93·5?	—	—
3	180·5	133·5?	92·1	—	112·2	117·4	93·9?	129	134·5	116·5	380	308	509?	—	—	—	—	—
4	187·5	132	96·2	136	114·8	105·1?	102·3	132	112·5	125·5	370	306	522	36·6	27·3	107·1	96·8?	74·9?
5	182·5	147	99·3	136·5	112·6	116·9	91·1	131	133	111	374	324	532	38·7	—	101·6	95·7?	66·9?
6	186·5	137·5	104·1	134	120·5	118·9	91·1	139	129	112	380	313	532	37·3	33·6	98·1	—	69·8?
7	194	136	93·7	133·5	108·0	132·0	—	120·5	148	—	—	309	535	—	31·5	108·9	101·9	74·2?
8	174	141	92·2	—	118·6	109·3	—	135·5	125	—	—	322	502	—	—	—	—	63·4
9	189·5	140	98·7	137	116·2	113·0	105·4	132·5	123·5	127	383	307	529	35·2	28·6	100·2	90·4	81·7
10	186	139·5	99·5	143·5	115·4	120·2	96·9	134	135·5	119	388	324	520	35·3	31·4	103·8	—	—
11	193?	143	96·2	139	119·5	—	—	135	—	—	—	310	539	—	33·5	106·2	105·5	78·8
12	177·5	133	95·0	135·5	110·3	116·9	91·4	125·5	131	108·5	366	308	505	40·7	—	99·9	91·1?	73·1
13	189·5	142	93·4	137	115·8	119·7	94·8	128	132·5	111·5	373	306	529	40·3	31·0	105·2	—	—
14	181·5	137·5	99·7	135	110·5	119·5	94·4	127·5	138·5	110·5	376	310	516	37·4	32·3	101·5	94·3?	68·7
15	194·5	147	97·4	142	118·8	123·9	103·5	134·5	140	129	403	313	535	36·3	28·7	100·9	—	—
16	185	141	94·1	—	121·6	116·8	—	142	132·5	—	—	324	525	—	—	—	—	—
17	179	129·5	87·7	136	117·0	111·6	92·7	134	127·5	108·5	371	305	497	34·6	25·9	101·2	—	—
18	180	133	92·1	140	111·0	123·3	90·0	126·5	137	110	373	314	504	35·3	30·0	103·4	93·2?	68·5?
19	178	145·5	100·4	134	112·0	105·4	103·4	127	114·5	125·5	368	317	521	38·6	30·2	97·7	—	—
20	187	—	100·0	129·5	109·2	120·6	95·2	123·5?	134	119·5	376?	300?	518?	39·4	27·9	99·8	—	—
21	180?	135·5?	86·6	132	112·0	111·6	—	127	120	—	—	299	494?	—	—	—	—	—
22	191	145·5	101·3	—	111·7	118·4	—	129	131	—	—	324	547	—	—	—	—	—
23	185·5	136	92·8	137	110·5	109·4	102·2	123·5	120	125·5	369	314	522	37·8	31·5	106·1	—	—
24	188	142	91·6	—	119·5	129·2	102·3	137·5	143	119	399	325	534?	—	—	—	—	—
25	188	132·5	89·8	139	111·2	113·8	99·8	124·5	128	121·5	375	302	516	37·7	29·0	106·5	98·5	75·5
26	181·5	138	94·4	126·5	109·9	107·3	92·0	123·5	118	115	356	298	510?	38·4	29·0	101·3	87·0?	68·0
27	190·5	137	98·7	136·5	113·0	116·7	104·2	125·5	127·5	126	378	306	528	37·1	32·3	105·6	100·0	75·8
28	184·5	140	95·6	136	107·5	117·3	100·5	120·5	135	122	377	309	519	33·6	28·6	106·3	102·7	70·6
29	186·5	144	96·2	137·5	114·5	121·8	99·7	127·5	137	118	382	321	529	38·6	29·2	99·0	96·8?	73·5?
30	191·5	141	91·2	135·5	112·6	121·2	96·0	125	134	122	381	310	531	40·0?	30·4?	104·0?	90·5?	73·0?
31	187	137	99·3	—	114·2	116·3	96·0	132·5	127·5	118	377	311	519	—	31·0	—	—	70·4
32	180·5	135	89·0	130	105·7	112·9	92·5	118	126·5	111·5	356	295?	500?	39·3	31·3	100·3	97·1	73·3
33	179	128·5	98·2	134·5	116·0	107·4	93·5	134·5	123	107	364	301	501	36·0	29·0	100·2	—	—
34	190	137·5	—	131·5	117·0?	—	—	134?	—	—	385?	304	526?	37·0	28·8	103·5	—	—
35	192	—	90·5	137·5	115·7	123·1?	98·1	133·5	136·5?	117	387?	318	539	37·7	28·6	103·6	—	—
36	194·5	135·5	96·1	140·5	116·5	120·0	99·0	138	131	127	395	321	537	38·5	28·8	104·4	97·4?	67·3?
37	185	138·5	93·5	120	105·6	117·1?	97·9?	121	134	118·5	373	296?	520?	37·6	28·2	95·8	—	—
38	174	131·5	94·4	—	113·2	113·8	86·8	131	127·5	103·5	362	302	495	—	—	—	—	—
39	187·5	142·5	93·5	131	116·6	118·5	99·0	135·5	131	118·5	386	315	532	35·7	31·9	97·1	92·9	72·8
40	183·5	135	95·3	134	118·4	117·7	91·7	139	134·5	113·5	386	308	514	33·9	31·8	97·8	—	—
41	187·5	141·5	91·6	134·5	—	118·0?	90·9?	—	132?	118	—	303	519	44·9?	33·1	—	—	—
42	188	142	98·4	137	115·6	115·3	100·4	132·5	125	125	383	—	534	36·1	28·6	100·3	91·1?	71·2?
43	182	136·5	94·7	135·5	114·2	121·2	90·8	129·5	135·5	107·5	373	—	518	32·0	27·9	104·3	—	—
44	185·5	143·5	91·1	129	105·5	96·1	107·9?	117	103	142·5	362	309?	520?	—	28·5	—	—	—
45	182·5	141·5	94·7	131	112·7	119·2	93·3	124	135	112·5	371	307	519	33·4	28·6	102·9	—	—
46	177·5	142	96·9	143·5	120·5	108·0	95·1	138·5	123	110·5	371	318	511	37·2	29·4	103·0	88·4?	74·3?
47	188·5	139	92·6	136	119·1	115·8	98·1	136	128·5	114	378	311	521	38·3	31·8	99·8	—	—
48	184	130	98·2	135	111·2	125·3	91·7	125·5	142·5	111	379	305	516	35·6	31·5	101·0	—	—
49	186	132·5	—	132	116·0	114·3	91·9	135	128·5	116	380	307?	526?	36·0	31·1	103·1	95·3	70·6
50	188	143	105·8	139·5	118·2	111·2	97·5	139	124	121·5	385	322	536	41·6	—	102·0	93·4?	77·3
51	178	129·5	97·2	133	110·3	110·6	92·2	127·5	119	115	360	307	498	36·6	—	104·7	99·5	76·1
52	184	133	97·1	—	107·0?	116·1	100·0	117·5?	128·5	121	366?	304?	518	—	30·7	—	—	—
53	178·5	131	94·2	136·5	113·3	121·5	93·0	133	134	109	375	304	502	35·8	29·0	97·2	89·0?	62·5?
54	181·5	141·5	95·0	125·5	109·7?	111·2	—	125?	121·5	—	—	305	521	—	—	95·7	—	71·3?
55	185	133	97·2	136·5	107·6	120·0	99·5	125	134·5	123·5	383	310	521	31·8	28·5	104·0	—	—
56	183	142	102·0	138·5	117·4	115·3	97·0	135·5	132·5	115	382	319	523	34·8	31·0	104·6	97·3	66·9
57	184	137·5	95·3	135	112·3?	113·3	100·2	126·5?	124·5	121·5	373?	312	521	37·0	32·9	101·9?	95·8	69·0?
58	191	141·5?	98·5	—	115·1	119·0	100·3	128·5	134·5	115	377	315?	535?	—	—	—	—	—
59	173	136	88·2	130	111·3	104·9	98·0	129	118	117	363	303	489	36·4	28·7	94·7	91·4?	59·8?

LB	*GL*	*G'H*	*J*	*GB*	*NB*	*NH,L*	*SC*	*SS*	O_2L	O_1L	G_1'	G_2	Ml_1	Ml_2	*C* (*ml*)	*S* (*ml*)	*OH*	100
100·2	95·9	70·1?	132?	91·6	26·3	53·8	11·3	5·0	—	41·6	45·7	44·9	—	50·5	—	—	112·7	71
93·5?	—	—	122·5	90·5	24·0	46·7	12·2	6·2	32·3	39·1	—	—	57·0	49·5	48·9	11·6	117·3	80
—	—	—	—	—	26·4	51·0	10·7	5·8	29·3	39·2	50·6	—	51·5?	48·5	47·1	10·8	116·0	74
107·1	96·8?	74·9?	—	—	—	54·0	10·0	6·5	36·9	43·8	—	—	—	—	—	—	113·1	70
101·6	95·7?	66·9?	—	97·1	24·2	—	8·2	4·1	34·7	42·1	46·4?	—	—	47·5	—	—	118·1	80
98·1	—	69·8?	—	94·2	25·4	53·3	8·0	4·0	34·2	42·9	47·5	—	61·0?	48·0	53·0?	13·6?	115·0	73
108·9	101·9	74·2?	—	101·3	28·4	52·8	13·9	6·8	32·6	40·1	47·2?	—	—	50·5	—	—	115·9	70
—	—	63·4	134·5?	90·7	27·3	48·9	9·0	3·9	32·2	43·0	—	40·1	62·5?	49·0	56·3?	12·1?	119·8	81
100·2	90·4	81·7	—	98·9	24·4?	57·2	12·8	6·1	37·9	43·2	44·1	—	61·5	51·5	54·0	12·9	111·4	73
103·8	—	—	—	105·0	27·6	49·8	9·1	4·1	32·0	42·9	—	—	62·5	50·0	56·2	9·9	121·5	75
106·2	105·5	78·8	129	103·1	26·2	—	7·0	5·3	35·6	40·7	51·4	45·5	59·5	53·0	51·2	13·2	—	74
99·9	91·1?	73·1	—	91·9?	24·9	51·8	13·0	5·4	32·7?	42·8	—	—	—	—	48·6?	11·0?	114·9	74
105·2	—	—	—	96·7	23·6	55·7	13·1	6·3	32·4	42·5	45·3	39·1?	—	51·5	—	—	115·0	74
101·5	94·3?	68·7?	—	—	25·0	50·5	11·5	4·7	34·5?	42·7?	44·9	35·8	—	—	—	—	116·1	75
100·9	—	—	127·5	93·3	27·9	52·4	16·5	6·5	32·2	41·8?	42·2	—	63·0	51·0	54·0	11·9	118·0	75
—	—	—	—	94·4	23·5	47·3	9·5	3·8	30·7	39·3	42·2	—	59·0	46·5	49·5	13·6	121·0	76
101·2	—	—	—	—	—	—	11·2	4·0	27·8	39·0	—	—	—	47·5	—	—	117·2	72
103·4	93·2?	68·5?	126·5	91·3	26·2	48·9	10·0	—	34·0	44·8	48·8	—	59·0	55·0	50·2	12·0	119·0	73
97·7	—	—	133	96·9	23·9	48·8	9·6	4·3	33·0	42·3	41·7	—	—	52·5	57·1	12·3	114·6	81
99·8	—	—	—	—	29·4	53·5	13·0	4·9	28·7	39·6	—	—	66·0	52·0	56·8	12·5	—	—
—	—	—	—	—	26·3	50·8	—	—	28·6	36·8	—	—	—	—	—	—	—	75
—	—	—	—	—	—	—	10·2	5·7	—	—	—	—	—	—	—	—	—	76
106·1	—	—	—	—	22·3	53·2	9·9	7·2	36·1?	—	44·1	39·5	—	—	—	—	118·2	73
—	—	—	—	—	—	—	11·9	5·4	—	—	—	—	—	—	—	—	—	75
106·5	98·5	75·5	—	—	24·2?	52·4	—	—	31·8?	41·7	47·8	40·0?	—	—	—	—	114·0	76
101·3	87·0?	68·0?	—	—	—	49·7	13·7	5·6	32·2	41·5	—	—	—	49·0	—	—	110·6	76
105·6	100·0	75·8	—	104·8	22·6	52·7	11·3	4·5	35·0	42·3	49·6	40·1	66·0	49·0	57·2	13·2	116·5	71
106·3	102·7	70·6	125·5	96·8	28·8	55·0	14·5	7·8	32·7	39·2	50·0	43·2?	65·0	50·5	54·8	14·0	117·2	75
99·0	96·8?	73·5?	129	91·9	—	53·0?	12·0	5·6	—	—	—	41·2	63·0	54·5	54·6	13·1	119·8	77
104·0?	90·5?	73·0?	—	86·5	25·5	52·2	8·8	4·1	32·6	42·3	—	—	56·0	52·5	51·0	11·3	117·0	75
—	—	70·4	—	—	25·0	53·3	10·7	5·9	33·9	41·6	46·4?	—	—	—	—	—	117·2	73
100·3	97·1	73·3	132	93·2	25·3	54·5	7·6	4·2	36·0	44·7	—	42·4?	64·0	50·5	55·0	13·4	107·6	74
100·2	—	—	130	84·3	24·5	45·1	10·0?	3·1	33·2	40·5	—	—	60·5	49·0	54·9	10·3	113·0	75
103·5	—	—	—	—	—	—	10·8	3·4	32·5	41·0	—	—	57·0	46·0	47·6	13·3	—	75
103·6	—	—	128	—	—	—	9·4	4·3	35·0	43·2	—	—	62·0	54·5	56·0	12·0	—	—
104·4	97·4?	67·3?	—	87·3	25·8	49·0	11·9	7·1	33·0	42·8	50·4	—	—	50·5	—	—	117·0	66
95·8	—	—	—	92·3	—	50·4	8·3	4·2	32·7	41·1	42·5	39·7	—	—	—	—	—	74
—	—	—	—	95·0?	—	—	—	—	31·9	40·5	—	39·6	—	44·5?	—	—	—	73
97·1	92·9	72·8	130	99·0	25·9	53·0?	—	—	35·5	39·8	46·8	39·9?	56·0	47·5	49·0	12·6	113·3	77
97·8	—	—	—	—	—	—	9·2	3·4	—	—	—	—	—	—	—	—	—	74
—	—	—	—	—	—	—	—	—	—	—	—	—	—	49·5	—	—	—	74
100·3	91·1?	71·2?	—	—	28·6	51·4	11·2	3·9	33·3	—	44·8	—	—	—	—	—	—	74
104·3	—	—	140·5	—	—	—	10·0	6·1	31·6	43·3	—	—	67·5	—	59·9	12·7	—	74
—	—	—	—	—	—	—	11·6	4·2	—	—	—	—	—	—	—	—	—	74
102·9	—	—	—	94·4	—	—	12·0	5·4	—	43·5?	—	—	—	53·0	—	—	—	74
103·0	88·4?	74·3?	131	93·5	24·9	55·4	10·0	6·3	34·0	42·2	—	—	55·0	48·5	49·8	9·5	118·7	8
99·8	—	—	129·5	91·2	28·2	51·1	—	—	30·6	41·0	51·7	—	66·5	59·5	55·5	12·9	116·3	74
101·0	—	—	128	86·7	28·3	53·8	12·6	5·3	31·5	43·7	—	—	61·0	56·5	53·5	13·8	118·0	74
103·1	95·3	70·6	134·5	95·3	25·5	50·3	9·8	4·5	34·2	44·4	47·1	—	61·0	53·5	54·7?	9·1?	113·5	74
102·0	93·4?	77·3	—	—	—	56·7	11·9?	5·8	35·9	43·8	—	—	61·0	54·5	52·9	11·0	118·8	74
104·7	99·5	76·1	—	—	—	52·8	9·2	5·1	34·7	41·9	49·2	44·9	—	49·5	—	—	114·9	74
—	—	—	—	—	24·8	52·1?	—	—	32·1	42·3	46·3	—	53·0	45·0	48·1	9·0	116·7	74
97·2	89·0?	62·5?	—	—	25·9	48·5	9·7	4·4	32·1	41·8	—	41·1	58·0	45·5	51·0	12·3	112·8	74
95·7	—	71·3?	—	94·6	25·8	53·5	—	—	33·8	41·0	—	—	—	52·0	—	—	—	74
104·0	—	—	—	98·2	23·3	48·0	10·2	7·3	32·5	42·2	51·0	42·5	65·5	51·0	56·7	14·4	117·2	74
104·6	97·3	66·9	—	99·7	26·8	47·4	11·1	4·3	31·4	41·9	47·1	43·6	—	51·5	—	—	—	74
101·9?	95·8	69·0?	—	98·7	27·1?	50·6	—	—	29·6	39·7	44·1?	37·3	—	—	—	—	119·6	74
—	—	—	—	109·8	24·4	54·7	10·1	3·8	34·8	43·1	50·1?	—	—	48·0	—	—	—	74
94·7	91·4?	59·8?	—	91·2	27·7	42·9	12·3	4·9	26·0	41·0	44·9	—	—	46·0	—	—	114·6	74

easurements of Lachish Skulls. **I.** ***Male Crania from Tomb*** **120**

$\frac{S}{ml}$	OH	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	$Oc.I.$	$100\frac{fmb}{fml}$	$100\frac{G'H}{GB}$	$100\frac{NB}{NH}, L$	$100\frac{O_2}{O_1}, L$	$100\frac{G_2}{G_1'}$	$100\frac{SS}{SC}$	$100\frac{Ml_2}{Ml_1}$	$100\frac{S}{C}(ml)$	
—	112·7	71·5	72·3	98·9	−0·8	61·9	85·5	76·5	48·9	—	98·3	44·2	—	—	8
1·6	117·3	80·9	76·1	106·3	4·8	62·1	77·3	—	51·4	82·6	—	50·8	86·8	23·7	
0·8	116·0	74·0?	—	—	—	57·5	—	—	51·8	74·7	—	54·2	94·2	22·9	8
—	113·1	70·4	72·5	97·1	−2·1	58·2	74·6	—	—	84·2	—	65·0	—	—	8
—	118·1	80·5	74·8	107·7	5·8	58·8	—	68·9	—	82·4	—	50·0	—	—	8
3·6?	115·0	73·7	71·8	102·6	1·9	58·1	90·1	74·1	47·7	79·7	—	50·0	78·7	25·7	8
—	115·9	70·1	68·8	101·9	1·3	—	—	73·2	53·8	81·3	—	48·9	—	—	8
2·1?	119·8	81·0	—	—	—	—	—	69·9	55·8	74·9	—	43·3	78·4	21·5	8
2·9	111·4	73·9	72·3	102·2	1·6	59·6	81·3	82·6	42·7	87·7	—	47·7	83·7	23·9	8
9·9	121·5	75·0	77·2	97·2	−2·2	58·2	89·0	—	55·4	74·6	—	45·1	80·0	17·6	
3·2	—	74·1?	72·0?	102·9	2·1	—	—	76·4	—	87·5	88·5	75·7	89·1	25·8	8
1·0?	114·9	74·9	76·3	98·2	−1·4	61·0	—	79·5	48·1	76·4	—	41·5	—	22·6	8
—	115·0	74·9	72·3	103·6	2·6	62·1	76·9	—	42·4	76·2	86·3	48·1	—	—	
—	116·1	75·8	74·4	101·9	1·4	62·6	86·4	—	49·5	80·8	79·7	40·9	—	—	8
1·9	118·0	75·6	73·0	103·5	2·6	57·2	79·1	—	53·2	77·0	—	39·4	81·0	22·0	
3·6	121·0	76·2	—	—	—	—	—	—	49·7	78·1	—	40·0	78·8	27·5	
—	117·2	72·3	76·0	95·2	−3·6	62·7	74·9	—	—	71·3	—	35·7	—	—	
2·0	119·0	73·9	77·8	95·0	−3·9	58·5	85·0	75·0	53·6	75·9	—	—	93·2	23·9	8
2·3	114·6	81·7	75·3	108·6	6·5	59·0	78·2	—	49·0	78·0	—	44·8	—	21·5	
2·5	—	—	69·3	—	—	56·8	70·8	—	55·0	72·5	—	37·7	78·8	22·0	
—	—	75·3?	73·3?	102·7?	1·9?	—	—	—	51·8	77·7	—	—	—	—	
—	—	76·2	—	—	—	—	—	—	—	—	—	55·9	—	—	
—	118·2	73·3	73·9	99·3	−0·5	58·2	83·3	—	41·9	—	89·6	72·7	—	—	
—	—	75·5	—	—	—	63·4	—	—	—	—	—	45·4	—	—	
—	114·0	70·5	73·9	95·3	−3·5	58·8	76·9	—	46·2	76·3	83·7	—	—	—	
—	110·6	76·0	69·7	109·1	6·3	57·1	75·5	—	—	77·6	—	40·9	—	—	
3·2	116·5	71·9	71·7	100·4	0·3	59·4	87·1	72·3	42·9	82·7	80·8	39·8	74·2	23·1	
4·0	117·2	75·9	73·7	102·9	2·2	59·0	85·1	72·9	52·4	83·4	86·4	53·8	77·7	25·5	
3·1	119·8	77·2	73·7	104·7	3·5	61·3	75·6	80·0	—	—	—	46·7	86·5	24·0	
1·3	117·0	73·6	70·8	104·1	2·9	56·2	76·0	84·4	48·9	77·1	—	46·6	93·7	22·2	
—	117·2	73·3	—	—	—	58·1	—	—	46·9	81·5	—	55·1	—	—	
3·4	107·6	74·8	72·0	103·8	2·8	59·6	79·6	78·6	46·4	80·5	—	55·3	78·9	24·4	
0·3	113·0	71·8	75·1	95·5	−3·4	65·8	80·6	—	54·3	82·0	—	31·0	81·0	18·8	
3·3	—	72·4	69·2	104·6	3·2	—	77·8	—	—	79·3	—	31·5	80·7	27·9	
2·0	—	—	71·6	—	—	60·5	75·9	—	—	81·0	—	45·7	87·9	21·4	
—	117·0	69·7	72·2	96·4	−2·6	55·8	74·8	77·1	52·7	77·1	—	59·7	—	—	
—	—	74·9	64·9	115·4	10·0	59·3	75·0	—	—	79·6	93·4	50·6	—	—	
—	—	75·6	—	—	—	60·6	—	—	—	78·8	—	—	—	—	
2·6	113·3	76·0	69·9	108·8	6·1	60·2	89·4	73·5	48·9	89·2	85·3	—	84·8	25·7	
—	—	73·6	73·0	100·7	0·5	57·6	93·8	—	—	—	—	37·0	—	—	
—	—	75·5	71·7	105·2	3·7	55·3	73·7	—	—	—	—	—	—	—	
—	—	75·5	72·9	103·6	2·7	57·3	79·2	—	55·6	—	—	34·8	—	—	
2·7	—	75·0	74·5	100·7	0·6	61·3	87·2	—	—	73·0	—	61·0	—	21·2	
—	—	77·4	69·5	111·2	7·8	54·7	—	—	—	—	—	36·2	—	—	
—	—	77·5	71·8	108·0	5·8	59·7	85·6	—	—	—	—	45·0	—	—	
9·5	118·7	80·0	80·8	99·0	−0·8	63·5	79·0	79·5	44·9	80·6	—	63·0	88·2	19·1	
2·9	116·3	73·7	72·1	102·2	1·6	63·5	83·0	—	55·2	74·6	—	—	89·5	23·2	
3·8	118·0	70·7	73·4	96·3	−2·7	59·3	88·5	—	52·6	72·1	—	42·0	92·6	25·8	
9·1?	113·5	71·2	71·0	100·3	0·3	56·5	86·4	74·1	50·7	77·0	—	45·9	87·7	16·6	
1·0	118·8	76·1	74·2	102·5	1·9	57·2	—	—	—	82·0	—	48·7	89·3	20·8	
—	114·9	72·8	74·7	97·4	−2·0	57·2	—	—	—	82·8	91·3	55·4	—	—	
9·0	116·7	72·3	—	—	—	59·3	—	—	47·6	75·9	—	—	84·9	18·7	
2·3	112·8	73·4	76·5	96·0	−3·1	62·4	81·0	—	53·4	76·8	—	45·4	78·4	24·1	
—	—	78·0	69·1	112·7	8·8	—	—	75·4	48·2	82·4	—	—	—	—	
4·4	117·2	71·9	73·8	97·4	−1·9	57·5	89·6	—	48·5	77·0	83·3	71·6	77·9	25·4	
—	—	77·6	75·7	102·5	1·9	61·1	89·1	67·1	56·5	74·9	92·6	38·7	—	—	
—	119·6	74·7	73·4	101·9	1·4	59·1	88·9	69·9	53·6	74·6	84·6	—	—	—	
—	—	74·1	—	—	—	65·4	—	—	44·6	80·7	—	37·6	—	—	
—	114·6	78·6	75·1	104·6	3·5	60·5	78·8	65·6	64·6	63·4	—	39·8	—	—	

$100\frac{SS}{SC}$	$100\frac{Ml_2}{Ml_1}$	$100\frac{S}{C}(ml)$	$P\angle$	$N\angle$	$A\angle$	$B\angle$	No.	Remarks
44·2	—	—	85°·5	65°·7?	72°·5?	41°·8?	1	Diseased area R parietal
50·8	86·8	23·7	—	—	—	—	2	Sagittal closed, lambdoid beginning to close; single large epipteric bone R; 5 molar before death
54·2	94·2	22·9	82°·5	—	—	—	3	Nasal bones fused; 1 molar lost before death
65·0	—	—	88°·5	61°·1?	76°·2?	42°·7?	4	Coronal closing, sagittal beginning to close; occipito-mastoid L obliterated; palate def
50·0	—	—	84°·5	65°·1?	75°·3?	39°·6?	5	Two molars lost before death; large wound on frontal
50·0	78·7	25·7	85°·0	—	—	—	6	Coronal and sagittal beginning to close; 5 molars lost before death; fused nasal b surface of parietal bones diseased?
48·9	—	—	87°·5	64°·1?	74°·8?	41°·1?	7	Coronal and sagittal beginning to close; fused nasal bones; probably no teeth lost death
43·3	78·4	21·5	88°·0	—	—	—	8	Coronal beginning to close; 1 molar lost before death
47·7	83·7	23·9	86°·5	58°·7	71°·1	50°·2	9	Coronal and sagittal closing
45·1	80·0	17·6	—	—	—	—	10	Single epipteric bone L; 1 incisor and 1 molar lost before death; M3's absent; a cavity at root of M 1, L
75·7	89·1	25·8	81°·0	67°·6	68°·7	43°·7	11	—
41·5	—	22·6	87°·0	61°·1?	74°·3?	44°·6?	12	Coronal and sagittal beginning to close; single epipteric bone R; 1 molar lost before M3's probably absent
48·1	—	—	—	—	—	—	13	Large ossicle at asterion R, and another above it between temporal squama and pa 2 epipteric bones R
40·9	—	—	85°·0	63°·8?	75°·2?	40°·9?	14	Single epipteric bones R and L; apparently 3 teeth lost before death, unusual abso of alveolar margin; ? injury on malar bone R
39·4	81·0	22·0	—	—	—	—	15	Coronal beginning to close; fused nasal bones, very broad
40·0	78·8	27·5	—	—	—	—	16	Coronal beginning to close, sagittal and lambdoid closed; most teeth lost before dea
35·7	—	—	—	—	—	—	17	Coronal and sagittal beginning to close; palate defective; ? healed wound R parieta
—	93·2	23·9	83°·0	61°·7?	78°·1?	40°·2?	18	Coronal and lambdoid beginning to close, sagittal closed; 2 molars lost before death
44·8	—	21·5	—	—	—	—	19	*Os épactal*; 2 or 3 molars lost before death, M3's apparently absent
37·7	78·8	22·0	—	—	—	—	20	Sagittal closing, lambdoid beginning to close; palate defective
—	—	—	—	—	—	—	21	Single epipteric bones R and L; fused nasal bones; 1 molar lost before death apparently absent; single pre-condyle L
55·9	—	—	—	—	—	—	22	Palate defective
72·7	—	—	—	—	—	—	23	*Os épactal*; 3rd molars absent
45·4	—	—	—	—	—	—	24	Sagittal closing; 2 wormian bones in coronal L; large single, or multiple, epipteric R and L; single large ossicle between temporal squama and parietal R and L; defective
—	—	—	84°·0	62°·8	74°·3	42°·9	25	Coronal and sagittal beginning to close
40·9	—	—	95°·0?	57°·8?	70°·8?	41°·5?	26	Palate defective
39·8	74·2	23·1	86°·5?	64°·4	72°·5	43°·1	27	Metopic; metopic, coronal, and sagittal beginning to close
53·8	77·7	25·5	83°·5	67°·3	73°·1	39°·6	28	M 3, R below alveolar margin
46·7	86·5	24·0	84°·5?	66°·2?	69°·7?	44°·6?	29	Sagittal and lambdoid closing
46·6	93·7	22·2	93°·0?	58°·4?	78°·2?	43°·3?	30	Coronal beginning to close; 3 or 4 molars lost before death; 2 small exostoses on fr
55·1	—	—	90°·5?	—	—	—	31	Coronal and sagittal beginning to close; 1 or more molars lost before death
55·3	78·9	24·4	80°·5	65°·7	70°·7	43°·6	32	Sagittal beginning to close; single epipteric bone L
31·0	81·0	18·8	—	—	—	—	33	Coronal and sagittal beginning to close; nearly all teeth lost before death
31·5	80·7	27·9	—	—	—	—	34	Sagittal and lambdoid closing; single large epipteric bone L; palate missing; 2 wounds L parietal
45·7	87·9	21·4	—	—	—	—	35	Coronal and sagittal beginning to close; palate defective
59·7	—	—	—	64°·9?	76°·2?	38°·8?	36	Wormian bone in coronal R; small epipteric bone R; single small ossicles between p and temporal squama R and L; palate defective
50·6	—	—	—	—	—	—	37	—
—	—	—	—	—	—	—	38	M3's absent
—	84·8	25·7	—	64°·2	70°·7	45°·0	39	Two supernumerary bones between temporal squama and parietal L; M 1, R lost death, large abscess cavity at root of M 1, L
37·0	—	—	—	—	—	—	40	Single epipteric bone L; palate defective
—	—	—	—	—	—	—	41	Occipito-mastoid L obliterated; palate missing
34·8	—	—	—	61°·4?	75°·8?	42°·7?	42	Coronal and sagittal beginning to close; numerous wormian bones in lambdoid
61·0	—	21·2	—	—	—	—	43	Coronal and sagittal closing; palate missing
36·2	—	—	—	—	—	—	44	Palate missing
45·0	—	—	—	—	—	—	45	Coronal and sagittal beginning to close; palate missing
63·0	88·2	19·1	89°·0	56°·4?	79°·0?	44°·6?	46	Coronal closing, sagittal and lambdoid nearly obliterated; 1 molar lost before deat
—	89·5	23·2	—	—	—	—	47	Sagittal beginning to close; 1 molar lost before death; healed wound on malar bon
42·0	92·6	25·8	—	—	—	—	48	Metopic; coronal and sagittal beginning to close; single epipteric bone L; some te before death
45·9	87·7	16·6	87°·5	63°·3	75°·1	41°·5	49	Coronal and sagittal beginning to close; fused nasal bones
48·7	89·3	20·8	—	60°·9?	72°·8?	46°·3?	50	Metopic; sagittal closing; large epipteric bone R; palate defective
55·4	—	—	87°·0	64°·5	71°·9	43°·6	51	—
—	84·9	18·7	—	—	—	—	52	Single epipteric bones R and L; M 3, L apparently absent
45·4	78·4	24·1	87°·5?	63°·3?	77°·9?	38°·8?	53	Coronal and sagittal beginning to close
—	—	—	—	—	—	—	54	One molar lost before death
71·6	77·9	25·4	—	—	—	—	55	Small epipteric bone L
38·7	—	—	87°·0	64°·9	76°·8	38°·3	56	One premolar lost before death
—	—	—	—	64°·8?	74·°5?	40°·7?	57	Coronal and lambdoid beginning to close; sagittal closing; small epipteric bone R
37·6	—	—	—	—	—	—	58	—
39·8	—	—	85°·0?	68°·1?	74·°4?	37°·4?	59	Superior half R side of coronal obliterated, sagittal beginning to close; large bone L

bone R; 5 molars lost
erated; palate defective
th; fused nasal bones;
ly no teeth lost before
; M3's absent; abscess
nolar lost before death,
l squama and parietal;
th, unusual absorption
th lost before death wound R parietal lost before death t
lost before death, M3's
nultiple, epipteric bones arietal R and L; palate
ll exostoses on frontal ore death
e death palate missing; 2 healed
ssicles between parietals
al L; M 1, R lost before
s in lambdoid
r lost before death und on malar bone L bone L; some teeth lost
e
epipteric bone R
to close; large epipteric

10	186	139·5	99·5	145·5	115·4	120·2	95·7	31	33·5									
11	193?	143	96·2	139	119·5	—	—	135	—	—	—	310	539	—	33·5	106·2	10
12	177·5	133	95·0	135·5	110·3	116·9	91·4	125·5	131	108·5	366	308	505	40·7	—	99·9	9
13	189·5	142	93·4	137	115·8	119·7	94·8	128	132·5	111·5	373	306	529	40·3	31·0	105·2	
14	181·5	137·5	99·7	135	110·5	119·5	94·4	127·5	138·5	110·5	376	310	516	37·4	32·3	101·5	9
15	194·5	147	97·4	142	118·8	123·9	103·5	134·5	140	129	403	313	535	36·3	28·7	100·9	
16	185	141	94·1	—	121·6	116·8	—	142	132·5	—	—	324	525	—	—	—	
17	179	129·5	87·7	136	117·0	111·6	92·7	134	127·5	108·5	371	305	497	34·6	25·9	101·2	
18	180	133	92·1	140	111·0	123·3	90·0	126·5	137	110	373	314	504	35·3	30·0	103·4	9
19	178	145·5	100·4	134	112·0	105·4	103·4	127	114·5	125·5	368	317	521	38·6	30·2	97·7	
20	187	—	100·0	129·5	109·2	120·6	95·2	123·5?	134	119·5	376?	300?	518?	39·4	27·9	99·8	
21	180?	135·5?	86·6	132	112·0	111·6	—	127	120	—	—	299	494?	—	—	—	
22	191	145·5	101·3	—	111·7	118·4	—	129	131	—	—	324	547	—	—	—	
23	185·5	136	92·8	137	110·5	109·4	102·2	123·5	120	125·5	369	314	522	37·8	31·5	106·1	
24	188	142	91·6	—	119·5	129·2	102·3	137·5	143	119	399	325	534?	—	—	—	
25	188	132·5	89·8	139	111·2	113·8	99·8	124·5	128	121·5	375	302	516	37·7	29·0	106·5	9
26	181·5	138	94·4	126·5	109·9	107·3	92·0	123·5	118	115	356	298	510?	38·4	29·0	101·3	8
27	190·5	137	98·7	136·5	113·0	116·7	104·2	125·5	127·5	126	378	306	528	37·1	32·3	105·6	10
28	184·5	140	95·6	136	107·5	117·3	100·5	120·5	135	122	377	309	519	33·6	28·6	106·3	10
29	186·5	144	96·2	137·5	114·5	121·8	99·7	127·5	137	118	382	321	529	38·6	29·2	99·0	9
30	191·5	141	91·2	135·5	112·6	121·2	96·0	125	134	122	381	310	531	40·0?	30·4?	104·0?	9
31	187	137	99·3	—	114·2	116·3	96·0	132·5	127·5	118	377	311	519	—	31·0	—	
32	180·5	135	89·0	130	105·7	112·9	92·5	118	126·5	111·5	356	295?	500?	39·3	31·3	100·3	9
33	179	128·5	98·2	134·5	116·0	107·4	93·5	134·5	123	107	364	301	501	36·0	29·0	100·2	
34	190	137·5	—	131·5	117·0?	—	—	134?	—	—	385?	304	526?	37·0	28·8	103·5	
35	192	—	90·5	137·5	115·7	123·1?	98·1	133·5	136·5?	117	387?	318	539	37·7	28·6	103·6	
36	194·5	135·5	96·1	140·5	116·5	120·0	99·0	138	131	127	395	321	537	38·5	28·8	104·4	9
37	185	138·5	93·5	120	105·6	117·1?	97·9?	121	134	118·5	373	296?	520?	37·6	28·2	95·8	
38	174	131·5	94·4	—	113·2	113·8	86·8	131	127·5	103·5	362	302	495	—	—	—	
39	187·5	142·5	93·5	131	116·6	118·5	99·0	135·5	131	118·5	386	315	532	35·7	31·9	97·1	9
40	183·5	135	95·3	134	118·4	117·7	91·7	139	134·5	113·5	386	308	514	33·9	31·8	97·8	
41	187·5	141·5	91·6	134·5	—	118·0?	90·9?	—	132?	118	—	303	519	44·9?	33·1	—	
42	188	142	98·4	137	115·6	115·3	100·4	132·5	125	125	383	—	534	36·1	28·6	100·3	9
43	182	136·5	94·7	135·5	114·2	121·2	90·8	129·5	135·5	107·5	373	—	518	32·0	27·9	104·3	
44	185·5	143·5	91·1	129	105·5	96·1	107·9?	117	103	142·5	362	309?	520?	—	28·5	—	
45	182·5	141·5	94·7	131	112·7	119·2	93·3	124	135	112·5	371	307	519	33·4	28·6	102·9	
46	177·5	142	96·9	143·5	120·5	108·0	95·1	138·5	123	110·5	371	318	511	37·2	29·4	103·0	8
47	188·5	139	92·6	136	119·1	115·8	98·1	136	128·5	114	378	311	521	38·3	31·8	99·8	
48	184	130	98·2	135	111·2	125·3	91·7	125·5	142·5	111	379	305	516	35·6	31·5	101·0	
49	186	132·5	—	132	116·0	114·3	91·9	135	128·5	116	380	307?	526?	36·0	31·1	103·1	9
50	188	143	105·8	139·5	118·2	111·2	97·5	139	124	121·5	385	322	536	41·6	—	102·0	9
51	178	129·5	97·2	133	110·3	110·6	92·2	127·5	119	115	360	307	498	36·6	—	104·7	9
52	184	133	97·1	—	107·0?	116·1	100·0	117·5?	128·5	121	366?	304?	518	—	30·7	—	
53	178·5	131	94·2	136·5	113·3	121·5	93·0	133	134	109	375	304	502	35·8	29·0	97·2	8
54	181·5	141·5	95·0	125·5	109·7?	111·2	—	125?	121·5	—	—	305	521	—	—	95·7	
55	185	133	97·2	136·5	107·6	120·0	99·5	125	134·5	123·5	383	310	521	31·8	28·5	104·0	9
56	183	142	102·0	138·5	117·4	115·3	97·0	135·5	132·5	115	382	319	523	34·8	31·0	104·6	9
57	184	137·5	95·3	135	112·3?	113·3	100·2	126·5?	124·5	121·5	373?	312	521	37·0	32·9	101·9?	9
58	191	141·5?	98·5	—	115·1	119·0	100·3	128·5	134·5	115	377	315?	535?	—	—	—	
59	173	136	88·2	130	111·3	104·9	98·0	129	118	117	363	303	489	36·4	28·7	94·7	9
60	181·5	135	100·6	137·5	112·2	112·1	94·0	127	122·5?	114	363?	309	509	38·0	32·1	105·9	
61	185	136·5	97·3	133	110·5	—	—	126	—	—	381	306	514?	36·5	29·9	97·3	9
62	191·5	142·5	96·9	139·5	120·3	122·3	90·7	139	139·5	113	392	323	528	39·0	29·0	104·3	9
63	182	136·5	99·2	134·5	108·5?	118·9	96·0	121·5?	132	115	369	314	517	40·1	34·6	101·3?	
64	178	139·5	91·9	135·5	116·2?	118·0?	90·7	135?	133·5?	106·5	374?	318	509?	35·7	28·4	97·6	
65	184·5	141·5	96·4	124?	116·1	—	—	135	—	—	369	312	524	34·3	25·5	100·3	
66	178·5	131·5	93·8	134	109·9	117·3	93·4	127·5	134	114	376	307	509	31·4	29·1	101·6	9
67	196·5?	—	95·7	136	116·8	119·2	107·8	132 ?	129·5?	132·5?	395?	317?	543	41·0?	31·2?	106·4	10
68	181·5	134·5	95·1	—	—	—	—	—	—	—	374?	306?	513	34·1	31·7	99·3	
69	175·5	134	86·6	126	106·0	112·5?	94·0?	122	126·5	116	364	291	496	29·9	28·6	96·0	9
70	185	140	90·6	130?	111·2	122·5?	94·1?	126·5	136	116·5	379	310	519	37·0	29·6	95·5?	
71	184·5	142	94·3	134	106·3	115·7	95·3	119·5	129	114·5	364	311	520	37·0	30·0	109·4	10
72	187	133·5	90·0?	136	117·6	112·5	100·1	139·5	127	127	393	302	516	34·9	29·2	97·4	8

3·5	106·2	105·5	78·8	129	103·1	26·2	—	7·6	5·3	35·6	40·7	51·4	45·5	59·5	53·6	51·2	13·2
—	99·9	91·1?	73·1	—	91·9?	24·9	51·8	13·0	5·4	32·7?	42·8	—	—	—	—	48·6?	11·0
1·0	105·2	—	—	—	96·7	23·6	55·7	13·1	6·3	32·4	42·5	45·3	39·1?	—	51·5	—	—
2·3	101·5	94·3?	68·7?	—	—	25·0	50·5	11·5	4·7	34·5?	42·7?	44·9	35·8	—	—	—	—
8·7	100·9	—	—	127·5	93·3	27·9	52·4	16·5	6·5	32·2	41·8?	42·2	—	63·0	51·0	54·0	11·9
—	—	—	—	—	94·4	23·5	47·3	9·5	3·8	30·7	39·3	42·2	—	59·0	46·5	49·5	13·6
5·9	101·2	—	—	—	—	—	—	11·2	4·0	27·8	39·0	—	—	—	47·5	—	—
0·0	103·4	93·2?	68·5?	126·5	91·3	26·2	48·9	10·0	—	34·0	44·8	48·8	—	59·0	55·0	50·2	12·0
0·2	97·7	—	—	133	96·9	23·9	48·8	9·6	4·3	33·0	42·3	41·7	—	—	52·5	57·1	12·3
7·9	99·8	—	—	—	—	29·4	53·5	13·0	4·9	28·7	39·6	—	—	66·0	52·0	56·8	12·5
—	—	—	—	—	—	26·3	50·8	—	—	28·6	36·8	—	—	—	—	—	—
—	—	—	—	—	—	—	—	10·2	5·7	—	—	—	—	—	—	—	—
1·5	106·1	—	—	—	—	22·3	53·2	9·9	7·2	36·1?	—	44·1	39·5	—	—	—	—
—	—	—	—	—	—	—	—	11·9	5·4	—	—	—	—	—	—	—	—
9·0	106·5	98·5	75·5	—	—	24·2?	52·4	—	—	31·8?	41·7	47·8	40·0?	—	—	—	—
9·0	101·3	87·0?	68·0?	—	—	—	49·7	13·7	5·6	32·2	41·5	—	—	—	49·0	—	—
2·3	105·6	100·0	75·8	—	104·8	22·6	52·7	11·3	4·5	35·0	42·3	49·6	40·1	66·0	49·0	57·2	13·2
8·6	106·3	102·7	70·6	125·5	96·8	28·8	55·0	14·5	7·8	32·7	39·2	50·0	43·2?	65·0	50·5	54·8	14·0
9·2	99·0	96·8?	73·5?	129	91·9	—	53·0?	12·0	5·6	—	—	—	41·2	63·0	54·5	54·6	13·1
0·4?	104·0?	90·5?	73·0?	—	86·5	25·5	52·2	8·8	4·1	32·6	42·3	—	—	56·0	52·5	51·0	11·3
1·0	—	—	70·4	—	—	25·0	53·3	10·7	5·9	33·9	41·6	46·4?	—	—	—	—	—
1·3	100·3	97·1	73·3	132	93·2	25·3	54·5	7·6	4·2	36·0	44·7	—	42·4?	64·0	50·5	55·0	13·4
9·0	100·2	—	—	130	84·3	24·5	45·1	10·0?	3·1	33·2	40·5	—	—	60·5	49·0	54·9	10·3
8·8	103·5	—	—	—	—	—	—	10·8	3·4	32·5	41·0	—	—	57·0	46·0	47·6	13·3
8·6	103·6	—	—	128	—	—	—	9·4	4·3	35·0	43·2	—	—	62·0	54·5	56·0	12·0
8·8	104·4	97·4?	67·3?	—	87·3	25·8	49·0	11·9	7·1	33·0	42·8	50·4	—	—	50·5	—	—
8·2	95·8	—	—	—	92·3	—	50·4	8·3	4·2	32·7	41·1	42·5	39·7	—	—	—	—
—	—	—	—	—	95·0?	—	—	—	—	31·9	40·5	—	39·6	—	44·5?	—	—
1·9	97·1	92·9	72·8	130	99·0	25·9	53·0?	—	—	35·5	39·8	46·8	39·9?	56·0	47·5	49·0	12·6
1·8	97·8	—	—	—	—	—	—	9·2	3·4	—	—	—	—	—	—	—	—
3·1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	49·5	—	—
8·6	100·3	91·1?	71·2?	—	—	28·6	51·4	11·2	3·9	33·3	—	44·8	—	—	—	—	—
7·9	104·3	—	—	140·5	—	—	—	10·0	6·1	31·6	43·3	—	—	67·5	—	59·9	12·7
8·5	—	—	—	—	—	—	—	11·6	4·2	—	—	—	—	—	—	—	—
8·6	102·9	—	—	—	94·4	—	—	12·0	5·4	—	43·5?	—	—	—	53·0	—	—
9·4	103·0	88·4?	74·3?	131	93·5	24·9	55·4	10·0	6·3	34·0	42·2	—	—	55·0	48·5	49·8	9·5
1·8	99·8	—	—	129·5	91·2	28·2	51·1	—	—	30·6	41·0	51·7	—	66·5	59·5	55·5	12·6
1·5	101·0	—	—	128	86·7	28·3	53·8	12·6	5·3	31·5	43·7	—	—	61·0	56·5	53·5	13·8
1·1	103·1	95·3	70·6	134·5	95·3	25·5	50·3	9·8	4·5	34·2	44·4	47·1	—	61·0	53·5	54·7?	9·5
—	102·0	93·4?	77·3	—	—	—	56·7	11·9?	5·8	35·9	43·8	—	—	61·0	54·5	52·9	11·6
—	104·7	99·5	76·1	—	—	—	52·8	9·2	5·1	34·7	41·9	49·2	44·9	—	49·5	—	—
0·7	—	—	—	—	—	24·8	52·1?	—	—	32·1	42·3	46·3	—	53·0	45·0	48·1	9·6
9·0	97·2	89·0?	62·5?	—	—	25·9	48·5	9·7	4·4	32·1	41·8	—	41·1	58·0	45·5	51·0	12·1
—	95·7	—	71·3?	—	94·6	25·8	53·5	—	—	33·8	41·0	—	—	—	52·0	—	—
8·5	104·0	—	—	—	98·2	23·3	48·0	10·2	7·3	32·5	42·2	51·0	42·5	65·5	51·0	56·7	14·1
1·0	104·6	97·3	66·9	—	99·7	26·8	47·4	11·1	4·3	31·4	41·9	47·1	43·6	—	51·5	—	—
2·9	101·9?	95·8	69·0?	—	98·7	27·1?	50·6	—	—	29·6	39·7	44·1?	37·3	—	—	—	—
—	—	—	—	—	109·8	24·4	54·7	10·1	3·8	34·8	43·1	50·1?	—	—	48·0	—	—
8·7	94·7	91·4?	59·8?	—	91·2	27·7	42·9	12·3	4·9	26·0	41·0	44·9	—	—	46·0	—	—
2·1	105·9	—	—	142	103·3	27·3	51·5	6·8	3·2	32·5	45·1	49·1	—	65·0	51·0	59·9	14·1
9·9	97·3	93·0	74·5?	—	—	24·0	53·5	11·6	5·7	—	—	48·2	—	—	—	—	—
9·0	104·3	92·5	70·5	—	86·5	25·8	51·1	9·5	3·9	33·0	41·2	—	—	—	54·0	—	—
4·6	101·3?	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8·4	97·6	—	—	124·5	101·7	—	—	11·2	3·9	31·2	39·5	—	—	—	50·5	—	—
5·5	100·3	—	—	—	88·0	—	—	8·6	4·9	—	40·3?	—	—	—	49·5	—	—
9·1	101·6	99·0	68·4	128	95·0	24·8	49·1	11·2	5·3	31·0	38·9	47·6	—	65·0	50·0	53·3	15·1
1·2?	106·4	103·7	71·1	—	—	—	—	—	—	32·0	42·2	49·8?	—	—	52·0	—	—
1·7	99·3	—	—	—	—	—	—	11·0	5·5	34·1	—	—	—	—	—	—	—
8·6	96·0	99·0	64·6	—	94·2	22·7	46·3	9·2	2·9	29·7	38·2	47·1	39·9	—	49·0	—	—
9·6	95·5?	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0·0	109·4	107·8	65·0	—	—	—	48·1	11·1?	—	31·1	41·7	—	—	67·0	48·5	56·9	17·1
9·2	97·4	87·7?	72·4?	125·5	88·0	—	49·7	5·7	2·8	32·9	39·6	47·3	36·5	58·5	51·0	52·4	11·1

—	48·6?	11·0?	114·9	74·9	76·3	98·2	−1·4	61·0	—	79·5	48·1	76·4	—	41·5	—
1·5	—	—	115·0	74·9	72·3	103·6	2·6	62·1	76·9	—	42·4	76·2	86·3	48·1	—
—	—	—	116·1	75·8	74·4	101·9	1·4	62·6	86·4	—	49·5	80·8	79·7	40·9	—
1·0	54·0	11·9	118·0	75·6	73·0	103·5	2·6	57·2	79·1	—	53·2	77·0	—	39·4	81·0
6·5	49·5	13·6	121·0	76·2	—	—	—	—	—	—	49·7	78·1	—	40·0	78·8
7·5	—	—	117·2	72·3	76·0	95·2	−3·6	62·7	74·9	—	—	71·3	—	35·7	—
5·0	50·2	12·0	119·0	73·9	77·8	95·0	−3·9	58·5	85·0	75·0	53·6	75·9	—	—	93·2
2·5	57·1	12·3	114·6	81·7	75·3	108·6	6·5	59·0	78·2	—	49·0	78·0	—	44·8	—
2·0	56·8	12·5	—	—	69·3	—	—	56·8	70·8	—	55·0	72·5	—	37·7	78·8
—	—	—	—	75·3?	73·3?	102·7?	1·9?	—	—	—	51·8	77·7	—	—	—
—	—	—	—	76·2	—	—	—	—	—	—	—	—	—	55·9	—
—	—	—	118·2	73·3	73·9	99·3	−0·5	58·2	83·3	—	41·9	—	89·6	72·7	—
—	—	—	—	75·5	—	—	—	63·4	—	—	—	—	—	45·4	—
—	—	—	114·0	70·5	73·9	95·3	−3·5	58·8	76·9	—	46·2	76·3	83·7	—	—
9·0	—	—	110·6	76·0	69·7	109·1	6·3	57·1	75·5	—	—	77·6	—	40·9	—
9·0	57·2	13·2	116·5	71·9	71·7	100·4	0·3	59·4	87·1	72·3	42·9	82·7	80·8	39·8	74·2
0·5	54·8	14·0	117·2	75·9	73·7	102·9	2·2	59·0	85·1	72·9	52·4	83·4	86·4	53·8	77·7
4·5	54·6	13·1	119·8	77·2	73·7	104·7	3·5	61·3	75·6	80·0	—	—	—	46·7	80·5
2·5	51·0	11·3	117·0	73·6	70·8	104·1	2·9	56·2	76·0	84·4	48·9	77·1	—	46·6	93·7
—	—	—	117·2	73·3	—	—	—	58·1	—	—	46·9	81·5	—	55·1	—
0·5	55·0	13·4	107·6	74·8	72·0	103·8	2·8	59·6	79·6	78·6	46·4	80·5	—	55·3	78·9
9·0	54·9	10·3	113·0	71·8	75·1	95·5	−3·4	65·8	80·6	—	54·3	82·0	—	31·0	81·0
6·0	47·6	13·3	—	72·4	69·2	104·6	3·2	—	77·8	—	—	79·3	—	31·5	80·7
4·5	56·0	12·0	—	—	71·6	—	—	60·5	75·9	—	—	81·0	—	45·7	87·9
0·5	—	—	117·0	69·7	72·2	96·4	−2·6	55·8	74·8	77·1	52·7	77·1	—	59·7	—
—	—	—	—	74·9	64·9	115·4	10·0	59·3	75·0	—	—	79·6	93·4	50·6	—
4·5?	—	—	—	75·6	—	—	—	60·6	—	—	—	78·8	—	—	—
7·5	49·0	12·6	113·3	76·0	69·9	108·8	6·1	60·2	89·4	73·5	48·9	89·2	85·3	—	84·8
—	—	—	—	73·6	73·0	100·7	0·5	57·6	93·8	—	—	—	—	37·0	—
9·5	—	—	—	75·5	71·7	105·2	3·7	55·3	73·7	—	—	—	—	—	—
—	—	—	—	75·5	72·9	103·6	2·7	57·3	79·2	—	55·6	—	—	34·8	—
—	59·9	12·7	—	75·0	74·5	100·7	0·6	61·3	87·2	—	—	73·0	—	61·0	—
—	—	—	—	77·4	69·5	111·2	7·8	54·7	—	—	—	—	—	36·2	—
3·0	—	—	—	77·5	71·8	108·0	5·8	59·7	85·6	—	—	—	—	45·0	—
8·5	49·8	9·5	118·7	80·0	80·8	99·0	−0·8	63·5	79·0	79·5	44·9	80·6	—	63·0	88·2
9·5	55·5	12·9	116·3	73·7	72·1	102·2	1·6	63·5	83·0	—	55·2	74·6	—	—	89·5
6·5	53·5	13·8	118·0	70·7	73·4	96·3	−2·7	59·3	88·5	—	52·6	72·1	—	42·0	92·6
3·5	54·7?	9·1?	113·5	71·2	71·0	100·3	0·3	56·5	86·4	74·1	50·7	77·0	—	45·9	87·7
4·5	52·9	11·0	118·8	76·1	74·2	102·5	1·9	57·2	—	—	—	82·0	—	48·7	89·3
9·5	—	—	114·9	72·8	74·7	97·4	−2·0	57·2	—	—	—	82·8	91·3	55·4	—
5·0	48·1	9·0	116·7	72·3	—	—	—	59·3	—	—	47·6	75·9	—	—	84·9
5·5	51·0	12·3	112·8	73·4	76·5	96·0	−3·1	62·4	81·0	—	53·4	76·8	—	45·4	78·4
2·0	—	—	—	78·0	69·1	112·7	8·8	—	—	75·4	48·2	82·4	—	—	—
1·0	56·7	14·4	117·2	71·9	73·8	97·4	−1·9	57·5	89·6	—	48·5	77·0	83·3	71·6	77·9
1·5	—	—	—	77·6	75·7	102·5	1·9	61·1	89·1	67·1	56·5	74·9	92·6	38·7	—
—	—	—	119·6	74·7	73·4	101·9	1·4	59·1	88·9	69·9	53·6	74·6	84·6	—	—
8·0	—	—	—	74·1	—	—	—	65·4	—	—	44·6	80·7	—	37·6	—
6·0	—	—	114·6	78·6	75·1	104·6	3·5	60·5	78·8	65·6	64·6	63·4	—	39·8	—
1·0	59·9	14·1	116·3	74·4	75·8	98·2	−1·4	59·1	84·5	—	53·0	72·1	—	47·1	78·5
—	—	—	—	73·8	71·9	102·6	1·9	—	81·9	—	44·9	—	—	49·1	—
4·0	—	—	121·1	74·4	72·8	102·2	1·6	57·2	74·4	81·5	50·5	80·1	—	41·1	—
—	—	—	—	75·0	73·9	101·5	1·1	60·2	86·3	—	—	—	—	—	—
0·5	—	—	—	78·4	76·1	103·0	2·2	62·2	79·6	—	—	79·0	—	34·8	—
9·5	—	—	—	76·7	67·2	114·1	9·5	—	74·3	—	—	—	—	57·0	—
0·0	53·3	15·3	116·8	73·7	75·1	98·1	−1·4	58·6	92·7	72·0	50·5	79·7	—	47·3	76·9
2·0	—	—	—	—	69·2	—	—	58·1	76·1	—	—	75·8	—	—	—
—	—	—	—	74·1	—	—	—	—	93·0	—	—	—	—	50·0	—
9·0	—	—	108·7	76·4	71·8	106·3	4·6	57·8	95·7	68·6	49·0	77·7	84·7	31·5	—
—	—	—	—	75·7	70·3	107·7	5·4	57·6	80·0	—	—	—	—	—	—
8·5	56·9	17·8	115·8	77·0	72·6	106·0	4·3	59·9	81·1	—	—	74·6	—	—	72·4
1·0	52·4	11·0	112·7	71·4	72·7	98·2	−1·3	56·3	83·7	82·3	—	83·1	77·2	49·1	87·2

88·5	75·7	89·1	25·8	81°·0	67°·6	68°·7	43°·7	11	—
—	41·5	—	22·6	87°·0	61°·1?	74°·3?	44°·6?	12	Coronal and sagittal beginning to close; single epipteric M3's probably absent
86·3	48·1	—	—	—	—	—	—	13	Large ossicle at asterion R, and another above it betwe 2 epipteric bones R
79·7	40·9	—	—	85°·0	63°·8?	75°·2?	40°·9?	14	Single epipteric bones R and L; apparently 3 teeth los of alveolar margin; ? injury on malar bone R
—	39·4	81·0	22·0	—	—	—	—	15	Coronal beginning to close; fused nasal bones, very bro
—	40·0	78·8	27·5	—	—	—	—	16	Coronal beginning to close, sagittal and lambdoid close
—	35·7	—	—	—	—	—	—	17	Coronal and sagittal beginning to close; palate defectiv
—	—	93·2	23·9	83°·0	61°·7?	8°·1?	40°·2?	18	Coronal and lambdoid beginning to close, sagittal close
—	44·8	—	21·5	—	—	—	—	19	*Os épactal*; 2 or 3 molars lost before death, M3's appar
—	37·7	78·8	22·0	—	—	—	—	20	Sagittal closing, lambdoid beginning to close; palate de
—	—	—	—	—	—	—	—	21	Single epipteric bones R and L; fused nasal bones apparently absent; single pre-condyle L
—	55·9	—	—	—	—	—	—	22	Palate defective
89·6	72·7	—	—	—	—	—	—	23	*Os épactal*; 3rd molars absent
—	45·4	—	—	—	—	—	—	24	Sagittal closing; 2 wormian bones in coronal L; large R and L; single large ossicle between temporal squ defective
83·7	—	—	—	84°·0	62°·8	74°·3	42°·9	25	Coronal and sagittal beginning to close
—	40·9	—	—	95°·0?	57°·8?	70°·8?	41°·5?	26	Palate defective
80·8	39·8	74·2	23·1	86°·5?	64°·4	72°·5	43°·1	27	Metopic; metopic, coronal, and sagittal beginning to cl
86·4	53·8	77·7	25·5	83°·5	67°·3	73°·1	39°·6	28	M 3, R below alveolar margin
—	46·7	86·5	24·0	84°·5?	66°·2?	69°·7?	44°·6?	29	Sagittal and lambdoid closing
—	46·6	93·7	22·2	93°·0?	58°·4?	78°·2?	43°·3?	30	Coronal beginning to close; 3 or 4 molars lost before d
—	55·1	—	—	90°·5?	—	—	—	31	Coronal and sagittal beginning to close; 1 or more mol
—	55·3	78·9	24·4	80°·5	65°·7	70°·7	43°·6	32	Sagittal beginning to close; single epipteric bone L
—	31·0	81·0	18·8	—	—	—	—	33	Coronal and sagittal beginning to close; nearly all teet
—	31·5	80·7	27·9	—	—	—	—	34	Sagittal and lambdoid closing; single large epipteric wounds L parietal
—	45·7	87·9	21·4	—	—	—	—	35	Coronal and sagittal beginning to close; palate defecti
—	59·7	—	—	—	64°·9?	76°·2?	38°·8?	36	Wormian bone in coronal R; small epipteric bone R; si and temporal squama R and L; palate defective
93·4	50·6	—	—	—	—	—	—	37	—
—	—	—	—	—	—	—	—	38	M3's absent
85·3	—	84·8	25·7	—	64°·2	70°·7	45°·0	39	Two supernumerary bones between temporal squama death, large abscess cavity at root of M 1, L
—	37·0	—	—	—	—	—	—	40	Single epipteric bone L; palate defective
—	—	—	—	—	—	—	—	41	Occipito-mastoid L obliterated; palate missing
—	34·8	—	—	—	61°·4?	75°·8?	42°·7?	42	Coronal and sagittal beginning to close; numerous wo
—	61·0	—	21·2	—	—	—	—	43	Coronal and sagittal closing; palate missing
—	36·2	—	—	—	—	—	—	44	Palate missing
—	45·0	—	—	—	—	—	—	45	Coronal and sagittal beginning to close; palate missin
—	63·0	88·2	19·1	89°·0	56°·4?	79°·0?	44°·6?	46	Coronal closing, sagittal and lambdoid nearly obliterat
—	—	89·5	23·2	—	—	—	—	47	Sagittal beginning to close; 1 molar lost before death;
—	42·0	92·6	25·8	—	—	—	—	48	Metopic; coronal and sagittal beginning to close; sing before death
—	45·9	87·7	16·6	87°·5	63°·3	75°·1	41°·5	49	Coronal and sagittal beginning to close; fused nasal b
—	48·7	89·3	20·8	—	60°·9?	72°·8?	46°·3?	50	Metopic; sagittal closing; large epipteric bone R; pala
91·3	55·4	—	—	87°·0	64°·5	71°·9	43°·6	51	—
—	—	84·9	18·7	—	—	—	—	52	Single epipteric bones R and L; M 3, L apparently ab
—	45·4	78·4	24·1	87°·5?	63°·3?	77°·9?	38°·8?	53	Coronal and sagittal beginning to close
—	—	—	—	—	—	—	—	54	One molar lost before death
83·3	71·6	77·9	25·4	—	—	—	—	55	Small epipteric bone L
92·6	38·7	—	—	87°·0	64°·9	76°·8	38°·3	56	One premolar lost before death
84·6	—	—	—	—	64°·8?	74°·5?	40°·7?	57	Coronal and lambdoid beginning to close; sagittal clo
—	37·6	—	—	—	—	—	—	58	—
—	39·8	—	—	85°·0?	68°·1?	74°·4?	37°·4?	59	Superior half R side of coronal obliterated, sagittal bone L
—	47·1	78·5	23·5	—	—	—	—	60	Coronal and sagittal beginning to close; 2 large *ossa* 5 teeth lost before death
—	49·1	—	—	84°·5?	63°·8?	70°·1?	46°·1?	61	Coronal closing, sagittal nearly obliterated, lambdoid death
—	41·1	—	—	92°·0	60°·0	78°·3	41°·6	62	Occipito-mastoid obliterated R and L; 3 teeth lost be
—	—	—	—	—	—	—	—	63	Palate defective; small wound on frontal
—	34·8	—	—	—	—	—	—	64	Coronal and sagittal closing, lambdoid beginning to c
—	57·0	—	—	—	—	—	—	65	Coronal closed, sagittal obliterated, lambdoid closing;
—	47·3	76·9	28·7	83°·0	67°·9	72°·1	39°·9	66	Sagittal beginning to close; traces of horizontal sut epipteric bones L, 1 R; small supernumerary bone be L; 3 molars lost before death
—	—	—	—	86°·0	68°·0	72°·4	39°·5	67	—
—	50·0	—	—	—	—	—	—	68	Coronal and sagittal nearly obliterated, lambdoid clo
84·7	31·5	—	—	82°·0	73°·0	68°·2	38°·7	69	Third molars at level of alveolar margin, but basal su
—	—	—	—	—	—	—	—	70	Coronal and lambdoid closing, sagittal nearly obliter
—	—	72·4	31·3	86°·5	71°·0	74°·0	34°·9	71	Coronal closing; trace of horizontal suture of interpa
77·2	49·1	87·2	21·0	—	59°·8?	74°·5?	45°·7?	72	Diseased areas (?) on frontal; M 3, R absent, crown facing backwards

; single epipteric bone R; 1 molar lost before death,

er above it between temporal squama and parietal;

rently 3 teeth lost before death, unusual absorption
r bone R
l bones, very broad
d lambdoid closed; most teeth lost before death
e; palate defective; ? healed wound R parietal
ose, sagittal closed; 2 molars lost before death
eath, M_3's apparently absent
o close; palate defective
sed nasal bones; 1 molar lost before death, M_3's
e L

coronal L; large single, or multiple, epipteric bones
en temporal squama and parietal R and L; palate

e

al beginning to close

lars lost before death; 2 small exostoses on frontal
se; 1 or more molars lost before death
pteric bone L
se; nearly all teeth lost before death
le large epipteric bone L; palate missing; 2 healed

se; palate defective
ipteric bone R; single small ossicles between parietals
ate defective

—

temporal squama and parietal L; M 1, R lost before
f M 1, L
tive
te missing
se; numerous wormian bones in lambdoid
nissing

se; palate missing
d nearly obliterated; 1 molar lost before death
ost before death; healed wound on malar bone L
ing to close; single epipteric bone L; some teeth lost

se; fused nasal bones
eric bone R; palate defective

—

L apparently absent
se

close; sagittal closing; small epipteric bone R

—

literated, sagittal beginning to close; large epipteric

lose; 2 large *ossa triangularia* meeting below lambda;

erated, lambdoid half obliterated; 2 molars lost before

L; 3 teeth lost before death
ontal
id beginning to close; palate defective
lambdoid closing; fused nasal bones; palate missing
of horizontal suture across malar bones R and L; 2
numerary bone between temporal squama and parietal

—

ed, lambdoid closing; palate missing
rgin, but basal suture obliterated
tal nearly obliterated; palate missing
suture of interparietal bone R
R absent, crown of M 3 L below alveolar margin and

No.	L	B	B'	H'	S_1'	S_2'	S_3'	S_1	S_2	S_3	S	$\beta Q'$	U	fml	f
73	175	132·5	91·5?	125·5?	103·9	102·5	92·1	117	113	109·5	339	285	494?	39·0	3
74	185·5	144	97·4	140·5	113·2?	112·6?	102·2	132	129	121·5	382	314	536	41·9	3
75	179	140	101·2	124·5	107·8?	118·7	88·9	126	134	107	366	311?	518?	36·8	3
76	185·5	127·5	89·8	132	108·2	—	—	124·5	—	—	377	291	513	34·7	3
77	190·5	130	97·0	132	116·6	116·1	99·9	136·5	130·5	123	389	305	529	35·9	
78	187	136	91·3	136·5?	112·0	112·2	—	129	125·5	—	—	314	520	—	2
79	180·5	138·5	94·0	130	116·2	115·0	94·5	135·5	130·5	121	387	303	518	35·6	3
80	184·5	137	95·1	139·5	114·6	117·9	101·2	128·5	133	118·5	379	319	511	38·7	3
81	189·5	143	97·9	139·5	116·2	125·1	98·0	133?	137·5?	121	392?	321?	532	35·4	2
82	185	142·5	97·6	140	107·1	117·7	99·0	126	130·5	121	378	320?	—	42·9	3
83	192·5	146·5	107·0	141·5	117·8	126·9	103·7	134·5	140	127	401	327	556	38·8	3
84	177	138·5	100·3	138	117·7	113·7?	—	136·5	126·5	—	—	321	503	—	
85	176?	139·5	92·8	121	109·5	108·0?	88·2	124	123·5?	104·5	353	308	503	35·9	3
86	182·5	137	99·3	136	118·5	111·0	92·3	137·5	122	116·5	375	319	529	38·4	
87	177	132	93·7	133	117·5?	107·8	97·0	133	120	114·5	367	306	496	33·8	
88	181·5	135	92·9	—	112·3	119·2	98·4	125	137	112?	374?	311	521	—	2
89	182	127·5	95·3	120·5	106·5	116·3	89·0	122	134	107·5	364	290	506	37·4	2
90	186·5	132·5	93·8	131·5	115·4	120·0	89·7	132	134·5	112	378	307	519	34·9	2
91	191	149	92·8	138·5	124·0	117·4	104·6	140·5	129	130	400	325	535	36·3	
92	175·5	129·5	94·6	130·5	103·9	113·1	91·0	116·5	128	108	352	295	495	35·7	
93	172·5	131	95·4	130	102·3?	111·8	95·2	118·5?	123	113·5	355	298	492	31·0	
94	175	129	88·5	131	107·0	118·0	90·2	120	135·5	106	361	300	498	35·4	
95	181·5	130	94·9	133·5	110·9	113·3	99·7	126	125·5	117	368	297	510	37·3	
96	178·5	135	100·0	137·5	111·1	114·2	93·7	128·5	125	111·5	366	308	506	39·9	
97	186·5	149·5	95·5	128·5	114·2	115·7	—	130	128·5	—	—	314	535	—	
98	175·5	147	94·9	138·5	117·3	118·1	97·4	138·5	132·5	117	388	334	516	34·8	
99	176·5	135·5	95·9	133	105·9	113·0	97·6	120	129	113	362	310	504	40·6	
100	179	132	92·6	128·5	110·5	111·1	91·5	128·5	125	112	365	296	504	36·9	
101	184·5	141	89·0	126	111·9	115·2	102·7	125·5	129	126	381	305	521	35·9	
102	178	138	90·6	125·5	109·4	110·8	96·3	125·5	123	113	362	300	503	33·4	
103	178	135	90·0	133·5	108·0	107·1	101·0	122	118·5	117·5	358	294	502	37·2	
104	184·5	138·5?	96·7	132·5?	116·3?	123·8?	—	—	142?	—	—	321?	521	—	
105	178·5	134	95·0	135·5	106·6	116·5	96·4	123·5	133	113	369	304	502	36·0	
106	172	129	93·1	132	104·7	112·6	92·4	115·5	124·5	107	348	291	484	38·0	
107	187·5	128	94·0	—	109·1	127·7	94·0?	122·5	146	116	384	309	503	—	
108	179	131?	100·3	129·5?	106·9	111·3?	98·2?	118·5	124?	116?	358	—	498	36·0	
109	177	143	97·6	—	107·9	115·8	96·3	125	132·5	121	378	314	518	—	
110	180	132·5	90·3	138	116·9	110·4	95·9	132	123	114	368	301	503	39·5	
111	184	131	101·2	135·5	109·3	113·8	98·4?	125	126·5	118·5	369	302	516	41·6?	
112	179	132·5	96·5	143	116·1?	111·5	98·5	134	124·5	116	374	308	510	39·4	
113	191·5	137	99·1	137	113·0	123·6	101·9	129	137	118	384	304	532	38·1	
114	188·5	137	95·1	136	116·0	—	—	134·5	—	—	384	307	525	35·2	
115	186·5	138·5	98·3	136	111·1	109·3?	102·9?	128·0	125·5	123	376	310	523	38·6	
116	187	137	86·0	137·0?	113·1	108·3	102·6	132	118	125	375	311	521	40·5?	
117	178	131	95·3	132·5	108·9	116·2	91·5	125	129·5	108·5	362	300	505	37·2	
118	180·5	138	95·2	131	112·5	114·7	95·9	130	130	118·5	378	—	511?	32·4	
119	176	134·5	98·8	130·5	111·8	114·0?	92·3?	132	125·5	110	367?	—	505	36·0	
120	182	131·5	99·2	122·5	108·5	115·0	91·2	120	127·5	112	359	296	511	35·8	
121	—	129	—	126	108·9	—	—	125·5	—	—	—	—	—	—	
122	177	127	85·4?	129·5	112·6	107·9	84·3?	129·5?	122·5	100	351?	—	488?	37·8?	
123	176·5	132	86·0	129·5	116·0	102·8	94·4	132·5	115·5	110	357	294	498	38·2	
124	172·5	133	91·0	134	111·1	110·0	93·9	125·5	121	109	355	308	489	33·8	
125	177·5	130·5	98·7	133·5	109·0	103·1?	101·0?	121·5	114	116·5	352	298	494	37·9	
126	174?	137	96·0	135	112·1	109·9?	—	126·5	126	—	—	301	497?	—	
127	181	133·5	94·2	132·5?	117·3	100·4?	89·6?	132·5	124·5	103	360	308	504	—	
128	178	142	100·0	133·5	101·0	121·2	98·5	124	136·5	118	378	318	510	32·0	
129	180	138·5	92·2	141	116·0	117·9	96·1	135·5	132	113	381	311	510	37·7	
130	181·5	140	92·6	—	108·0	120·2	—	121	134·5	—	—	309	511	—	
131	177	135	93·0	128	108·0	112·9	92·9	125	125·5	116	366	303	495	33·2	
132	183	133	90·9	136·5	111·0	121·3	92·2	130	130·5	112	365	307	503	33·2	

	U	*fml*	*fmb*	*LB*	*GL*	*G'H*	*J*	*GB*	*NB*	*NH,L*	*SC*	*SS*	O_2L	O_1L	G_1'	G_2	Ml_1	Ml_2	*C(ml)*	*S(ml)*	*OH*
	494?	39·0	33·8?	103·3	—	—	—	96·9	—	51·9	11·2	5·9	33·2	42·0	44·0?	—	—	46·0	—	—	—
	536	41·9	32·0	105·6	—	—	136	99·1	24·7	53·8	10·3	5·4	34·7	42·9	—	—	62·0	48·5	54·8	11·9	116·3
?	518?	36·8	32·1	100·8?	96·0?	71·3?	—	97·7	—	53·5	—	—	37·7	44·6	—	—	56·0	55·0	48·1	11·5	113·0
	513	34·7	30·0	99·8	90·6	63·2	—	92·2	24·8	49·8	7·8	6·5	31·1	40·1	—	—	—	48·5	—	—	—
	529	35·9	29·1	100·0	—	—	—	—	—	—	11·2	6·8	—	—	—	—	—	—	—	—	—
	520	—	—	—	—	—	—	—	27·0	52·6	12·9	6·9	30·0	41·1	—	—	—	—	—	—	118·1
	518	35·6	30·4	95·5	90·9	72·8	126·5	92·2	23·9	51·9	11·0	6·6	33·1	40·2	49·2	—	62·5	51·5	53·8	13·4	113·4
	511	38·7	30·5	103·2	93·0	73·3	—	94·3	—	50·5	13·3	8·2	32·0	43·2	50·2?	—	62·5?	54·0?	56·0?	11·4?	121·8
?	532	35·4	28·3	102·2	—	—	—	—	—	—	13·0	6·0	28·0	41·1	—	—	—	—	—	—	—
?	—	42·9	35·6	105·6	—	—	—	93·5	—	—	9·0	4·1	31·4	41·9	—	—	—	50·0	—	—	—
	556	38·8	33·1	104·0	99·6	71·8	135	—	—	—	14·1	5·8	35·0	42·2	49·3?	40·7?	—	—	—	—	122·6
	503	—	—	99·0	92·0	64·4	—	—	25·0	47·1	8·2	4·3	34·0	43·5	45·1	—	—	52·0	—	—	119·5
3	503	35·9	31·1	98·2	87·3?	70·8	—	93·3	24·1	54·2	10·2	5·0	34·1	44·8	—	—	—	52·0	—	—	—
)	529	38·4	—	96·7	92·1	69·2	128·5	—	26·4	52·3	10·1	6·1	33·0	41·5	45·7	—	51·5	49·5	45·1	10·9	121·7
5	496	33·8	28·6	100·7?	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	116·1
:	521	—	—	—	—	72·5?	—	99·5	27·0	52·5	11·9	5·3	30·9	43·8	53·5	—	65·0	54·0	54·7	13·2	118·1
)	506	37·4	29·2	98·1	—	—	—	—	22·5?	—	6·9	2·8	34·4	39·2	—	—	63·0	49·5	55·1	14·1	107·8
7	519	34·9	27·5	101·8	101·1	63·8	133·5	98·4	24·4	48·0	8·7	5·2	30·9	44·0	52·1?	43·1	65·0	51·5	55·1	11·9	114·7
5	535	36·3	28·4	98·9	91·0	78·2	—	97·1	24·0	54·7	10·8	4·9	31·2	40·2	44·1	—	62·0	54·0	55·4	10·3	120·8
5	495	35·7	28·1	97·0	95·1	66·4	—	91·7	24·2	48·8	8·2	2·6	29·9	37·4	48·2	—	—	48·0	—	—	115·0
8	492	31·0	29·0	97·3?	—	—	—	94·6	23·4?	51·8?	—	—	34·2	40·0	43·2	—	50·5	44·5	44·6	11·4	112·2
0	498	35·4	27·9	97·2	96·9	70·4	—	96·0	25·9	51·7	8·3	3·4	35·1	41·8	46·4	39·3	52·0	51·0	45·9	9·1	114·6
7	510	37·3	31·2	100·3	—	—	—	—	—	—	8·1	3·0	—	—	—	—	—	—	—	—	—
8	506	39·9	32·5	101·0	—	—	—	—	—	—	12·1	4·7	—	—	—	—	—	—	—	—	—
4	535	—	—	97·7	92·3	65·1	—	—	24·1	48·9	10·9	5·6	—	—	48·4	38·9	—	—	—	—	112·8
4	516	34·8	29·9	93·0	—	—	120	91·2	27·3	51·8?	12·5	4·7	29·0	40·2	42·9	—	53·5	44·5	47·6	11·0	122·2
0	504	40·6	31·9	98·8	—	—	—	—	—	—	10·5	4·5	—	—	—	—	—	—	—	—	—
6	504	36·9	33·1?	98·5	—	—	120·5	87·7	22·8	52·5	8·7	4·7	31·1	40·6	—	—	57·0	46·5	50·6	10·3	108·0
5	521	35·9	30·6	96·1	91·4	75·8	127·5?	94·5	23·3	54·3	12·2	6·0	35·0	38·8	51·1	40·3	—	50·5	—	—	111·8
0	503	33·4	26·2	94·5	94·2	63·5	—	—	24·5	44·2	9·9	5·2	30·3	40·0	44·2	42·3?	61·0	47·5	53·6	12·5	109·5
4	502	37·2	29·9	103·2	103·0	71·0	127·5	89·8	—	—	10·9	6·4	33·8	42·7	53·1	41·0	58·5?	47·5	51·5	11·1	109·4
1?	521	—	—	101·0	97·5	71·6	125	92·0	24·1	49·2	12·0	7·2	32·6	42·1	48·1	—	56·5	53·5	49·0	11·2	—
4	502	36·0	31·4?	101·1	97·4	66·9	122	89·0?	23·9	48·8?	12·9	5·1	29·4	38·3	—	—	60·5	44·0	54·4	10·1	113·6
1	484	38·0	27·8	97·9	89·5	67·1	122	87·2	26·9	49·0	13·0	7·0	—	40·0	—	—	58·0	55·0	51·5	11·8	109·4
9	503	—	—	—	—	—	—	—	—	—	10·0	5·6	—	—	—	—	—	—	—	—	—
	498	36·0	30·0	99·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
4	518	—	27·7	—	—	—	126	90·9	26·2	—	10·4	5·3	33·3	41·6	—	—	—	53·5	—	—	115·5
1	503	39·5	32·3	100·2	—	—	—	—	—	—	9·1	5·1	—	—	—	—	—	—	—	—	—
2	516	41·6?	—	101·2	92·2	74·1	—	92·0	24·0?	53·9	9·8	3·5	33·9	43·0	44·5	39·6	—	42·5	—	—	113·1
8	510	39·4	30·1	103·0?	90·3	74·5?	126	97·6	—	52·6?	—	—	36·8	40·3	—	—	51·5	45·0	46·4	9·1	116·9
4	532	38·1	35·4	105·1	—	—	—	—	28·4	55·4	11·5	5·2	33·3	42·7	—	39·2?	—	—	—	—	116·5
7	525	35·2	28·3	99·2	92·2	68·9	—	103·2	28·0	52·3	10·3	4·7	31·9	41·9	—	—	—	52·5	—	—	—
0	523	38·6	30·0	103·5	100·7	75·2	131·5	95·1	23·9	52·5	10·7	5·5	32·6	40·1	48·9	43·3	62·5	49·0	55·0	11·2	—
1	521	40·5?	31·7	100·5?	—	—	—	96·5	25·5	50·1?	—	—	30·6?	40·5?	—	38·7	—	45·5	—	—	—
0	505	37·2	31·7	95·7	—	—	—	—	—	—	11·2	5·0	—	—	—	—	—	—	—	—	—
—	511?	32·4	33·2	99·1	—	—	—	—	—	—	7·1	2·7	33·0	42·1	—	—	61·0	45·5	53·2	14·3	—
—	505	36·0	29·5	96·0	—	—	124	93·8	25·9?	—	—	—	28·5	40·0	42·8?	36·6	—	44·0?	—	—	—
6	511	35·8	28·8	97·9	—	—	—	89·1	23·3?	50·9	—	—	36·1	42·2	50·1	—	65·0	51·0	57·1?	13·0?	110·8
—	—	—	—	99·9	90·7	66·8	—	93·7	23·4	50·0	11·1	4·3	—	—	44·7	—	57·0	—	50·2	12·1	—
—	488?	37·8?	—	102·5	93·1	66·2	—	—	25·0	49·4	7·5	4·9	31·0?	—	42·2	—	—	—	—	—	—
4	498	38·2	33·4	94·0	—	—	125·5	96·2	29·2	55·1	8·2	3·3	35·9	40·0	—	—	59·0?	47·0	51·0	12·5	—
8	489	33·8	28·4	100·0	—	—	—	91·4	26·8	53·2	8·4	4·9	34·0	41·9	—	—	52·5	46·0	46·6?	10·0?	—
8	494	37·9	32·9	102·2	—	—	—	95·2	24·6	53·1	10·6	6·1	33·0	40·8	45·8	41·7	—	48·0	—	—	114·2
1	497?	—	—	101·2	100·3	74·0	—	97·8	28·7	56·1	8·6	3·9	33·9	41·0?	—	42·6	60·5	49·5	55·3	11·1	—
8	504	—	29·5	—	—	—	121·5	93·4	—	—	8·6	3·7	33·4	41·8	—	38·5?	—	53·0	—	—	114·5
8	510	32·0	30·0	97·9	91·0	66·0	—	—	25·9?	50·0	10·0	4·4	33·1	—	44·5	—	—	—	—	—	110·0
1	510	37·7	32·2	97·0	86·0	66·7	—	91·0	23·1	49·6	5·9	2·1	31·1	40·0	42·1	38·1	59·0	43·5	51·3	11·8	114·1
9	511	—	—	—	—	—	—	93·1	21·9?	49·9	10·7	4·6	35·0	38·9	48·7	40·0?	—	43·5	—	—	115·2
3	495	33·2	26·9	96·3	88·0?	67·6?	—	93·4	25·9	49·7	10·6	6·0	34·1	40·4	—	—	63·0	46·5	52·9	13·0	104

APPENDIX II. *Individual Measurements of Lachish Skulls.* II. *Male (cont.: Nos*

J	*GB*	*NB*	*NH,L*	*SC*	*SS*	O_2L	O_1L	G_1'	G_2	Ml_1	Ml_2	*C(ml)*	*S(ml)*	*OH*	$100\frac{B}{L}$	$100\frac{H'}{L}$
—	96·9	—	51·9	11·2	5·9	33·2	42·0	44·0?	—	—	46·0	—	—	—	75·7	71·7
36	99·1	24·7	53·8	10·3	5·4	34·7	42·9	—	—	62·0	48·5	54·8	11·9	116·3	77·6	75·7
—	97·7	—	53·5	—	—	37·7	44·6	—	—	56·0	55·0	48·1	11·5	113·0	78·2	69·6
—	92·2	24·8	49·8	7·8	6·5	31·1	40·1	—	—	—	48·5	—	—	—	68·7	71·2
—	—	—	—	11·2	6·8	—	—	—	—	—	—	—	—	—	68·2	69·3
—	—	27·0	52·6	12·9	6·9	30·0	41·1	—	—	—	—	—	—	118·1	72·7	73·0
26·5	92·2	23·9	51·9	11·0	6·6	33·1	40·2	49·2	—	62·5	51·5	53·8	13·4	113·4	76·7	72·0
—	94·3	—	50·5	13·3	8·2	32·0	43·2	50·2?	—	62·5?	54·0?	56·0?	11·4?	121·8	74·3	75·6
—	—	—	—	13·0	6·0	28·0	41·1	—	—	—	—	—	—	—	75·4	73·6
—	93·5	—	—	9·0	4·1	31·4	41·9	—	—	—	50·0	—	—	—	77·0	75·7
35	—	—	—	14·1	5·8	35·0	42·2	49·3?	40·7?	—	—	—	—	122·6	76·1	73·5
—	—	25·0	47·1	8·2	4·3	34·0	43·5	45·1	—	—	52·0	—	—	119·5	78·2	78·0
—	93·3	24·1	54·2	10·2	5·0	34·1	44·8	—	—	—	52·0	—	—	—	79·3	68·7
28·5	—	26·4	52·3	10·1	6·1	33·0	41·5	45·7	—	51·5	49·5	45·1	10·9	121·7	75·1	74·5
—	—	—	—	—	—	—	—	—	—	—	—	—	—	116·1	74·6	75·1
—	99·5	27·0	52·5	11·9	5·3	30·9	43·8	53·5	—	65·0	54·0	54·7	13·2	118·1	74·4	—
—	—	22·5?	—	6·9	2·8	34·4	39·2	—	—	63·0	49·5	55·1	14·1	107·8	70·1	66·2
33·5	98·4	24·4	48·0	8·7	5·2	30·9	44·0	52·1?	43·1	65·0	51·5	55·1	11·9	114·7	71·0	70·5
—	97·1	24·0	54·7	10·8	4·9	31·2	40·2	44·1	—	62·0	54·0	55·4	10·3	120·8	78·0	72·5
—	91·7	24·2	48·8	8·2	2·6	29·9	37·4	48·2	—	—	48·0	—	—	115·0	73·8	74·4
—	94·6	23·4?	51·8?	—	—	34·2	40·0	43·2	—	50·5	44·5	44·6	11·4	112·2	75·9	75·4
—	96·0	25·9	51·7	8·3	3·4	35·1	41·8	46·4	39·3	52·0	51·0	45·9	9·1	114·6	73·7	74·9
—	—	—	—	8·1	3·0	—	—	—	—	—	—	—	—	—	71·6	73·6
—	—	—	—	12·1	4·7	—	—	—	—	—	—	—	—	—	75·6	77·0
—	—	24·1	48·9	10·9	5·6	—	—	48·4	38·9	—	—	—	—	112·8	80·2	68·9
120	91·2	27·3	51·8?	12·5	4·7	29·0	40·2	42·9	—	53·5	44·5	47·6	11·0	122·2	83·8	78·9
—	—	—	—	10·5	4·5	—	—	—	—	—	—	—	—	—	76·8	75·4
120·5	87·7	22·8	52·5	8·7	4·7	31·1	40·6	—	—	57·0	46·5	50·6	10·3	108·0	73·7	71·8
127·5?	94·5	23·3	54·3	12·2	6·0	35·0	38·8	51·1	40·3	—	50·5	—	—	111·8	76·4	68·3
—	—	24·5	44·2	9·9	5·2	30·3	40·0	44·2	42·3?	61·0	47·5	53·6	12·5	109·5	77·5	70·5
127·5	89·8	—	—	10·9	6·4	33·8	42·7	53·1	41·0	58·5?	47·5	51·5	11·1	109·4	75·8	75·0
125	92·0	24·1	49·2	12·0	7·2	32·6	42·1	48·1	—	56·5	53·5	49·0	11·2	—	75·1	71·8
122	89·0?	23·9	48·8?	12·9	5·1	29·4	38·3	—	—	60·5	44·0	54·4	10·1	113·6	75·1	75·9
122	87·2	26·9	49·0	13·0	7·0	—	40·0	—	—	58·0	55·0	51·5	11·8	109·4	75·0	76·7
—	—	—	—	10·0	5·6	—	—	—	—	—	—	—	—	—	68·3	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	73·2	72·3
126	90·9	26·2	—	10·4	5·3	33·3	41·6	—	—	—	53·5	—	—	115·5	80·8	—
—	—	—	—	9·1	5·1	—	—	—	—	—	—	—	—	—	73·6	76·7
—	92·0	24·0?	53·9	9·8	3·5	33·9	43·0	44·5	39·6	—	42·5	—	—	113·1	71·2	73·6
126	97·6	—	52·6?	—	—	36·8	40·3	—	—	51·5	45·0	46·4	9·1	116·9	74·0	79·9
—	—	28·4	55·4	11·5	5·2	33·3	42·7	—	39·2?	—	—	—	—	116·5	71·5	71·5
—	103·2	28·0	52·3	10·3	4·7	31·9	41·9	—	—	—	52·5	—	—	—	72·7	72·1
131·5	95·1	23·9	52·5	10·7	5·5	32·6	40·1	48·9	43·3	62·5	49·0	55·0	11·2	—	74·3	72·9
—	96·5	25·5	50·1?	—	—	30·6?	40·5?	—	38·7	—	45·5	—	—	—	73·3	73·3
—	—	—	—	11·2	5·0	—	—	—	—	—	—	—	—	—	73·6	74·4
—	—	—	—	7·1	2·7	33·0	42·1	—	—	61·0	45·5	53·2	14·3	—	76·5	72·6
124	93·8	25·9?	—	—	—	28·5	40·0	42·8?	36·6	—	44·0?	—	—	—	76·4	74·1
—	89·1	23·3?	50·9	—	—	36·1	42·2	50·1	—	65·0	51·0	57·1?	13·0?	110·8	72·3	67·3
—	93·7	23·4	50·0	11·1	4·3	—	—	44·7	—	57·0	—	50·2	12·1	—	—	—
—	—	25·0	49·4	7·5	4·9	31·0?	—	42·2	—	—	—	—	—	—	71·8	73·2
125·5	96·2	29·2	55·1	8·2	3·3	35·9	40·0	—	—	59·0?	47·0	51·0	12·5	—	74·8	73·4
—	91·4	26·8	53·2	8·4	4·9	34·0	41·9	—	—	52·5	46·0	46·6?	10·0?	—	77·1	77·7
—	95·2	24·6	53·1	10·6	6·1	33·0	40·8	45·8	41·7	—	48·0	—	—	114·2	73·5	75·2
—	97·8	28·7	56·1	8·6	3·9	33·9	41·0?	—	42·6	60·5	49·5	55·3	11·1	—	78·7	77·6
121·5	93·4	—	—	8·6	3·7	33·4	41·8	—	38·5?	—	53·0	—	—	114·5	73·8	73·2
—	—	25·9?	50·0	10·0	4·4	33·1	—	44·5	—	—	—	—	—	119·0	79·8	75·0
—	91·0	23·1	49·6	5·9	2·1	31·1	40·0	42·1	38·1	59·0	43·5	51·3	11·8	114·1	76·9	78·3
—	93·1	21·9?	49·9	10·7	4·6	35·0	38·9	48·7	40·0?	—	43·5	—	—	115·2	77·1	—
—	92·4	25·9	49·7	10·6	6·0	34·1	40·4	—	—	63·0	46·5	52·9	14·6	109·7	76·3	72·3
126	89·5	24·3	47·0	9·0	4·4	34·1	40·3	46·6	40·8	56·0?	43·0	45·3?	8·1?	113·1	72·5	73·7

ale (cont.: Nos. 73–115) and Queried Male (Nos. 116–143) Crania from Tomb 120

$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	*Oc.I.*	$100\frac{fmb}{fml}$	$100\frac{G'H}{GB}$	$100\frac{NB}{NH'}, L$	$100\frac{O_2}{O_1}, L$	$100\frac{G_2}{G_1'}$	$100\frac{SS}{SC}$	$100\frac{Ml_2}{Ml_1}$	$100\frac{S}{C}(ml)$
5·7	71·7	105·6	4·0	—	86·7	—	—	79·0	—	52·7	—	—
7·6	75·7	102·5	1·9	60·9	76·4	—	45·9	80·9	—	52·4	78·2	21·7
8·2	69·6	112·4	8·7	59·7	87·2	73·0	—	84·5	—	—	98·2	23·9
8·7	71·2	96·6	−2·4	—	86·5	68·5	49·8	77·6	—	83·3	—	—
8·2	69·3	98·5	−1·0	58·0	81·1	—	—	—	—	60·7	—	—
2·7	73·0	99·6	−0·3	—	—	—	51·3	73·0	—	53·5	—	—
6·7	72·0	106·5	4·7	55·9	85·4	79·0	46·1	82·3	—	60·0	82·4	24·9
4·3	75·6	98·2	−1·4	62·6	78·8	77·7	—	74·1	—	61·7	86·4	20·4
5·4	73·6	102·5	1·8	57·8	79·9	—	—	68·1	—	46·2	—	—
7·0	75·7	101·8	1·4	58·5	83·0	—	—	74·9	—	45·6	—	—
6·1	73·5	103·5	2·6	58·4	85·3	—	—	82·9	82·6	41·1	—	—
8·2	78·0	100·4	0·3	—	—	—	53·1	78·2	—	52·4	—	—
9·3	68·7	115·3	10·5	61·2	86·6	75·9	44·5	78·2	—	49·0	—	—
5·1	74·5	100·7	0·6	56·5	—	—	50·5	79·5	—	60·4	96·1	24·2
4·6	75·1	99·2	−0·6	61·7	84·6	—	—	—	—	—	—	—
4·4	—	—	—	66·7	—	72·9	51·4	70·5	—	44·5	83·1	24·1
0·1	66·2	105·8	3·9	59·4	78·1	—	—	87·8	—	40·6	78·6	25·6
1·0	70·5	100·8	0·5	57·1	78·8	64·8	50·8	70·2	82·7	59·8	79·2	21·6
8·0	72·5	107·6	5·5	57·4	78·2	80·5	43·9	77·6	—	45·4	87·1	18·6
3·8	74·4	99·2	−0·6	61·0	78·7	72·4	49·6	79·9	—	31·7	—	—
5·9	75·4	100·8	0·6	60·6	93·5	—	45·2	85·5	—	—	88·1	25·6
3·7	74·9	98·5	−1·1	62·1	78·8	73·3	50·1	84·0	84·7	41·0	98·1	19·8
1·6	73·6	97·4	−1·9	62·2	83·6	—	—	—	—	37·0	—	—
5·6	77·0	98·2	−1·4	60·8	81·5	—	—	—	—	38·8	—	—
0·2	68·9	116·3	11·3	—	—	—	49·3	—	80·4	51·4	—	—
3·8	78·9	106·1	4·8	59·9	85·9	—	52·7	72·1	—	37·6	83·2	23·1
6·8	75·4	101·9	1·4	64·0	78·6	—	—	—	—	42·9	—	—
3·7	71·8	102·7	2·0	58·4	89·7	—	43·4	76·6	—	54·0	81·6	20·4
6·4	68·3	111·9	8·1	58·2	80·5	80·2	42·9	90·2	78·9	49·2	—	—
7·5	70·5	110·0	7·0	62·3	78·4	—	55·4	75·7	95·7	52·5	77·9	23·3
5·8	75·0	101·1	0·8	63·4	80·4	79·1	—	79·2	77·2	58·7	81·2	21·6
5·1	71·8	104·5	3·3	—	—	77·8	49·0	77·4	—	60·0	94·7	22·9
5·1	75·9	98·9	−0·8	62·4	87·2	75·2	49·0	76·8	—	39·5	72·7	18·6
5·0	76·7	97·7	−1·7	64·0	73·2	76·9	54·9	—	—	53·8	94·8	22·9
8·3	—	—	—	57·8	—	—	—	—	—	56·0	—	—
3·2	72·3	101·2	0·8	61·6	83·3	—	—	—	—	—	—	—
0·8	—	—	—	56·8	—	—	—	80·0	—	51·0	—	—
3·6	76·7	96·0	−3·1	60·9	81·8	—	—	—	—	56·0	—	—
1·2	73·6	96·7	−2·4	59·7	—	80·5	44·5	78·8	89·0	35·7	—	—
4·0	79·9	92·7	−5·9	61·9	76·4	76·3	—	91·3	—	—	87·4	19·6
1·5	71·5	100·0	0	64·0	92·9	—	51·3	78·0	—	45·2	—	—
2·7	72·1	100·7	0·5	—	80·4	66·8	53·5	76·1	—	45·6	—	—
4·3	72·9	101·8	1·3	60·4	77·7	79·1	45·5	81·3	88·5	51·4	78·4	20·4
3·3	73·3	100·0	0	58·8	78·3	—	50·9	75·6	—	—	—	—
3·6	74·4	98·9	−0·8	61·1	85·2	—	—	—	—	44·6	—	—
6·5	72·6	105·3	3·9	57·7	102·5	—	—	78·4	—	38·0	74·6	26·9
6·4	74·1	103·1	2·3	60·6	81·9	—	—	71·3	85·5	—	—	—
2·3	67·3	107·3	4·9	58·2	80·4	—	45·8	85·5	—	—	78·5	22·8
—	—	102·4	—	—	—	71·3	46·8	—	—	38·7	—	24·1
1·8	73·2	98·1	1·4	61·1	—	—	50·6	—	—	65·3	—	—
4·8	73·4	101·9	1·4	63·2	87·4	—	53·0	89·7	—	40·2	79·7	24·5
7·1	77·7	99·3	−0·6	63·6	84·0	—	50·4	81·1	—	58·3	87·6	21·5
3·5	75·2	97·8	−1·7	64·6	86·8	—	46·3	80·9	91·0	57·5	—	—
8·7	77·6	101·5	1·1	—	—	75·7	51·2	82·7	—	45·3	81·8	20·1
3·8	73·2	100·8	0·6	65·0	—	—	—	79·9	—	43·0	—	—
9·8	75·0	106·4	4·8	60·2	93·7	—	51·8	—	—	44·0	—	—
6·9	78·3	98·2	−1·4	62·1	85·4	73·3	46·6	77·7	96·5	35·6	73·7	23·0
7·1	—	—	—	—	—	—	43·9	90·0	82·1	43·0	—	—
6·3	72·3	105·5	4·0	57·1	81·0	73·2	52·1	84·4	—	56·6	73·8	27·6
5·7	75·7	98·0	−3·0	64·7	85·9	71·5	51·7	84·4	97·0	44·4	86·6	17·9

$100\frac{S}{O}(ml)$	$P\angle$	$N\angle$	$A\angle$	$B\angle$	No.	Remarks
—	—	—	—	—	73	Coronal and lambdoid partly obliterated, sagittal obliterated; fused nasal bones; palate defective
21·7	—	—	—	—	74	Metopic; sagittal beginning to close; all molars lost before death
23·9	—	65°·3?	72°·9?	41°·8?	75	Sagittal closing; M 3, L apparently absent, abscess cavities at roots of M 1, L and M 2, L
—	86°·0	62°·8	78°·8	38°·3	76	Coronal and lambdoid closing, sagittal nearly obliterated; several teeth lost before death; several abscess cavities (?); small wound R parietal
—	—	—	—	—	77	Coronal and sagittal closing; occipito-mastoid R obliterated; palate missing
—	—	—	—	—	78	Sagittal nearly obliterated, lambdoid closing; 4 or 5 teeth lost before death, several abscess cavities (?); large epipteric bone L
24·9	86°·0	63°·6	70°·5	45°·9	79	Coronal and lambdoid closed, sagittal nearly obliterated; 2 or more molars lost before death; small pre-condyle R
20·4	89°·5	60°·8	75°·8	43°·4	80	Fronto-temporal articulation R (L pterion normal); palate defective
—	—	—	—	—	81	Single epipteric bones R and L; palate defective
—	—	—	—	—	82	Metopic; metopic and sagittal beginning to close; single epipteric bone L; L jugular foramen exceptionally large, R exceptionally small; palate defective; small wound on frontal
—	82°·5	65°·9	72°·9	41°·1	83	Sagittal beginning to close; close approach to pterion in *K*, L; palate defective
—	86°·0	64°·4	76°·5	39°·1	84	Coronal and lambdoid beginning to close, sagittal closing; palate defective; healed wound on superciliary ridge, and on L parietal
—	93°·0	59°·5?	76°·1?	44°·4?	85	Coronal partly obliterated, sagittal obliterated, lambdoid closed; 2 or more molars lost before death; trace of horizontal suture across malar bone R
24·2	86°·5	64°·9	72°·1	42°·9	86	Large epipteric bone R; wormian bone between temporal squama and parietal R above mastoid process; trace of metopic suture above nasion; 3 molars lost before death
—	—	—	—	—	87	Coronal and sagittal beginning to close; palate defective; small pre-condyle L
24·1	—	—	—	—	88	Two molars lost before death
25·6	—	—	—	—	89	Two epipteric bones R and 2 L; palate defective
21·6	84°·0	71°·0	72°·4	36°·5	90	Occipito-mastoid L obliterated, and posterior half of temporal L fused to parietal; small healed wound frontal
18·6	90°·5	60°·4	71°·1	48°·4	91	All sutures beginning to close; greater wing of sphenoid fused to parietal and frontal R and L; 1 molar and 1 premolar lost before death
—	83°·0?	68°·0	71°·5	40°·5	92	Sagittal closed; single epipteric bones R and L; 3 molars and 1 pre-molar lost before death; healed injury to margin of nasal aperture
25·6	—	—	—	—	93	Small epipteric bone R
19·8	80°·5	68°·3	69°·1	42°·6	94	*Os triangulare* R
—	—	—	—	—	95	Coronal beginning to close; palate missing; fused nasal bones
—	—	—	—	—	96	Metopic; palate missing
—	85°·5	65°·6	74°·5	39°·9	97	Trace of horizontal suture on malar bone R; 2 small pre-condyles; wound on frontal
23·1	—	—	—	—	98	*Os triangulare* R; M 3's absent; 2 small wounds on frontal
—	—	—	—	—	99	Palate missing; trace of horizontal suture on malar bone R
20·4	—	—	—	—	100	All sutures closing; posterior halves of temporal squamae fused to parietals R and L; most teeth lost before death
—	85°·5	62°·8	69°·7	47°·4	101	Sagittal beginning to close; fused nasal bones
23·3	84°·5	70°·0	70°·6	39°·4	102	Coronal and sagittal-beginning to close
21·6	80°·5	69°·6	70°·1	40°·2	103	Pre-condyle L; occipital condyle R extending to median plane
22·9	86°·0	66°·1	71°·7	42°·3	104	Palate defective
18·6	83°·0	66°·6	72°·4	41°·1	105	Metopic; metopic closing; 3 molars lost before death
22·9	84°·5	62°·4	76°·0	41°·7	106	Coronal and sagittal beginning to close; most molars lost before death, abscess cavities at site of M 2, L and M 3, L; small wounds frontal and parietal L
—	—	—	—	—	107	Palate missing; pre-condyle R
—	—	—	—	—	108	Sagittal beginning to close; palate missing; blunt spine projecting from margin of foramen magnum immediately to R of opisthion; severe wounds on frontal and parietals, probably cause of death
—	—	—	—	—	109	Coronal beginning to close, sagittal closed; palate defective
—	—	—	—	—	110	Sagittal beginning to close; palate missing
—	85°·5	61°·0	74°·2	44°·8	111	M 3's absent; canine R below and obliquely situated with regard to alveolar margin
19·6	85°·0?	58°·6?	76°·7?	44°·8?	112	Palate defective
—	—	—	—	—	113	Sagittal nearly obliterated, lambdoid closing
—	85°·0	63°·5	74°·7	41°·9	114	Coronal and lambdoid beginning to close, sagittal nearly obliterated; trepanned; wound on frontal
20·4	84°·0	66°·3	70°·5	43°·2	115	Epipteric bone L; abscess cavity at site of roots of M 1, L; trepanned
—	—	—	—	—	116	Sagittal beginning to close
—	—	—	—	—	117	Palate missing; jugular foramen R divided by arch
26·9	—	—	—	—	118	Sagittal beginning to close; 4 epipteric bones L, pterion R defective; palate missing
—	—	—	—	—	119	Metopic; single epipteric bone L and 2 R; M 3's absent
22·8	—	—	—	—	120	Two or 3 molars lost before death; depressed wound on temporal squama L; small pre-condyle L
24·1	—	62°·8	76°·2	40°·9	121	Aged skull; greater part of occipital and R temporal missing; coronal and sagittal obliterated, lambdoid probably closed; 2 or 3 molars lost before death
—	—	62°·7	78°·1	39°·2	122	L temporal missing; sagittal closing
24·5	—	—	—	—	123	Coronal beginning to close, sagittal closing; large single epipteric bones R and L; palate defective
21·5	—	—	—	—	124	Coronal beginning to close; palate defective
—	—	—	—	—	125	Coronal beginning to close; *os pentagonale*
20·1	79°·5	67°·7	69°·2	43°·1	126	Occipital bone defective; single epipteric bone R
—	—	—	—	—	127	Sagittal closed, lambdoid closing; *os triangulare* L; palate defective
—	90°·5	64°·6	74°·4	41°·0	128	Coronal closing, sagittal closed
23·0	85°·5	60°·0	77°·8	42°·2	129	Metopic
—	—	—	—	—	130	Single epipteric bone L; occipital condyle R extending to median plane
27·6	—	62°·1?	75°·8?	42°·1?	131	Single epipteric bone R; palate defective
17·9	78°·0	64°·9	74°·9	40°·1	132	Diastema between canine and 1st premolar R and L, M 3's absent
—	—	—	—	—	133	Coronal and lambdoid closing, sagittal closed; palate defective

79	180·5	138·5	94·0	130	116·2	115·0	94·5	135·5	130·5	121	387	303	518	35·6
80	184·5	137	95·1	139·5	114·6	117·9	101·2	128·5	133	118·5	379	319	511	38·7
81	189·5	143	97·9	139·5	116·2	125·1	98·0	133?	137·5?	121	392?	321?	532	35·4
82	185	142·5	97·6	140	107·1	117·7	99·0	126	130·5	121	378	320?	—	42·9
83	192·5	146·5	107·0	141·5	117·8	126·9	103·7	134·5	140	127	401	327	556	38·8
84	177	138·5	100·3	138	117·7	113·7?	—	136·5	126·5	—	—	321	503	—
85	176?	139·5	92·8	121	109·5	108·0?	88·2	124	123·5?	104·5	353	308	503	35·9
86	182·5	137	99·3	136	118·5	111·0	92·3	137·5	122	116·5	375	319	529	38·4
87	177	132	93·7	133	117·5?	107·8	97·0	133	120	114·5	367	306	496	33·8
88	181·5	135	92·9	—	112·3	119·2	98·4	125	137	112?	374?	311	521	—
89	182	127·5	95·3	120·5	106·5	116·3	89·0	122	134	107·5	364	290	506	37·4
90	186·5	132·5	93·8	131·5	115·4	120·0	89·7	132	134·5	112	378	307	519	34·9
91	191	149	92·8	138·5	124·0	117·4	104·6	140·5	129	130	400	325	535	36·3
92	175·5	129·5	94·6	130·5	103·9	113·1	91·0	116·5	128	108	352	295	495	35·7
93	172·5	131	95·4	130	102·3?	111·8	95·2	118·5?	123	113·5	355	298	492	31·0
94	175	129	88·5	131	107·0	118·0	90·2	120	135·5	106	361	300	498	35·4
95	181·5	130	94·9	133·5	110·9	113·3	99·7	126	125·5	117	368	297	510	37·3
96	178·5	135	100·0	137·5	111·1	114·2	93·7	128·5	125	111·5	366	308	506	39·9
97	186·5	149·5	95·5	128·5	114·2	115·7	—	130	128·5	—	—	314	535	—
98	175·5	147	94·9	138·5	117·3	118·1	97·4	138·5	132·5	117	388	334	516	34·8
99	176·5	135·5	95·9	133	105·9	113·0	97·6	120	129	113	362	310	504	40·6
100	179	132	92·6	128·5	110·5	111·1	91·5	128·5	125	112	365	296	504	36·9
101	184·5	141	89·0	126	111·9	115·2	102·7	125·5	129	126	381	305	521	35·9
102	178	138	90·6	125·5	109·4	110·8	96·3	125·5	123	113	362	300	503	33·4
103	178	135	90·0	133·5	108·0	107·1	101·0	122	118·5	117·5	358	294	502	37·1
104	184·5	138·5?	96·7	132·5?	116·3?	123·8?	—	—	142?	—	—	321?	521	—
105	178·5	134	95·0	135·5	106·6	116·5	96·4	123·5	133	113	369	304	502	36·6
106	172	129	93·1	132	104·7	112·6	92·4	115·5	124·5	107	348	291	484	38·6
107	187·5	128	94·0	—	109·1	127·7	94·0?	122·5	146	116	384	309	503	—
108	179	131?	100·3	129·5?	106·9	111·3?	98·2?	118·5	124?	116?	358	—	498	36·
109	177	143	97·6	—	107·9	115·8	96·3	125	132·5	121	378	314	518	—
110	180	132·5	90·3	138	116·9	110·4	95·9	132	123	114	368	301	503	39·
111	184	131	101·2	135·5	109·3	113·8	98·4?	125	126·5	118·5	369	302	516	41·
112	179	132·5	96·5	143	116·1?	111·5	98·5	134	124·5	116	374	308	510	39·
113	191·5	137	99·1	137	113·0	123·6	101·9	129	137	118	384	304	532	38·
114	188·5	137	95·1	136	116·0	—	—	134·5	—	—	384	307	525	35·
115	186·5	138·5	98·3	136	111·1	109·3?	102·9?	128·0	125·5	123	376	310	523	38·
116	187	137	86·0	137·0?	113·1	108·3	102·6	132	118	125	375	311	521	40·
117	178	131	95·3	132·5	108·9	116·2	91·5	125	129·5	108·5	362	300	505	37·
118	180·5	138	95·2	131	112·5	114·7	95·9	130	130	118·5	378	—	511?	32·
119	176	134·5	98·8	130·5	111·8	114·0?	92·3?	132	125·5	110	367?	—	505	36·
120	182	131·5	99·2	122·5	108·5	115·0	91·2	120	127·5	112	359	296	511	35·
121	—	129	—	126	108·9	—	—	125·5	—	—	—	—	—	—
122	177	127	85·4?	129·5	112·6	107·9	84·3?	129·5?	122·5	100	351?	—	488?	37·
123	176·5	132	86·0	129·5	116·0	102·8	94·4	132·5	115·5	110	357	294	498	38·
124	172·5	133	91·0	134	111·1	110·0	93·9	125·5	121	109	355	308	489	33·
125	177·5	130·5	98·7	133·5	109·0	103·1?	101·0?	121·5	114	116·5	352	298	494	37·
126	174?	137	96·0	135	112·1	109·9?	—	126·5	126	—	—	301	497?	—
127	181	133·5	94·2	132·5?	117·3	100·4?	89·6?	132·5	124·5	103	360	308	504	—
128	178	142	100·0	133·5	101·9	121·2	98·5	124	136·5	118	378	318	510	32·
129	180	138·5	92·2	141	116·0	117·9	96·1	135·5	132	113	381	311	510	37·
130	181·5	140	92·6	—	108·0	120·2	—	121	134·5	—	—	309	511	—
131	177	135	93·0	128	108·0	112·9	92·9	125	125·5	116	366	303	495	33·
132	183	133	96·9	138·5	111·0	121·3	97·2	130	139·5	112	381	301	510	41·
133	186·5	141	94·0	134·5	—	111·2	104·0	—	124	124·5	—	311	523	40·
134	179·5	133·5	96·2	129·5	112·0	111·4	91·8	130	126·5	110·5	366	301	508	39·
135	191·5	135	97·1	132	118·9?	114·3	99·8	139·5?	128·5	122·5	391?	307	526	39·
136	182·5	145	97·3	140·5	117·0	112·4	97·1	134	124	117	375	320	522	40·
137	179	139	95·2?	137	112·2?	122·7?	97·0?	129	138	117·5	385	316	513	39·
138	177	129	89·2	128	108·4	99·8	93·1	124	108·5	120	352	281	490	37·
139	187·5	128	92·7	—	110·2	122·8	99·0?	125	139·5	120	385	296	502	—
140	190·5	132	94·8	134	118·0	116·8	93·0	135	128	115	378	302	524	40·
141	181	126	90·1	129·5	112·6	111·2	92·5	127·5	124·5	107	359	289	497	38·
142	173·5	131·5	89·9	131	112·0	100·9	103·8	130	108	129	366	309	495	37·
143	197·5	138	95·0	143	119·9	120·1	106·5	138·5	133·5	136·5?	408?	318	540	39·

303	518	35·6	30·4	95·5	90·9	72·8	126·5	92·2	23·9	51·9	11·0	6·6	33·1	40·2	49·2	—	62·5	51·5	53·8	13·4	
319	511	38·7	30·5	103·2	93·0	73·3	—	94·3	—	50·5	13·3	8·2	32·0	43·2	50·2?	—	62·5?	54·0?	56·0?	11·4?	
321?	532	35·4	28·3	102·2	—	—	—	—	—	—	13·0	6·0	28·0	41·1	—	—	—	—	—	—	
320?	—	42·9	35·6	105·6	—	—	—	93·5	—	—	9·0	4·1	31·4	41·9	—	—	—	50·0	—	—	
327	556	38·8	33·1	104·0	99·6	71·8	135	—	—	—	14·1	5·8	35·0	42·2	49·3?	40·7?	—	—	—	—	
321	503	—	—	99·0	92·0	64·4	—	—	25·0	47·1	8·2	4·3	34·0	43·5	45·1	—	—	52·0	—	—	
308	503	35·9	31·1	98·2	87·3?	70·8	—	93·3	24·1	54·2	10·2	5·0	34·1	44·8	—	—	—	52·0	—	—	
319	529	38·4	—	96·7	92·1	69·2	128·5	—	26·4	52·3	10·1	6·1	33·0	41·5	45·7	—	51·5	49·5	45·1	10·9	
306	496	33·8	28·6	100·7?	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
311	521	—	—	—	—	72·5?	—	99·5	27·0	52·5	11·9	5·3	30·9	43·8	53·5	—	65·0	54·0	54·7	13·2	
290	506	37·4	29·2	98·1	—	—	—	—	22·5?	—	6·9	2·8	34·4	39·2	—	—	63·0	49·5	55·1	14·1	
307	519	34·9	27·5	101·8	101·1	63·8	133·5	98·4	24·4	48·0	8·7	5·2	30·9	44·0	52·1?	43·1	65·0	51·5	55·1	11·9	
325	535	36·3	28·4	98·9	91·0	78·2	—	97·1	24·0	54·7	10·8	4·9	31·2	40·2	44·1	—	62·0	54·0	55·4	10·3	
295	495	35·7	28·1	97·0	95·1	66·4	—	91·7	24·2	48·8	8·2	2·6	29·9	37·4	48·2	—	—	48·0	—	—	
298	492	31·0	29·0	97·3?	—	—	—	94·6	23·4?	51·8?	—	—	34·2	40·0	43·2	—	50·5	44·5	44·6	11·4	
300	498	35·4	27·9	97·2	96·9	70·4	—	96·0	25·9	51·7	8·3	3·4	35·1	41·8	46·4	39·3	52·0	51·0	45·9	9·1	
297	510	37·3	31·2	100·3	—	—	—	—	—	—	8·1	3·0	—	—	—	—	—	—	—	—	
308	506	39·9	32·5	101·0	—	—	—	—	—	—	12·1	4·7	—	—	—	—	—	—	—	—	
314	535	—	—	97·7	92·3	65·1	—	—	24·1	48·9	10·9	5·6	—	—	48·4	38·9	—	—	—	—	
334	516	34·8	29·9	93·0	—	—	120	91·2	27·3	51·8?	12·5	4·7	29·0	40·2	42·9	—	53·5	44·5	47·6	11·0	
310	504	40·6	31·9	98·8	—	—	—	—	—	—	10·5	4·5	—	—	—	—	—	—	—	—	
296	504	36·9	33·1?	98·5	—	—	120·5	87·7	22·8	52·5	8·7	4·7	31·1	40·6	—	—	57·0	46·5	50·6	10·3	
305	521	35·9	30·6	96·1	91·4	75·8	127·5?	94·5	23·3	54·3	12·2	6·0	35·0	38·8	51·1	40·3	—	50·5	—	—	
300	503	33·4	26·2	94·5	94·2	63·5	—	—	24·5	44·2	9·9	5·2	30·3	40·0	44·2	42·3?	61·0	47·5	53·6	12·5	
294	502	37·2	29·9	103·2	103·0	71·0	127·5	89·8	—	—	10·9	6·4	33·8	42·7	53·1	41·0	58·5?	47·5	51·5	11·1	
321?	521	—	—	101·0	97·5	71·6	125	92·0	24·1	49·2	12·0	7·2	32·6	42·1	48·1	—	56·5	53·5	49·0	11·2	
304	502	36·0	31·4?	101·1	97·4	66·9	122	89·0?	23·9	48·8?	12·9	5·1	29·4	38·3	—	—	60·5	44·0	54·4	10·1	
291	484	38·0	27·8	97·9	89·5	67·1	122	87·2	26·9	49·0	13·0	7·0	—	40·0	—	—	58·0	55·0	51·5	11·8	
309	503	—	—	—	—	—	—	—	—	—	10·0	5·6	—	—	—	—	—	—	—	—	
—	498	36·0	30·0	99·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
314	518	—	27·7	—	—	—	126	90·9	26·2	—	10·4	5·3	33·3	41·6	—	—	—	53·5	—	—	
301	503	39·5	32·3	100·2	—	—	—	—	—	—	9·1	5·1	—	—	—	—	—	—	—	—	
302	516	41·6?	—	101·2	92·2	74·1	—	92·0	24·0?	53·9	9·8	3·5	33·9	43·0	44·5	39·6	—	42·5	—	—	
308	510	39·4	30·1	103·0?	90·3	74·5?	126	97·6	—	52·6?	—	—	36·8	40·3	—	—	51·5	45·0	46·4	9·1	
304	532	38·1	35·4	105·1	—	—	—	—	28·4	55·4	11·5	5·2	33·3	42·7	—	39·2?	—	—	—	—	
307	525	35·2	28·3	99·2	92·2	68·9	—	103·2	28·0	52·3	10·3	4·7	31·9	41·9	—	—	—	52·5	—	—	
310	523	38·6	30·0	103·5	100·7	75·2	131·5	95·1	23·9	52·5	10·7	5·5	32·6	40·1	48·9	43·3	62·5	49·0	55·0	11·2	
311	521	40·5?	31·7	100·5?	—	—	—	96·5	25·5	50·1?	—	—	30·6?	40·5?	—	38·7	—	45·5	—	—	
300	505	37·2	31·7	95·7	—	—	—	—	—	—	11·2	5·0	—	—	—	—	—	—	—	—	
—	511?	32·4	33·2	99·1	—	—	—	—	—	—	7·1	2·7	33·0	42·1	—	—	61·0	45·5	53·2	14·3	
—	505	36·0	29·5	96·0	—	—	124	93·8	25·9?	—	—	—	28·5	40·0	42·8?	36·6	—	44·0?	—	—	
296	511	35·8	28·8	97·9	—	—	—	89·1	23·3?	50·9	—	—	36·1	42·2	50·1	—	65·0	51·0	57·1?	13·0?	
—	—	—	—	99·9	90·7	66·8	—	93·7	23·4	50·0	11·1	4·3	—	—	44·7	—	57·0	—	50·2	12·1	
—	488?	37·8?	—	102·5	93·1	66·2	—	—	25·0	49·4	7·5	4·9	31·0?	—	42·2	—	—	—	—	—	
294	498	38·2	33·4	94·0	—	—	125·5	96·2	29·2	55·1	8·2	3·3	35·9	40·0	—	—	59·0?	47·0	51·0	12·5	
308	489	33·8	28·4	100·0	—	—	—	91·4	26·8	53·2	8·4	4·9	34·0	41·9	—	—	52·5	46·0	46·6?	10·0?	
298	494	37·9	32·9	102·2	—	—	—	95·2	24·6	53·1	10·6	6·1	33·0	40·8	45·8	41·7	—	48·0	—	—	
301	497?	—	—	101·2	100·3	74·0	—	97·8	28·7	56·1	8·6	3·9	33·9	41·0?	—	42·6	60·5	49·5	55·3	11·1	
308	504	—	29·5	—	—	—	121·5	93·4	—	—	8·6	3·7	33·4	41·8	—	38·5?	—	53·0	—	—	
318	510	32·0	30·0	97·9	91·0	66·0	—	—	25·9?	50·0	10·0	4·4	33·1	—	44·5	—	—	—	—	—	
311	510	37·7	32·2	97·0	86·0	66·7	—	91·0	23·1	49·6	5·9	2·1	31·1	40·0	42·1	38·1	59·0	43·5	51·3	11·8	
309	511	—	—	—	—	—	—	93·1	21·9?	49·9	10·7	4·6	35·0	38·9	48·7	40·0?	—	43·5	—	—	
303	495	33·2	26·9	96·3	88·0?	67·6?	—	92·4	25·9	49·7	10·6	6·0	34·1	40·4	—	—	63·0	46·5	52·9	14·6	
301	510	41·1	35·3	95·9	90·1	64·0	126	89·5	24·3	47·0	9·9	4·4	34·5	40·9	46·6	40·8	50·0?	43·0	45·3?	8·1?	
311	523	40·1	31·2	—	—	—	—	—	22·8	—	—	—	33·4	—	44·9?	—	—	—	—	—	
301	508	37·8	30·0	99·2	94·3	69·3	121·5	90·0	24·6	47·9	11·1	4·0	32·6	42·1	48·5?	39·9	60·0	47·5	52·3	12·8	
307	526	38·2	30·3?	99·1	90·3?	69·7	—	91·0	—	51·0	15·0	5·0	35·0	39·8	45·7	39·0	—	50·0	—	—	
320	522	40·1	31·2	102·2	—	—	—	97·2	27·4	—	11·6?	5·6?	36·2	41·2	—	38·1	59·5	51·0	52·6	11·0	
316	513	33·3	30·7	97·3	84·9?	70·1?	—	85·9	—	53·1?	12·1	5·1	—	—	—	—	—	45·5	—	—	
281	490	34·5	27·9	96·2	—	—	—	—	—	—	9·7	5·2	34·0?	39·2?	—	—	52·0	—	46·0	10·1	
296	502	—	—	—	—	—	—	88·9	23·3	50·1	9·2	5·3	32·6	42·0	47·6?	—	60·0	49·0	53·2	11·1	
302	524	40·7	32·8	102·2	97·5	72·9	122	93·0	24·4	52·6	10·3	5·2	32·9	41·0	50·9	40·8	57·0	48·0	50·9	11·0	
289	497	36·0	33·6	103·0	—	—	128·5?	90·6?	23·7	—	10·3	5·3	33·0	39·2	—	—	—	45·0	—	—	
309	495	34·8	27·3	91·1	85·1	64·8	—	90·0	23·9	49·6	9·3	5·2	32·1	39·0	—	—	—	47·0	—	—	
318	540	37·4	30·5	105·2	97·2	67·0	—	—	25·0	53·0	11·6	8·0	—	—	46·8?	—	—	—	—	—	

—	50·0	—	—	—	77·0	75·7	101·8	1·4	58·5	83·0	—	—	
—	—	—	—	122·6	76·1	73·5	103·5	2·6	58·4	85·3	—	—	
—	52·0	—	—	119·5	78·2	78·0	100·4	0·3	—	—	—	53·1	
—	52·0	—	—	—	79·3	68·7	115·3	10·5	61·2	86·6	75·9	44·5	
51·5	49·5	45·1	10·9	121·7	75·1	74·5	100·7	0·6	56·5	—	—	50·5	
—	—	—	—	116·1	74·6	75·1	99·2	−0·6	61·7	84·6	—	—	
65·0	54·0	54·7	13·2	118·1	74·4	—	—	—	66·7	—	72·9	51·4	
63·0	49·5	55·1	14·1	107·8	70·1	66·2	105·8	3·9	59·4	78·1	—	—	
65·0	51·5	55·1	11·9	114·7	71·0	70·5	100·8	0·5	57·1	78·8	64·8	50·8	
62·0	54·0	55·4	10·3	120·8	78·0	72·5	107·6	5·5	57·4	78·2	80·5	43·9	
—	48·0	—	—	115·0	73·8	74·4	99·2	−0·6	61·0	78·7	72·4	49·6	
50·5	44·5	44·6	11·4	112·2	75·9	75·4	100·8	0·6	60·6	93·5	—	45·2	
52·0	51·0	45·9	9·1	114·6	73·7	74·9	98·5	−1·1	62·1	78·8	73·3	50·1	
—	—	—	—	—	71·6	73·6	97·4	−1·9	62·2	83·6	—	—	
—	—	—	—	—	75·6	77·0	98·2	−1·4	60·8	81·5	—	—	
—	—	—	—	112·8	80·2	68·9	116·3	11·3	—	—	—	49·3	
53·5	44·5	47·6	11·0	122·2	83·8	78·9	106·1	4·8	59·9	85·9	—	52·7	
—	—	—	—	—	76·8	75·4	101·9	1·4	64·0	78·6	—	—	
57·0	46·5	50·6	10·3	108·0	73·7	71·8	102·7	2·0	58·4	89·7	—	43·4	
—	50·5	—	—	111·8	76·4	68·3	111·9	8·1	58·2	80·5	80·2	42·9	
61·0	47·5	53·6	12·5	109·5	77·5	70·5	110·0	7·0	62·3	78·4	—	55·4	
58·5?	47·5	51·5	11·1	109·4	75·8	75·0	101·1	0·8	63·4	80·4	79·1	—	
56·5	53·5	49·0	11·2	—	75·1	71·8	104·5	3·3	—	—	77·8	49·0	
60·5	44·0	54·4	10·1	113·6	75·1	75·9	98·9	−0·8	62·4	87·2	75·2	49·0	
58·0	55·0	51·5	11·8	109·4	75·0	76·7	97·7	−1·7	64·0	73·2	76·9	54·9	
—	—	—	—	—	68·3	—	—	—	57·8	—	—	—	
—	—	—	—	—	73·2	72·3	101·2	0·8	61·6	83·3	—	—	
—	53·5	—	—	115·5	80·8	—	—	—	56·8	—	—	—	
—	—	—	—	—	73·6	76·7	96·0	−3·1	60·9	81·8	—	—	
—	42·5	—	—	113·1	71·2	73·6	96·7	−2·4	59·7	—	80·5	44·5	
51·5	45·0	46·4	9·1	116·9	74·0	79·9	92·7	−5·9	61·9	76·4	76·3	—	
—	—	—	—	116·5	71·5	71·5	100·0	0	64·0	92·9	—	51·3	
—	52·5	—	—	—	72·7	72·1	100·7	0·5	—	80·4	66·8	53·5	
62·5	49·0	55·0	11·2	—	74·3	72·9	101·8	1·3	60·4	77·7	79·1	45·5	
—	45·5	—	—	—	73·3	73·3	100·0	0	58·8	78·3	—	50·9	
—	—	—	—	—	73·6	74·4	98·9	−0·8	61·1	85·2	—	—	
61·0	45·5	53·2	14·3	—	76·5	72·6	105·3	3·9	57·7	102·5	—	—	
—	44·0?	—	—	—	76·4	74·1	103·1	2·3	60·6	81·9	—	—	
65·0	51·0	57·1?	13·0?	110·8	72·3	67·3	107·3	4·9	58·2	80·4	—	45·8	
57·0	—	50·2	12·1	—	—	—	102·4	—	—	—	71·3	46·8	
—	—	—	—	—	71·8	73·2	98·1	1·4	61·1	—	—	50·6	
59·0?	47·0	51·0	12·5	—	74·8	73·4	101·9	1·4	63·2	87·4	—	53·0	
52·5	46·0	46·6?	10·0?	—	77·1	77·7	99·3	−0·6	63·6	84·0	—	50·4	
—	48·0	—	—	114·2	73·5	75·2	97·8	−1·7	64·6	86·8	—	46·3	
60·5	49·5	55·3	11·1	—	78·7	77·6	101·5	1·1	—	—	75·7	51·2	
—	53·0	—	—	114·5	73·8	73·2	100·8	0·6	65·0	—	—	—	
—	—	—	—	119·0	79·8	75·0	106·4	4·8	60·2	93·7	—	51·8	
59·0	43·5	51·3	11·8	114·1	76·9	78·3	98·2	−1·4	62·1	85·4	73·3	46·6	
—	43·5	—	—	115·2	77·1	—	—	—	—	—	—	43·9	
63·0	46·5	52·9	14·6	109·7	76·3	72·3	105·5	4·0	57·1	81·0	73·2	52·1	
50·0?	43·0	45·3?	8·1?	112·3	72·7	75·7	96·0	−3·0	64·7	85·9	71·5	51·7	
—	—	—	—	—	75·6	72·1	104·8	3·5	60·2	77·8	—	—	
60·0	47·5	52·3	12·8	113·0	74·4	72·1	103·1	2·2	59·7	79·4	77·0	51·4	
—	50·0	—	—	—	70·5	68·9	102·3	1·6	58·2	79·3	76·6	—	
59·5	51·0	52·6	11·0	118·1	79·3	77·0	103·2	2·5	59·6	77·8	—	—	
—	45·5	—	—	—	77·7	76·5	101·5	1·1	59·2	92·2	81·6	—	
52·0	—	46·0	10·1	—	72·9	72·3	100·8	0·6	55·6	80·9	—	—	
60·0	49·0	53·2	11·1	111·3	68·3	—	—	—	59·2	—	—	46·5	
57·0	48·0	50·9	11·0	113·3	69·3	70·3	98·6	−1·0	57·7	80·6	78·4	46·4	
—	45·0	—	—	106·2	69·6	71·5	97·3	−1·9	64·1	93·3	—	—	
—	47·0	—	—	114·6	75·8	75·6	100·4	0·3	57·4	78·4	72·0	48·2	
—	—	—	—	120·0	69·9	72·4	96·5	−2·5	55·8	81·6	—	47·2	

—	74·9	—	45·6	—	—	—	—	—	—	—	82
—	82·9	82·6	41·1	—	—	82°·5	65°·9	72°·9	41°·1	83
53·1	78·2	—	52·4	—	—	86°·0	64°·4	76°·5	39°·1	84
44·5	78·2	—	49·0	—	—	93°·0	59°·5?	76°·1?	44°·4?	85
50·5	79·5	—	60·4	96·1	24·2	86°·5	64°·9	72°·1	42°·9	86
—	—	—	—	—	—	—	—	—	—	87
51·4	70·5	—	44·5	83·1	24·1	—	—	—	—	88
—	87·8	—	40·6	78·6	25·6	—	—	—	—	89
50·8	70·2	82·7	59·8	79·2	21·6	84°·0	71°·0	72°·4	36°·5	90
43·9	77·6	—	45·4	87·1	18·6	90°·5	60°·4	71°·1	48°·4	91
49·6	79·9	—	31·7	—	—	83°·0?	68°·0	71°·5	40°·5	92
45·2	85·5	—	—	88·1	25·6	—	—	—	—	93
50·1	84·0	84·7	41·0	98·1	19·8	80°·5	68°·3	69°·1	42°·6	94
—	—	—	37·0	—	—	—	—	—	—	95
—	—	—	38·8	—	—	—	—	—	—	96
49·3	—	80·4	51·4	—	—	85°·5	65°·6	74°·5	39°·9	97
52·7	72·1	—	37·6	83·2	23·1	—	—	—	—	98
—	—	—	42·9	—	—	—	—	—	—	99
43·4	76·6	—	54·0	81·6	20·4	—	—	—	—	100
42·9	90·2	78·9	49·2	—	—	85°·5	62°·8	69°·7	47°·4	101
55·4	75·7	95·7	52·5	77·9	23·3	84°·5	70°·0	70°·6	39°·4	102
—	79·2	77·2	58·7	81·2	21·6	80°·5	69°·6	70°·1	40°·2	103
49·0	77·4	—	60·0	94·7	22·9	86°·0	66°·1	71°·7	42°·3	104
49·0	76·8	—	39·5	72·7	18·6	83°·0	66°·6	72°·4	41°·1	105
54·9	—	—	53·8	94·8	22·9	84°·5	62°·4	76°·0	41°·7	106
—	—	—	56·0	—	—	—	—	—	—	107
—	—	—	—	—	—	—	—	—	—	108
—	80·0	—	51·0	—	—	—	—	—	—	109
—	—	—	56·0	—	—	—	—	—	—	110
44·5	78·8	89·0	35·7	—	—	85°·5	61°·0	74°·2	44°·8	111
—	91·3	—	—	87·4	19·6	85°·0?	58°·6?	76°·7?	44°·8?	112
51·3	78·0	—	45·2	—	—	—	—	—	—	113
53·5	76·1	—	45·6	—	—	85°·0	63°·5	74°·7	41°·9	114
45·5	81·3	88·5	51·4	78·4	20·4	84°·0	66°·3	70°·5	43°·2	115
50·9	75·6	—	—	—	—	—	—	—	—	116
—	—	—	44·6	—	—	—	—	—	—	117
—	78·4	—	38·0	74·6	26·9	—	—	—	—	118
—	71·3	85·5	—	—	—	—	—	—	—	119
45·8	85·5	—	—	78·5	22·8	—	—	—	—	120
46·8	—	—	38·7	—	24·1	—	62°·8	76°·2	40°·9	121
50·6	—	—	65·3	—	—	—	62°·7	78°·1	39°·2	122
53·0	89·7	—	40·2	79·7	24·5	—	—	—	—	123
50·4	81·1	—	58·3	87·6	21·5	—	—	—	—	124
46·3	80·9	91·0	57·5	—	—	—	—	—	—	125
51·2	82·7	—	45·3	81·8	20·1	79°·5	67°·7	69°·2	43°·1	126
—	79·9	—	43·0	—	—	—	—	—	—	127
51·8	—	—	44·0	—	—	90°·5	64°·6	74°·4	41°·0	128
46·6	77·7	90·5	35·6	73·7	23·0	85°·5	60°·0	77°·8	42°·2	129
43·9	90·0	82·1	43·0	—	—	—	—	—	—	130
52·1	84·4	—	56·6	73·8	27·6	—	62°·1?	75°·8?	42°·1?	131
51·7	84·4	87·6	44·4	86·0	17·9	78°·0	64°·9	74°·9	40°·1	132
—	—	—	—	—	—	—	—	—	—	133
51·4	77·4	82·3	36·0	79·2	24·5	86°·0	64°·4	73°·7	42°·0	134
—	87·9	85·3	33·3	—	—	87°·0?	61°·8?	75°·5?	42°·8?	135
—	87·9	—	48·3	85·7	20·9	—	—	—	—	136
—	—	—	42·1	—	—	—	58°·1?	77°·4?	44°·5?	137
—	86·7	—	53·6	—	22·0	—	—	—	—	138
46·5	77·6	—	57·6	81·7	20·9	—	—	—	—	139
46·4	80·2	80·2	50·5	84·2	21·6	—	65°·1	72°·1	42°·7	140
—	84·2	—	51·5	—	—	—	—	—	—	141
48·2	82·3	—	55·9	—	—	86°·5	63°·4	73°·7	42°·9	142
47·2	—	—	69·0	—	—	84°·5	64°·3	77°·4	38°·3	143

					foramen exceptionally large, R exceptionally small; palate defective; small wound on frontal
2°·5	65°·9	72°·9	41°·1	83	Sagittal beginning to close; close approach to pterion in *K*, L; palate defective
6°·0	64°·4	76°·5	39°·1	84	Coronal and lambdoid beginning to close, sagittal closing; palate defective; healed wound on superciliary ridge, and on L parietal
3°·0	59°·5?	76°·1?	44°·4?	85	Coronal partly obliterated, sagittal obliterated, lambdoid closed; 2 or more molars lost before death; trace of horizontal suture across malar bone R
6°·5	64°·9	72°·1	42°·9	86	Large epipteric bone R; wormian bone between temporal squama and parietal R above mastoid process; trace of metopic suture above nasion; 3 molars lost before death
—	—	—	—	87	Coronal and sagittal beginning to close; palate defective; small pre-condyle L
—	—	—	—	88	Two molars lost before death
—	—	—	—	89	Two epipteric bones R and 2 L; palate defective
4°·0	71°·0	72°·4	36°·5	90	Occipito-mastoid L obliterated, and posterior half of temporal L fused to parietal; small healed wound frontal
0°·5	60°·4	71°·1	48°·4	91	All sutures beginning to close; greater wing of sphenoid fused to parietal and frontal R and L; 1 molar and 1 premolar lost before death
3°·0?	68°·0	71°·5	40°·5	92	Sagittal closed; single epipteric bones R and L; 3 molars and 1 pre-molar lost before death; healed injury to margin of nasal aperture
—	—	—	—	93	Small epipteric bone R
0°·5	68°·3	69°·1	42°·6	94	*Os triangulare* R
—	—	—	—	95	Coronal beginning to close; palate missing; fused nasal bones
—	—	—	—	96	Metopic; palate missing
5°·5	65°·6	74°·5	39°·9	97	Trace of horizontal suture on malar bone R; 2 small pre-condyles; wound on frontal
—	—	—	—	98	*Os triangulare* R; M 3's absent; 2 small wounds on frontal
—	—	—	—	99	Palate missing; trace of horizontal suture on malar bone R
—	—	—	—	100	All sutures closing; posterior halves of temporal squamae fused to parietals R and L; most teeth lost before death
5°·5	62°·8	69°·7	47°·4	101	Sagittal beginning to close; fused nasal bones
4°·5	70°·0	70°·6	39°·4	102	Coronal and sagittal-beginning to close
0°·5	69°·6	70°·1	40°·2	103	Pre-condyle L; occipital condyle R extending to median plane
6°·0	66°·1	71°·7	42°·3	104	Palate defective
3°·0	66°·6	72°·4	41°·1	105	Metopic; metopic closing; 3 molars lost before death
4°·5	62°·4	76°·0	41°·7	106	Coronal and sagittal beginning to close; most molars lost before death, abscess cavities at site of M 2, L and M 3, L; small wounds frontal and parietal L
—	—	—	—	107	Palate missing; pre-condyle R
—	—	—	—	108	Sagittal beginning to close; palate missing; blunt spine projecting from margin of foramen magnum immediately to R of opisthion; severe wounds on frontal and parietals, probably cause of death
—	—	—	—	109	Coronal beginning to close, sagittal closed; palate defective
—	—	—	—	110	Sagittal beginning to close; palate missing
5°·5	61°·0	74°·2	44°·8	111	M 3's absent; canine R below and obliquely situated with regard to alveolar margin
5°·0?	58°·6?	76°·7?	44°·8?	112	Palate defective
—	—	—	—	113	Sagittal nearly obliterated, lambdoid closing
5°·0	63°·5	74°·7	41°·9	114	Coronal and lambdoid beginning to close, sagittal nearly obliterated; trepanned; wound on frontal
4°·0	66°·3	70°·5	43°·2	115	Epipteric bone L; abscess cavity at site of roots of M 1, L; trepanned
—	—	—	—	116	Sagittal beginning to close
—	—	—	—	117	Palate missing; jugular foramen R divided by arch
—	—	—	—	118	Sagittal beginning to close; 4 epipteric bones L, pterion R defective; palate missing
—	—	—	—	119	Metopic; single epipteric bone L and 2 R; M 3's absent
—	—	—	—	120	Two or 3 molars lost before death; depressed wound on temporal squama L; small pre-condyle L
—	62°·8	76°·2	40°·9	121	Aged skull; greater part of occipital and R temporal missing; coronal and sagittal obliterated, lambdoid probably closed; 2 or 3 molars lost before death
—	62°·7	78°·1	39°·2	122	L temporal missing; sagittal closing
—	—	—	—	123	Coronal beginning to close, sagittal closing; large single epipteric bones R and L; palate defective
—	—	—	—	124	Coronal beginning to close; palate defective
—	—	—	—	125	Coronal beginning to close; *os pentagonale*
9°·5	67°·7	69°·2	43°·1	126	Occipital bone defective; single epipteric bone R
—	—	—	—	127	Sagittal closed, lambdoid closing; *os triangulare* L; palate defective
0°·5	64°·6	74°·4	41°·0	128	Coronal closing, sagittal closed
5°·5	60°·0	77°·8	42°·2	129	Metopic
—	—	—	—	130	Single epipteric bone L; occipital condyle R extending to median plane
—	62°·1?	75°·8?	42°·1?	131	Single epipteric bone R; palate defective
8°·0	64°·9	74°·9	40°·1	132	Diastema between canine and 1st premolar R and L, M 3's absent
—	—	—	—	133	Coronal and lambdoid closing, sagittal closed; palate defective
6°·0	64°·4	73°·7	42°·0	134	One molar lost before death
7°·0?	61°·8?	75°·5?	42°·8?	135	—
—	—	—	—	136	Metopic; palate defective
—	58°·1?	77°·4?	44°·5?	137	Coronal closed, sagittal obliterated, lambdoid closing; 1 premolar and 1 molar lost before death; fused nasal bones
—	—	—	—	138	Metopic; palate missing
—	—	—	—	139	Coronal and lambdoid beginning to close, sagittal closed; 4 molars lost before death; temporal squama L in process of fusing to parietal
—	65°·1	72°·1	42°·7	140	Sagittal closed, lambdoid closing; M 3, R absent
—	—	—	—	141	Coronal and lambdoid beginning to close, sagittal closed; palate defective
6°·5	63°·4	73°·7	42°·9	142	Sagittal closing; single epipteric bone R; palate defective; small pre-condyle R; canine R below alveolar margin, and erupting obliquely behind lateral incisor
4°·5	64°·3	77°·4	38°·3	143	Abcess cavities at sites of I 1 and I 2, R, and M 2, R, and these teeth lost before death; zygomatic arch R bent inwards, possibly owing to injury in infancy

Tomb no.	Sex	No.	L	B	B'	H'	S_1'	S_2'	S_3'	S_1	S_2	S_3	S	$\beta Q'$	U
120	♂?	144	182	140	89·3	132	108·1	113·3	97·2	124	127·5	118	369	301	520
,,	,,	145	181·5	143·5	93·3	—	113·9	118·0	93·2?	124·5	131·5	113	369	321	516
,,	,,	146	193·5	139·5	97·2	135	117·3	124·2	100·6	133·5	138	124	396	312	534
,,	,,	147	187·5	—	92·2	—	107·1?	107·6?	—	123·5	122·5	—	—	—	—
107	♂	148	185	140	89·6	137·5	110·9	112·5	99·0	125	123·5	121	370	312	517
,,	,,	149	186	127·5	99·5	—	113·0	—	—	127·5	—	—	—	311	512
,,	,,	150	189	—	100·1	134	117·1	122·0	97·7	140	135·5	116	392	—	—
,,	,,	151	178·5	130	97·8	123·5	106·1	110·7	89·9?	120	125	110·5	356	289	500
,,	,,	152	184·5	136	94·5	134?	113·6?	112·8?	95·1	130·5?	—	114	—	309?	518?
,,	,,	153	180·5	136·5	97·4	133·5	108·0	112·6	95·5	125·5	126	115·5	366	302	516
,,	,,	154	179·5	—	93·1	133·5	107·5	116·0	99·0	119·5	132	113	365	—	—
,,	,,	155	191	136	91·7	—	113·0	115·3	101·4?	130	128·5	125	383?	308	529
,,	,,	156	185·5	151·5	111·6	137	107·9	107·9	101·2	124·5	122	125	371	331	537
,,	,,	157	188	138	90·4	134·5	120·2?	119·0?	93·1	135·5?	131·5?	110	378	312	515
,,	,,	158	180·5	141	104·2	138·5	111·6	120·7	96·0	127·5	—	111	—	320	524?
,,	,,	159	184	—	100·0	136·5	109·2	122·9	91·9	124	138	112·5	375	—	—
,,	,,	160	188·5	138	105·3	—	114·2	118·2	93·6	129	131	109·5	369	314	529
,,	,,	161	186·5	138·5	94·0	136·5	114·9?	122·6	95·0	131·5?	139	110·5	381	300	523
,,	,,	162	189	142	93·0	—	118·0	122·2	—	135	136·5	—	—	319	532
,,	,,	163	186	137	95·9	132	115·2	112·7	—	132	129	—	—	306	520
,,	♂?	164	184	132·5	99·0	120·5	113·0	108·1	96·1	129·5	117	125	371	292	510
,,	,,	165	177·5	135·5	98·1	136·5	114·3	113·1	96·2	132	126·5	113	371	305	513
,,	,,	166	190·5	140·5	97·0	139·5	122·7	114·1	101·3	145	130	122	397	318	530?
,,	,,	167	188·5	137	—	135·5	—	119·3	—	—	133	—	—	308	—
,,	,,	168	—	—	99·0	—	114·1?	115·7	96·2	—	128	115	—	—	—
,,	,,	169	175	137·5	93·0	126·5	107·1	117·0	85·4	123	130	107	360	306	505
,,	,,	170	183·5?	134·5?	91·8	—	111·0	111·7	97·3?	125	124	117·5?	366?	297	504
,,	,,	171	194	140	93·7	124	122·1	—	—	142	—	—	—	319	536
,,	,,	172	185	134	95·5	132	110·2	114·2	94·0	127	126·5	114	367	301	519
116	♂	173	192·5	135	95·8	132	116·3	—	—	136	—	—	—	313	536
,,	,,	174	182·5	125·5	99·1	—	111·3	117·1	91·9	128·5	130·5	110	368?	302	510
,,	,,	175	185	134·5	93·9	132	112·9	115·7	91·5	130·5	128·5	113	372	304	514
,,	,,	176	179	133·5	89·0	132·5	106·0	114·0	92·0	117·5	128	112	357	297	499
,,	,,	177	191·5	139	95·1	136	113·7	112·0	—	127·5	124·5	—	—	312	533
,,	,,	178	186·5	132·5	89·6	130	116·3	110·4	96·1	133	123·5	118	375	302	512
,,	,,	179	187·5	147	92·9?	143·5	116·1	120·2	95·0	—	135·5	114·5	—	322	535
,,	,,	180	182	131·5	86·9	137	113·9	113·3	100·1	130·5	130	120	380	296	503
,,	,,	181	188	135	91·4	140?	—	—	—	—	—	—	388	307	522
,,	♂?	182	175	129·5	95·2	123·5	113·0	115·1	80·0	129·5	132	97·5	360	308	496
,,	,,	183	184	140	100·1	—	118·6	113·4	—	136	131·5	—	—	312	520
,,	,,	184	184	140	97·1	122·5	111·0	117·9	85·8	128·5	133	100·5	362	310	518?
,,	,,	185	190·5	129	94·1	143	116·0	123·3	96·2	130·5	139	117	386	303	517
108	♂	186	187	146·5	96·1	143	125·9	115·1	94·1	145·5	128·5	—	—	332	532
,,	,,	187	189·5	137	94·2	129·5	113·0?	107·9?	100·0	129?	120	124·5	373	310	—
,,	,,	188	190	—	99·0	130	111·2	123·0	94·1	125·5	136	114	375	—	—
,,	,,	189	184	137	90·3	—	117·2	122·4	98·2?	134·5	137	113·5?	386?	322	514
,,	,,	190	187	134·5	88·0	140·5	119·9	123·2	—	137·5	144·5	—	—	319	515
120	♂	364	194	125	96·3	131	—	—	—	—	—	—	373	284	527?
,,	,,	365	182	131	93·9	122·5	—	—	—	—	—	—	384	291	507
,,	,,	366	174·5	136·5	93·0	131·5?	—	—	—	—	—	—	373	304	501
,,	,,	367	177·5	132·5	96·1	129·5	111·1	—	—	130	—	—	368	294	504
,,	,,	368	188	135·5	96·9	129·5?	—	—	—	—	—	—	401	312	529
,,	,,	369	190	131	93·0	127·5	—	—	—	—	—	—	381	300	530?
,,	♂?	370	186·5	134	93·2	134	112·2	—	—	128·5	—	—	377	297	511
,,	,,	371	187	141·5	96·1	126	123·3	—	—	141	—	—	380	324	535
,,	,,	372	182	132·5	97·1	127·5	—	—	—	—	—	—	373	297	510
,,	,,	373	181	134	96·1	—	—	—	—	—	—	—	—	320	513
,,	,,	374	192?	134·5	—	127·5	—	—	—	—	—	—	—	297	—

APPENDIX II. *Individual Measurements of Lachish Skulls.*

S_2	S_3	S	$\beta Q'$	U	*fml*	*fmb*	*LB*	*GL*	*G'H*	*J*	*GB*	*NB*	*NH,L*	*S*
127·5	118	369	301	520	38·0	32·6	101·6	—	—	—	—	—	—	
131·5	113	369	321	516	—	—	—	—	—	—	—	—	—	
138	124	396	312	534	38·4	—	100·0	—	—	—	—	—	—	
122·5	—	—	—	—	—	—	—	—	68·0?	—	—	—	51·1?	
123·5	121	370	312	517	40·0	33·7	102·0	—	—	—	92·1	—	52·9	
—	—	—	311	512	—	29·8	—	—	64·2	—	—	24·8	49·5	
135·5	116	392	—	—	37·3	31·1	101·0	97·9	68·9	128·5	—	25·0	53·8	
125	110·5	356	289	500	38·0	—	99·1	93·8	73·0	—	—	25·2	53·1	
—	114	—	309?	518?	39·0	30·1	105·0	97·2	73·2	—	—	27·8	54·0	
126	115·5	366	302	516	37·1	—	102·2	93·8	71·0	—	94·8	—	—	
132	113	365	—	—	37·0	—	99·9	—	—	—	95·2	25·5	51·8	
128·5	125	383?	308	529	—	—	—	—	720?	—	—	26·0	49·1?	
122	125	371	331	537	39·1	28·2	106·1	100·1	70·5	137?	108·3	29·8	53·3	
131·5?	110	378	312	515	36·9	31·7	100·6	—	—	—	—	—	—	
—	111	—	320	524?	38·2	29·1	101·0	95·4	74·2	—	—	23·1	53·9	
138	112·5	375	—	—	34·1	—	107·1	—	—	—	90·4	—	54·0	
131	109·5	369	314	529	—	—	—	—	—	—	96·1	—	—	
139	110·5	381	300	523	38·9	33·9	101·9?	94·0?	63·1?	—	—	—	—	
136·5	—	—	319	532	—	—	—	—	—	—	95·2	25·1?	55·0?	
129	—	—	306	520	—	—	101·3	—	—	—	—	23·8	53·1	
117	125	371	292	510	33·0	25·7	95·3	95·1	70·9	—	98·0	23·6	51·2	
126·5	113	371	305	513	33·0	29·8	100·0	85·0	70·2	130·5	99·6	24·7	51·8	
130	122	397	318	530?	37·0	—	101·0	—	—	—	95·8	24·3	53·0	
133	—	—	308	—	—	—	—	—	—	—	—	24·0	—	
128	115	—	—	—	—	31·8?	—	—	—	—	—	24·9	53·5?	
130	107	360	306	505	35·9	29·9	92·5	95·7	68·1	124·5	95·2	23·8	48·1	
124	117·5?	366?	297	504	—	—	—	—	—	—	—	—	48·0	
—	—	—	319	536	—	27·0	96·4	92·5	61·5	—	—	23·8	47·6	
126·5	114	367	301	519	38·0	29·0	103·0	94·9	63·0	—	—	25·3?	46·3	
—	—	—	313	536	—	—	96·1	—	—	—	102·5?	23·0	51·5?	
130·5	110	368?	302	510	—	—	—	—	—	—	—	—	—	
128·5	113	372	304	514	35·4	29·9	100·1	—	—	—	89·8	24·3	53·6	
128	112	357	297	499	39·2	30·7	100·9	—	—	—	—	—	—	
124·5	—	—	312	533	—	—	107·1	98·8	80·9	—	—	—	55·1	
123·5	118	375	302	512	37·2	27·1	96·1	—	—	—	—	24·0	54·1	
135·5	114·5	—	322	535	37·6	30·2	109·8	—	—	—	—	—	—	
130	120	380	296	503	38·6	33·1	100·2	85·2	77·3?	—	—	—	—	
—	—	388	307	522	39·4	33·0	101·9	89·0	73·1	—	—	27·7?	50·7	
132	97·5	360	308	496	33·0	—	95·1	93·5	66·4	—	—	27·5	48·9	
131·5	—	—	312	520	—	—	—	—	—	—	—	23·8	52·0	
133	100·5	362	310	518?	38·8	—	96·4	—	—	—	—	—	53·4	
139	117	386	303	517	38·0	28·3	105·6	—	—	—	97·0	24·0	54·0	
128·5	—	—	332	532	40·3	—	100·2	—	—	—	—	—	—	
120	124·5	373	310	—	39·3	33·0	102·2	93·0	70·3	—	90·0	23·0	52·9	
136	114	375	—	—	40·1	31·4	103·8	96·8	70·0	132	102·1	27·0	51·6	
137	113·5?	386?	322	514	—	—	—	—	74·0	—	—	—	52·0	
144·5	—	—	319	515	—	—	98·6	94·0	68·0	—	97·1	—	47·0	

C

—	—	373	284	527?	39·8	31·0	111·1	99·5	69·1	—	100·1	24·3	53·3	
—	—	384	291	507	32·9	27·7	90·5	—	—	—	—	—	—	
—	—	373	304	501	35·0	29·0	96·3	88·7?	67·9	131·5	96·0	23·5	49·1	
—	—	368	294	504	36·0	—	95·1	88·2	68·1	—	—	—	54·0	
—	—	401	312	529	35·9	27·0	95·1	—	—	—	—	—	—	
—	—	381	300	530?	37·5	33·0	101·1	—	—	—	—	—	—	
—	—	377	297	511	40·0	34·0	102·0	—	—	—	—	—	—	
—	—	380	324	535	38·0	27·9	101·1	—	—	—	—	—	—	
—	—	373	297	510	39·0	29·6	99·0	—	—	—	—	—	—	
—	—	—	320	513	—	—	—	—	—	—	—	—	—	
—	—	—	297	—	36·0	30·0	—	—	—	—	—	—	—	

B	*NB*	*NH,L*	*SC*	*SS*	O_2L	O_1L	G_1'	G_2	Ml_1	Ml_2	*C(ml)*	*S(ml)*	*OH*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	*Oc.I.*	
–	—	—	9·0	5·3	34·8	40·1	—	—	51·5	48·0	45·7	10·0	—	76·9	72·5	106·1	4·4	59·0	
–	—	—	12·1	5·2	33·0	41·9	—	—	—	54·0	—	—	—	79·1	—	—	—	59·2	
–	—	—	11·5?	5·6?	—	—	—	—	—	—	—	—	119·5	72·1	69·8	103·3	2·3	57·9	
·1	—	51·1?	—	—	30·0?	—	—	—	—	—	—	—	—	—	—	—	—	—	
–	—	52·9	9·1	5·0	36·6	41·7	47·4?	—	60·5	47·5	54·2	11·0	116·0	75·7	74·3	101·8	1·4	58·5	
–	24·8	49·5	11·0	5·4	—	41·1	—	—	—	46·0	—	—	—	68·5	—	—	—	—	
–	25·0	53·8	11·3	6·3	31·1	40·9	48·6?	—	—	—	—	—	—	—	70·9	—	—	61·0	
–	25·2	53·1	10·3	6·9	34·7	41·1	48·1	—	—	48·0	—	—	107·6	72·8	69·2	105·3	3·6	58·1	
-8	27·8	54·0	10·1	5·3	34·3?	42·3?	47·2	—	61·0	50·0	54·3	11·0	—	73·7	72·6	101·5	1·1	60·1	
·2	—	—	9·3	2·5	32·9	42·3	46·7	—	—	45·0	—	—	—	75·6	74·0	102·2	1·7	59·4	
	25·5	51·8	9·9	4·4	33·0	40·2	41·3	35·1?	—	50·5	—	—	—	—	74·4	—	—	66·3	
·3	26·0	49·1?	10·0	4·2	—	—	—	—	68·0	—	—	—	—	71·2	—	—	—	57·9	
	29·8	53·3	16·1	7·4	32·3	43·1	46·2	—	—	49·5	58·2	14·3	118·3	81·7	73·9	110·6	7·8	57·8	
	—	—	12·1	8·4	34·8	41·3	—	—	52·5	58·5	47·5	9·2	—	73·4	71·5	102·6	1·9	61·5	
4	23·1	53·9	9·0	6·4	36·1	—	—	43·8	—	—	—	—	—	78·1	76·7	101·8	1·4	64·2	
I	—	54·0	8·0	4·3	32·4	45·2	43·4	40·0	66·5	51·0	58·5	11·5	—	—	74·2	—	—	58·4	
	—	—	14·1	7·0	36·2	43·4	—	—	—	47·5	—	—	114·0	73·2	—	—	—	62·7	
2	25·1?	55·0?	12·0	6·7	32·2	41·1?	44·5	—	—	—	—	—	—	74·3	73·2	101·5	1·1	63·4	
	23·8	53·1	8·9	3·9	—	45·1?	—	43·6	—	50·0	—	—	—	75·1	—	—	—	—	
o	23·6	51·2	10·2	4·3	33·1	42·7	46·0	41·7	60·5	53·0	52·1	12·8	114·2	73·7	71·0	103·8	2·7	—	
6	24·7	51·8	9·2	4·2	34·9	42·4	39·9	35·8	63·5?	53·0	54·0	12·5	106·2	72·0	65·5	110·0	6·5	55·2	
8	24·3	53·0	13·1	5·8	30·3	40·1	—	—	—	44·5	—	—	113·0	76·3	76·9	99·3	−0·6	62·1	
	24·0	—	—	—	35·6	42·6	42·9?	—	55·0	49·5	48·9	10·5	120·1	73·8	73·2	100·7	0·5	59·7	
	24·9	53·5?	11·3	6·4	33·1	40·0	42·6?	—	58·5	52·5	51·1	11·6	—	72·7	71·9	101·1	0·8	—	
	23·8	48·1	9·6	3·5	—	41·6	50·7	40·6	—	46·5	—	—	111·9	—	—	—	—	60·4	
	—	48·0	9·5	4·0	34·0	40·0	—	—	—	—	—	—	—	78·6	72·3	108·7	6·3	56·9	
	23·8	47·6	10·0	4·8	—	—	43·5	—	—	—	—	—	—	73·3	—	—	—	59·4	
	25·3?	46·3	10·7	5·5	31·0	39·1	45·1	37·9	—	—	—	—	—	72·2	63·9	112·9	8·2	—	
?	23·0	51·5?	—	—	33·9	—	47·2	—	—	—	—	—	110·9	72·4	71·4	101·5	1·1	59·1	
	—	—	10·3	4·7	—	—	—	—	—	53·5	—	—	—	70·1	68·6	102·3	1·6	—	
	24·3	53·6	10·0	6·5	33·3	40·3	43·8	39·9	—	—	—	—	—	68·8	—	—	—	60·2	
	—	—	8·2	2·9	—	—	—	—	—	51·0	—	—	111·4	72·7	71·4	101·9	1·4	57·8	
	—	55·1	13·3	7·8	31·9	42·2	48·2	—	—	—	—	—	—	74·6	74·0	100·8	0·6	58·8	
	24·0	54·1	7·8	5·7	33·3	41·0	45·4?	40·0	59·0	57·0	—	—	116·2	72·6	71·0	102·2	1·6	—	
	—	—	—	—	—	—	—	41·4	—	56·5	52·1	10·7	111·2	71·0	69·7	101·9	1·3	58·2	
	27·7?	50·7	10·6	5·0	37·2	—	46·8	—	—	—	—	—	120·8	78·4	76·5	102·4	1·9	59·6	
	27·5	48·9	8·4	3·3	29·2	42·2	—	—	—	53·5	—	—	—	72·3	75·3	96·0	−3·0	60·1	
	23·8	52·0	9·3	6·6	32·8	43·6	44·0	—	—	51·5	—	—	—	71·8	74·5	96·4	−2·7	—	
	—	53·4	—	—	33·0	—	46·5	40·6?	—	—	—	—	115·5	74·0	70·6	104·9	3·4	58·7	
					38·1	43·3	—	—	—	—	—	—	113·5	76·1	—	—	—	—	
	24·0	54·0	9·1	5·2	33·8	41·0	48·1	—	59·5	53·0	—	—	—	76·1	66·6	114·3	9·5	62·6	
	—	—	9·5	6·3	—	—	—	—	—	52·5	52·9	11·9	113·3	67·7	75·1	90·2	−7·3	58·9	
	23·0	52·9	9·2	6·4	33·5	40·6	46·2	38·1	—	—	—	—	—	78·3	76·5	102·4	1·9	—	
	27·0	51·6	12·0	5·8	33·3	43·4	48·0	—	—	51·0	—	—	—	72·3	68·3	105·8	4·0	57·3	
	—	52·0	10·9	5·3	—	—	—	—	63·5	48·5	55·0	13·4	—	—	68·4	—	—	59·2	
	—	47·0	—	—	29·8	42·9	46·0	37·2	—	—	—	—	—	74·5	—	—	—	64·2	
							46·4	—	59·5	49·5	52·1	11·3	—	71·9	75·1	95·7	−3·2	—	

Crania and calvariae showing premature closing of the sagittal suture.

24·3	53·3	9·0	4·3	34·9	43·1	42·0?	—	—	46·5	—	—	—	64·4	67·5	95·4	−3·1	—		
—	—	6·2	3·6	—	—	—	—	—	—	—	—	—	72·0	67·3	106·9	4·7	—		
23·5	49·1	9·1	4·0	32·0	41·0	44·8	—	55·0	43·5	48·3	10·0	—	78·2	75·4	103·8	2·9	—		
—	54·0	8·0	—	30·8	40·1	42·1	40·9	55·5?	46·0	50·2	9·9	—	74·6	73·0	102·3	1·7	—		
—	—	—	—	—	—	—	—	—	—	—	—	—	72·1	68·9	104·6	3·2	—		
—	—	—	—	—	—	—	—	—	—	—	—	—	68·9	67·1	102·7	1·8	—		
—	—	—	—	—	—	—	—	—	—	—	—	—	71·8	71·8	100·0	0	—		
—	—	9·9	5·1	—	—	—	—	—	—	—	—	—	75·7	67·4	112·3	8·3	—		
—	—	—	—	—	—	—	—	—	—	—	—	—	72·8	70·1	103·9	2·7	—		
—	—	—	—	—	—	—	—	—	—	—	—	—	74·0	—	—	—	—		
													70·1	66·4	105·5	3·6	—		

e from Tombs 107, 116 *and* 108; *and anomalous Male and Queried Male Crania and Calvariae from Tomb* 12

100 $\frac{B}{H'}$	100 $\frac{B-H'}{L}$	*Oc.I.*	100 $\frac{fmb}{fml}$	100 $\frac{G'H}{GB}$	100 $\frac{NB}{NH'}$, *L*	100 $\frac{O_2}{O_1}$, *L*	100 $\frac{G_2}{G_1'}$	100 $\frac{SS}{SC}$	100 $\frac{Ml_2}{Ml_1}$	100 $\frac{S}{C}$(*ml*)	*P*∠	*N*∠
106·1	4·4	59·0	85·8	—	—	86·8	—	58·9	93·2	21·9	—	—
—	—	59·2	—	—	—	78·8	—	43·0	—	—	—	—
103·3	2·3	57·9	—	—	—	—	—	48·7	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—
101·8	1·4	58·5	84·3	—	—	87·8	—	54·9	78·5	28·2	—	—
—	—	—	—	—	50·1	—	—	49·1	—	—	—	—
—	—	61·0	83·4	—	46·5	76·0	—	55·8	—	—	84°·5	67°·0
105·3	3·6	58·1	—	—	47·5	84·4	—	67·0	—	—	86°·0?	63°·9
101·5	1·1	60·1	77·2	—	51·5	81·1	—	52·5	82·0	20·3	86°·0	63°·0
102·2	1·7	59·4	—	74·9	—	77·8	—	26·9	—	—	85°·5	62°·7
—	—	66·3	—	—	49·2	82·1	85·0	44·4	—	—	—	—
—	—	57·9	—	—	53·0	—	—	42·0	—	—	89°·0?	—
110·6	7·8	57·8	72·1	65·1	55·9	74·9	—	46·0	72·8	24·6	86°·5?	65°·2
102·6	1·9	61·5	85·9	—	—	84·3	—	69·4	111·4	19·4	—	—
101·8	1·4	64·2	76·2	—	42·9	—	—	71·1	—	—	80°·5	63°·9
—	—	58·4	—	—	—	71·7	92·2	53·7	76·7	19·7	—	—
—	—	62·7	—	—	—	83·4	—	49·6	—	—	—	—
101·5	1·1	63·4	87·1	—	—	—	—	—	—	—	—	64°·6
—	—	—	—	—	45·6	78·3	—	55·8	—	—	—	—
103·8	2·7	—	—	—	44·8	—	—	43·8	—	—	—	—
110·0	6·5	55·2	77·9	72·3	46·1	77·5	90·7	42·2	87·6	24·6	87°·0	67°·9
99·3	−0·6	62·1	90·3	70·5	47·7	82·3	89·7	45·7	83·5	23·1	91°·0?	56°·7
100·7	0·5	59·7	—	—	45·8	75·6	—	44·3	—	—	—	—
101·1	0·8	—	—	—	—	83·6	—	—	90·0	21·5	—	—
—	—	60·4	—	—	46·5	82·7	—	56·6	89·7	22·7	—	—
108·7	6·3	56·9	83·3	71·5	49·5	—	80·1	36·5	—	—	83°·0	71°·1
—	—	59·4	—	—	—	85·0	—	42·1	—	—	—	—
112·9	8·2	—	—	—	50·0	—	—	48·0	—	—	92°·0	67°·7
101·5	1·1	59·1	76·3	—	54·6	79·3	84·0	51·4	—	—	86°·5	64°·6
102·3	1·6	—	—	—	44·7	—	—	—	—	—	—	—
—	—	60·2	—	—	—	—	—	45·6	—	—	—	—
101·9	1·4	57·8	84·5	—	45·3	82·6	91·1	65·0	—	—	—	—
100·8	0·6	58·8	78·3	—	—	—	—	35·4	—	—	—	—
102·2	1·6	—	—	—	—	75·6	—	58·6	—	—	88°·5	61°·4
101·9	1·3	58·2	72·8	—	44·4	81·2	88·1	73·1	95·8	20·5	89°·0	—
102·4	1·9	59·6	80·3	—	—	—	—	—	—	—	—	—
96·0	−3·0	60·1	85·8	—	—	—	—	—	—	—	—	55°·6
96·4	−2·7	—	83·8	—	54·6	69·2	—	47·2	—	—	88°·5?	58°·2
104·9	3·4	58·7	—	—	56·2	75·2	—	39·3	—	—	89°·0	68°·0
—	—	—	—	—	45·8	—	87·3	71·0	—	—	—	—
114·3	9·5	62·6	—	—	—	88·0	—	—	—	—	—	—
90·2	−7·3	58·9	74·5	—	44·4	82·4	—	57·1	88·2	22·5	—	—
102·4	1·9	—	—	—	—	—	—	66·3	—	—	—	—
105·8	4·0	57·3	84·0	78·1	43·5	82·5	82·5	69·6	—	—	89°·0	61°·9
—	—	59·2	78·3	68·6	52·3	76·7	—	48·3	76·4	24·4	88°·5	64°·3
—	—	64·2	—	—	—	—	80·9	48·6	—	—	—	—
95·7	−3·2	—	—	70·0	—	69·5	—	—	83·2	21·7	84°·0	65°·8
95·4	−3·1	—	77·9	69·0	45·6	81·0	—	47·8	—	—	87°·0	61°·8
106·9	4·7	—	84·2	—	—	—	—	58·1	—	—	—	—
103·8	2·9	—	82·9	70·7	47·9	78·0	—	44·0	79·1	20·7	87°·5?	62°·4
102·3	1·7	—	—	—	—	76·8	97·1	—	82·9	19·7	86°·5	62°·8
104·6	3·2	—	75·2	—	—	—	—	—	—	—	—	—
102·7	1·8	—	88·0	—	—	—	—	—	—	—	—	—
100·0	0	—	85·0	—	—	—	—	—	—	—	—	—
112·3	8·3	—	73·4	—	—	—	—	—	—	—	—	—
103·9	2·7	—	75·9	—	—	—	—	51·5	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—
105·5	3·6	—	83·3	—	—	—	—	—	—	—	—	—

from Tomb 120

P∠	*N*∠	*A*∠	*B*∠	No.	Remarks
—	—	—	—	144	Two epipteric bones L and 3 R; palate missing
—	—	—	—	145	Several small wormian bones between temporal squama and parietal R
—	—	—	—	146	Palate defective
—	—	—	—	147	L temporal and parts of L parietal and occipital bones missing; coronal and lambdoid closed, sagittal obliterated; 3 or 4 molars lost before death
—	—	—	—	148	Coronal beginning to close, sagittal closed; several teeth lost before death
—	—	—	—	149	Coronal beginning to close; palate defective
84°·5	67°·0	72°·5	40°·4	150	Sagittal beginning to close; ossicle of bregma behind coronal suture; 5 molars lost before death; pre-condyle R
86°·0?	63°·9	71°·9	44°·3	151	Sagittal and lambdoid beginning to close
86°·0	63°·0	74°·8	42°·1	152	One premolar and 4 or 5 molars lost before death
85°·5	62°·7	75°·4	42°·0	153	Sagittal beginning to close; 1 or 2 molars lost before death
—	—	—	—	154	One molar lost before death; horizontal suture across malar bone R; malar bone L normal
89°·0?	—	—	—	155	Sagittal closing, lambdoid beginning to close; 5 molars lost before death
86°·5?	65°·2	74°·9	39°·8	156	Metopic; traces of horizontal suture of interparietal bones R and L; single epipteric bone L; 2 wormian bones between temporal squama and parietal L
—	—	—	—	157	Coronal closed, sagittal nearly obliterated, lambdoid beginning to close; palate missing
80°·5	63°·9	71°·9	44°·2	158	One incisor lost before death
—	—	—	—	159	Traces of horizontal suture of inter-parietal bones R and L
—	—	—	—	160	Coronal closing, sagittal closed, lambdoid beginning to close; palate defective
—	64°·6	78°·1	37°·4	161	Coronal beginning to close, sagittal closing; palate defective
—	—	—	—	162	Coronal and sagittal beginning to close; palate defective
—	—	—	—	163	Large epipteric bone L; 1 molar lost before death
87°·0	67°·9	68°·4	43°·7	164	Trace of horizontal suture of interparietal bones R
91°·0?	56°·7	79°·9	43°·4	165	Small healed injury at glabella
—	—	—	—	166	Palate defective
—	—	—	—	167	Sagittal beginning to close
—	—	—	—	168	Palate defective; small pre-condyle L
83°·0	71°·1	66°·6	42°·3	169	Traces of horizontal sutures of malar bones R and L; M 3, R absent
—	—	—	—	170	Coronal closing; palate defective
92°·0	67°·7	74°·5	37°·8	171	Coronal beginning to close, sagittal obliterated, lambdoid closed; occipito-mastoid L obliterated; posterior half of temporal squama L fused to parietal; 2 molars lost before death
86°·5	64°·6	78°·7	36°·8	172	—
—	—	—	—	173	All sutures closing; palate defective
—	—	—	—	174	Epipteric bone R; palate missing
—	—	—	—	175	—
—	—	—	—	176	Palate missing; small exostosis on supra-occipital
88°·5	61°·4	72°·7	46°·0	177	Coronal and sagittal closing; palate defective
89°·0	—	—	—	178	—
—	—	—	—	179	Coronal beginning to close, sagittal closing; palate defective
—	55°·6	76°·1	48°·4	180	Coronal closed, sagittal obliterated; palate defective
88°·5?	58°·2	77°·4	44°·3	181	Aged skull; coronal and lambdoid nearly obliterated, sagittal obliterated; palate defective
89°·0	68°·0	70°·9	41°·1	182	Three molars lost before death; trace of horizontal suture of interparietal bones R and L
—	—	—	—	183	Coronal beginning to close, sagittal closing
—	—	—	—	184	Lambdoid closing; occipito-mastoid obliterated R and L, parieto-mastoid L obliterated; palate defective
—	—	—	—	185	Small epipteric bone R
—	—	—	—	186	Coronal beginning to close; palate defective
89°·0	61°·9	76°·4	41°·8	187	—
88°·5	64°·3	75°·0	40°·7	188	Sagittal beginning to close; trace of horizontal suture on malar bone R; 1 molar lost before death, M 3, R apparently absent
—	—	—	—	189	Sagittal beginning to close
84°·0	65°·8	73°·1	41°·1	190	Coronal beginning to close, sagittal closing; palate defective; 2 wounds on frontal, probably sword-cuts
87°·0	61°·8	80°·5	37°·7	364	Sagittal completely obliterated, coronal and lambdoid open; large epipteric bone R; palate defective
—	—	—	—	365	Sagittal completely obliterated, coronal open, lambdoid nearly obliterated; large epipteric bone R; palate missing
87°·5?	62°·4	74°·9	42°·8	366	Sagittal completely obliterated, coronal and lambdoid beginning to close; articular facet lateral to occipital condyle L; wound external angular process of frontal L; all molars and 1 premolar lost before death
86°·5	62°·8	73°·8	43°·4	367	Sagittal completely obliterated, coronal and lambdoid open; 1 molar lost before death
—	—	—	—	368	Sagittal completely obliterated, coronal open, lambdoid closing
—	—	—	—	369	Sagittal completely obliterated, coronal and lambdoid open
—	—	—	—	370	Sagittal completely obliterated, coronal and lambdoid closing, single epipteric bone L; palate defective
—	—	—	—	371	Sagittal completely obliterated, coronal and lambdoid open; occipito-mastoid obliterated R and L
—	—	—	—	372	Metopic; sagittal completely obliterated, metopic and coronal beginning to close, lambdoid closed
—	—	—	—	373	Sagittal completely obliterated, coronal and lambdoid open
—	—	—	—	374	Sagittal completely obliterated, coronal and lambdoid open

„	„	168	—	—	99·0	—	114·1?	115·7	96·2	—	128	115	—	—	—	—	31·8	
„	„	169	175	137·5	93·0	126·5	107·1	117·0	85·4	123	130	107	360	306	505	35·9	29·9	
„	„	170	183·5?	134·5?	91·8	—	111·0	111·7	97·3?	125	124	117·5?	366?	297	504	—	—	
„	„	171	194	140	93·7	124	122·1	—	—	142	—	—	—	319	536	—	27·0	
„	„	172	185	134	95·5	132	110·2	114·2	94·0	127	126·5	114	367	301	519	38·0	29·0	
116	♂	173	192·5	135	95·8	132	116·3	—	—	136	—	—	—	313	536	—	—	
„	„	174	182·5	125·5	99·1	—	111·3	117·1	91·9	128·5	130·5	110	368?	302	510	—	—	
„	„	175	185	134·5	93·9	132	112·9	115·7	91·5	130·5	128·5	113	372	304	514	35·4	29·9	
„	„	176	179	133·5	89·0	132·5	106·0	114·0	92·0	117·5	128	112	357	297	499	39·2	30·7	
„	„	177	191·5	139	95·1	136	113·7	112·0	—	127·5	124·5	—	—	312	533	—	—	
„	„	178	186·5	132·5	89·6	130	116·3	110·4	96·1	133	123·5	118	375	302	512	37·2	27·1	
„	„	179	187·5	147	92·9?	143·5	116·1	120·2	95·0	—	135·5	114·5	—	322	535	37·6	30·2	
„	„	180	182	131·5	86·9	137	113·9	113·3	100·1	130·5	130	120	380	296	503	38·6	33·1	
„	„	181	188	135	91·4	140?	—	—	—	—	—	—	388	307	522	39·4	33·0	
„	♂?	182	175	129·5	95·2	123·5	113·0	115·1	80·0	129·5	132	97·5	360	308	496	33·0	—	
„	„	183	184	140	100·1	—	118·6	113·4	—	136	131·5	—	—	312	520	—	—	
„	„	184	184	140	97·1	122·5	111·0	117·9	85·8	128·5	133	100·5	362	310	518?	38·8	—	
„	„	185	190·5	129	94·1	143	116·0	123·3	96·2	130·5	139	117	386	303	517	38·0	28·3	
108	♂	186	187	146·5	96·1	143	125·9	115·1	94·1	145·5	128·5	—	—	332	532	40·3	—	
„	„	187	189·5	137	94·2	129·5	113·0?	107·9?	100·0	129?	120	124·5	373	310	—	39·3	33·0	
„	„	188	190	—	99·0	130	111·2	123·0	94·1	125·5	136	114	375	—	—	40·1	31·4	
„	„	189	184	137	90·3	—	117·2	122·4	98·2?	134·5	137	113·5?	386?	322	514	—	—	
„	„	190	187	134·5	88·0	140·5	119·9	123·2	—	137·5	144·5	—	—	319	515	—	—	

120	♂	364	194	125	96·3	131	—	—	—	—	—	—	373	284	527?	39·8	31·0	
„	„	365	182	131	93·9	122·5	—	—	—	—	—	—	384	291	507	32·9	27·7	
„	„	366	174·5	136·5	93·	131·5?	—	—	—	—	—	—	373	304	501	35·0	29·0	
„	„	367	177·5	132·5	96·1	129·5	111·1	—	—	130	—	—	368	294	504	36·0	—	
„	„	368	188	135·5	96·9	129·5?	—	—	—	—	—	—	401	312	529	35·9	27·0	
„	„	369	190	131	93·0	127·5	—	—	—	—	—	—	381	300	530?	37·5	33·0	
„	♂?	370	186·5	134	93·2	134	112·2	—	—	128·5	—	—	377	297	511	40·0	34·0	
„	„	371	187	141·5	96·1	126	123·3	—	—	141	—	—	380	324	535	38·0	27·9	
„	„	372	182	132·5	97·1	127·5	—	—	—	—	—	—	373	297	510	39·0	29·6	
„	„	373	181	134	96·1	—	—	—	—	—	—	—	—	320	513	—	—	
„	„	374	192?	134·5	—	127·5	—	—	—	—	—	—	—	297	—	36·0	30·0	

120	♂	375	176	132·5	94·3	131·5	111·9	111·9	100·0	123	128·5	112	364	307	493	36·6	28·5	
„	„	376	181·5	140	95·5	129	115·0	113·1	96·9	129·5	133	113	375	304	512	41·0	—	
„	„	377	181·5	136	96·0	139·5	118·0	118·0	105·0	127	137	123·5	387	311	510	34·2	29·0	
„	„	378	182	—	85·4	—	111·6	120·0	—	122	138	—	—	—	—	—	—	
„	„	379	179	138	103·9	137·5	115·3	111·1	97·2?	132	125	114?	371?	318	507	39·9?	32·5	
„	♂?	380	167·5	131·5	88·9	139·5	—	—	—	—	—	—	—	—	488	—	—	
„	„	381	175·5	131	96·4	142·5	120·3	96·9	109·0?	130	113	119	362	311	482	39·0	—	

120	♂	382	182·5	155·5	101·9	125	113·1	119·2	96·6	—	135	119	—	332?	537?	32·9	—

—	—	—	31·8?	—	—	—	—	—	24·9	53·5?	11·3	6·4	33·1	40·0	42·6?	—	58·5	52·
306	505	35·9	29·9	92·5	95·7	68·1	124·5	95·2	23·8	48·1	9·6	3·5	—	41·6	50·7	40·6	—	46·
297	504	—	—	—	—	—	—	—	—	48·0	9·5	4·0	34·0	40·0	—	—	—	—
319	536	—	27·0	96·4	92·5	61·5	—	—	23·8	47·6	10·0	4·8	—	—	43·5	—	—	—
301	519	38·0	29·0	103·0	94·9	63·0	—	—	25·3?	46·3	10·7	5·5	31·0	39·1	45·1	37·9	—	—
313	536	—	—	96·1	—	—	—	102·5?	23·0	51·5?	—	—	33·9	—	47·2	—	—	53
302	510	—	—	—	—	—	—	—	—	—	10·3	4·7	—	—	—	—	—	—
304	514	35·4	29·9	100·1	—	—	—	89·8	24·3	53·6	10·0	6·5	33·3	40·3	43·8	39·9	—	51
297	499	39·2	30·7	100·9	—	—	—	—	—	—	8·2	2·9	—	—	—	—	—	—
312	533	—	—	107·1	98·8	80·9	—	—	—	55·1	13·3	7·8	31·9	42·2	48·2	—	—	57
302	512	37·2	27·1	96·1	—	—	—	—	24·0	54·1	7·8	5·7	33·3	41·0	45·4?	40·0	59·0	56
322	535	37·6	30·2	109·8	—	—	—	—	—	—	—	—	—	—	—	41·4	—	—
296	503	38·6	33·1	100·2	85·2	77·3?	—	—	—	—	—	—	37·2	—	46·8	—	—	53
307	522	39·4	33·0	101·9	89·0	73·1	—	—	27·7?	50·7	10·6	5·0	29·2	42·2	—	—	—	51
308	496	33·0	—	95·1	93·5	66·4	—	—	27·5	48·9	8·4	3·3	32·8	43·6	44·0	—	—	—
312	520	—	—	—	—	—	—	—	23·8	52·0	9·3	6·6	33·0	—	46·5	40·6?	—	—
310	518?	38·8	—	96·4	—	—	—	—	—	53·4	—	—	38·1	43·3	—	—	—	53
303	517	38·0	28·3	105·6	—	—	—	97·0	24·0	54·0	9·1	5·2	33·8	41·0	48·1	—	59·5	52
332	532	40·3	—	100·2	—	—	—	—	—	—	9·5	6·3	—	—	—	—	—	—
310	—	39·3	33·0	102·2	93·0	70·3	—	90·0	23·0	52·9	9·2	6·4	33·5	40·6	46·2	38·1	—	5
—	—	40·1	31·4	103·8	96·8	70·0	132	102·1	27·0	51·6	12·0	5·8	33·3	43·4	48·0	—	63·5	4
322	514	—	—	—	—	74·0	—	—	—	52·0	10·9	5·3	—	—	46·0	37·2	—	—
319	515	—	—	98·6	94·0	68·0	—	97·1	—	47·0	—	—	29·8	42·9	46·4	—	59·5	4

Crania and calvariae showing premature closing of

284	527?	39·8	31·0	111·1	99·5	69·1	—	100·1	24·3	53·3	9·0	4·3	34·9	43·1	42·0?	—	—	4
291	507	32·9	27·7	90·5	—	—	—	—	—	—	6·2	3·6	—	—	—	—	—	
304	501	35·0	29·0	96·3	88·7?	67·9	131·5	96·0	23·5	49·1	9·1	4·0	32·0	41·0	44·8	—	55·0	4
294	504	36·0	—	95·1	88·2	68·1	—	—	—	54·0	8·0	—	30·8	40·1	42·1	40·9	55·5?	4
312	529	35·9	27·0	95·1	—	—	—	—	—	—	—	—	—	—	—	—	—	
300	530?	37·5	33·0	101·1	—	—	—	—	—	—	—	—	—	—	—	—	—	
297	511	40·0	34·0	102·0	—	—	—	—	—	—	—	—	—	—	—	—	—	
324	535	38·0	27·9	101·1	—	—	—	—	—	—	—	—	—	—	—	—	—	
297	510	39·0	29·6	99·0	—	—	—	—	—	—	9·9	5·1	—	—	—	—	—	
320	513	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
297	—	36·0	30·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	

Crania and calvariae known or suspected to be ar

307	493	36·6	28·5	95·3	—	—	—	—	—	—	—	—	—	—	—	—	—	
304	512	41·0	—	92·0	87·4	69·9	—	92·1	27·0	—	6·9?	—	—	45·7	—	—	—	
311	510	34·2	29·0	98·0	—	—	—	100·0	25·0	50·1	14·6	5·3	33·5	41·0	—	—	—	
—	—	—	—	—	—	—	—	—	—	—	10·0	5·9	—	—	—	—	—	
318	507	39·9?	32·5	100·0	95·4	61·1	130·5	100·1	25·0	46·8	5·1	—	33·0	41·2	45·6	—	57·5	
—	488	—	—	88·9	89·9	70·8	—	89·3?	21·1	49·4	7·4	3·4	35·7	39·1	42·0	33·8	—	
311	482	39·0	—	105·1	—	—	—	103·3	26·1	53·5	11·1	3·8	38·8?	39·5	45·2	—	—	

Anomalous cranium possibly affected by h

	332?	537?	32·9	—	88·0	—	—	—	—	—	51·0	—	—	34·2	39·8	—	—	62·0

—	58·5	52·5	51·1	11·6	—	—	—	—	—	60·4	—	—	40·5	52·7		
40·6	—	46·5	—	—	111·9	78·6	72·3	108·7	6·3	56·9	83·3	71·5	49·5	—		
—	—	—	—	—	—	73·3	—	—	—	59·4	—	—	—	85·0		
—	—	—	—	—	—	72·2	63·9	112·9	8·2	—	—	—	50·0	—		
37·9	—	—	—	—	110·9	72·4	71·4	101·5	1·1	59·1	76·3	—	54·6	79·3		
—	—	53·5	—	—	—	70·1	68·6	102·3	1·6	—	—	—	44·7	—		
—	—	—	—	—	—	68·8	-	—	—	60·2	—	—	—	—		
39·9	—	51·0	—	—	111·4	72·7	71·4	101·9	1·4	57·8	84·5	—	45·3	82·6		
—	—	—	—	—	—	74·6	74·0	100·8	0·6	58·8	78·3	—	—	—		
—	—	57·0	—	—	116·2	72·6	71·0	102·2	1·6	—	—	—	—	75·6		
40·0	59·0	56·5	52·1	10·7	111·2	71·0	69·7	101·9	1·3	58·2	72·8	—	44·4	81·2		
41·4	—	—	—	—	120·8	78·4	76·5	102·4	1·9	59·6	80·3	—	—	—		
—	—	53·5	—	—	—	72·3	75·3	96·0	−3·0	60·1	85·8	—	—	—		
—	—	51·5	—	—	—	71·8	74·5	96·4	−2·7	—	83·8	—	54·6	69·2		
—	—	—	—	—	115·5	74·0	70·6	104·9	3·4	58·7	—	—	56·2	75·2		
40·6?	—	—	—	—	113·5	76·1	—	—	—	—	—	—	45·8	—		
—	—	53·0	—	—	—	76·1	66·6	114·3	9·5	62·6	—	—	—	88·0		
—	59·5	52·5	52·9	11·9	113·3	67·7	75·1	90·2	−7·3	58·9	74·5	—	44·4	82·4		
—	—	—	—	—	—	78·3	76·5	102·4	1·9	—	—	—	—	—		
38·1	—	51·0	—	—	—	72·3	68·3	105·8	4·0	57·3	84·0	78·1	43·5	82·5		
—	63·5	48·5	55·0	13·4	—	—	68·4	—	—	59·2	78·3	68·6	52·3	76·7		
37·2	—	—	—	—	—	74·5	—	—	—	64·2	—	—	—	—		
—	59·5	49·5	52·1	11·3	—	71·9	75·1	95·7	−3·2	—	—	70·0	—	69·5		

nature closing of the sagittal suture.

? —	—	46·5	—	—	—	64·4	67·5	95·4	−3·1	—	77·9	69·0	45·6	81·0	
—	—	—	—	—	—	72·0	67·3	106·9	4·7	—	84·2	—	—	—	
—	55·0	43·5	48·3	10·0	—	78·2	75·4	103·8	2·9	—	82·9	70·7	47·9	78·0	
40·9	55·5?	46·0	50·2	9·9	—	74·6	73·0	102·3	1·7	—	—	—	—	76·8	
—	—	—	—	—	—	72·1	68·9	104·6	3·2	—	75·2	—	—	—	
—	—	—	—	—	—	68·9	67·1	102·7	1·8	—	88·0	—	—	—	
—	—	—	—	—	—	71·8	71·8	100·0	0	—	85·0	—	—	—	
—	—	—	—	—	—	75·7	67·4	112·3	8·3	—	73·4	—	—	—	
—	—	—	—	—	—	72·8	70·1	103·9	2·7	—	75·9	—	—	—	
—	—	—	—	—	—	74·0	—	—	—	—	—	—	—	—	
—	—	—	—	—	—	70·1	66·4	105·5	3·6	—	83·3	—	—	—	

uspected to be artificially deformed.

—	—	—	—	—	—	75·3	74·7	100·8	0·6	69·8	77·9	—	—	—	
—	—	50·5	—	—	—	77·1	71·1	108·5	6·1	63·1	—	75·9	—	—	
—	—	49·5	—	—	—	74·9	76·9	97·5	−1·9	62·1	84·8	—	49·9	81·7	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
5 —	57·5	43·5	52·2	10·8	—	77·1	76·8	100·4	0·3	62·3	81·5	61·0	53·4	80·1	
0 33·8	—	54·0?	—	—	—	78·5	83·3	94·3	−4·8	—	—	79·3	42·7	91·3	
2 —	—	51·5	—	—	—	74·6	81·2	91·9	−6·6	76·8	—	—	48·8	98·2	

bly affected by hydrocephaly.

-	—	62·0	52·5	54·1	10·0	—	85·2	68·5	124·4	16·7	57·9	—	—	—	85·9	

—	83·0	—	—	90·0	21·5	—	—	—	—	167	Sagittal beginning to close
46·5	82·7	—	56·6	89·7	22·7	—	—	—	—	168	Palate defective; small pre
49·5	—	80·1	36·5	—	—	83°·0	71°·1	66°·6	42°·3	169	Traces of horizontal suture
—	85·0	—	42·1	—	—	—	—	—	—	170	Coronal closing; palate def
50·0	—	—	48·0	—	—	92°·0	67°·7	74°·5	37°·8	171	Coronal beginning to close, literated; posterior half of
54·6	79·3	84·0	51·4	—	—	86°·5	64°·6	78°·7	36°·8	172	
44·7	—	—	—	—	—	—	—	—	—	173	All sutures closing; palate
—	—	—	45·6	—	—	—	—	—	—	174	Epipteric bone R; palate n
45·3	82·6	91·1	65·0	—	—	—	—	—	—	175	
—	—	—	35·4	—	—	—	—	—	—	176	Palate missing; small exos
—	75·6	—	58·6	—	—	88°·5	61°·4	72°·7	46°·0	177	Coronal and sagittal closin
44·4	81·2	88·1	73·1	95·8	20·5	89°·0	—	—	—	178	
—	—	—	—	—	—	—	—	—	—	179	Coronal beginning to close,
—	—	—	—	—	—	—	55°·6	76°·1	48°·4	180	Coronal closed, sagittal ob
54·6	69·2	—	47·2	—	—	88°·5?	58°·2	77°·4	44°·3	181	Aged skull; coronal and lar
56·2	75·2	—	39·3	—	—	89°·0	68°·0	70°·9	41°·1	182	Three molars lost before de
45·8	—	87·3	71·0	—	—	—	—	—	—	183	Coronal beginning to close,
—	88·0	—	—	—	—	—	—	—	—	184	Lambdoid closing; occipit palate defective
44·4	82·4	—	57·1	88·2	22·5	—	—	—	—	185	Small epipteric bone R
—	—	—	66·3	—	—	—	—	—	—	186	Coronal beginning to close
43·5	82·5	82·5	69·6	—	—	89°·0	61°·9	76°·4	41°·8	187	
52·3	76·7	—	48·3	76·4	24·4	88°·5	64°·3	75°·0	40°·7	188	Sagittal beginning to clos before death, M 3, R app
—	—	80·9	48·6	—	—	—	—	—	—	189	Sagittal beginning to close
—	69·5	—	—	83·2	21·7	84°·0	65°·8	73°·1	41°·1	190	Coronal beginning to close bably sword-cuts

45·6	81·0	—	47·8	—	—	87°·0	61°·8	80°·5	37°·7	364	Sagittal completely obli palate defective
—	—	—	58·1	—	—	—	—	—	—	365	Sagittal completely obliter bone R; palate missing
47·9	78·0	—	44·0	79·1	20·7	87°·5?	62°·4	74°·9	42°·8	366	Sagittal completely oblite lateral to occipital condy and 1 premolar lost befo
—	76·8	97·1	—	82·9	19·7	86°·5	62°·8	73°·8	43°·4	367	Sagittal completely oblite
—	—	—	—	—	—	—	—	—	—	368	Sagittal completely oblite
—	—	—	—	—	—	—	—	—	—	369	Sagittal completely oblite
—	—	—	—	—	—	—	—	—	—	370	Sagittal completely oblite palate defective
—	—	—	—	—	—	—	—	—	—	371	Sagittal completely oblite R and L
—	—	—	51·5	—	—	—	—	—	—	372	Metopic; sagittal complete closed
—	—	—	—	—	—	—	—	—	—	373	Sagittal completely oblite
—	—	—	—	—	—	—	—	—	—	374	Sagittal completely oblite

—	—	—	—	—	—	—	—	—	—	375	Face missing; all sutures b suggestion of frontal an coronal depression
—	—	—	—	—	—	—	63°·6	70°·8	45°·7	376	Sagittal beginning to clos frontal and occipital fla pression; no teeth lost b
49·9	81·7	—	36·3	—	—	—	—	—	—	377	All sutures open; single e deformation; no suggest molars lost before death
—	—	—	59·0	—	—	—	—	—	—	378	Face missing; coronal ope articulation R and L; su flattening due to artifici
53·4	80·1	—	—	75·7	20·7	—	67°·8	76°·2	36°·0	379	Metopic; all sutures open diastema, no teeth lost artificial interference; no
42·7	91·3	80·5	45·9	—	—	82°·0	66°·8	67°·0	46°·1	380	Coronal nearly obliterated mian bones at pterion R premolars on either side lost before death but po frontal bone (but occipit suture, but possibly (in depression
48·8	98·2	—	34·2	—	—	—	—	—	—	381	Metopic; sagittal beginni L and 2 R; large ossicle molar lost before death; flattened and post-coron

—	85·9	—	—	84·7	18·5	—	—	—	—	382	All sutures open; pteria r well-filled calvaria, poss

eginning to close
fective; small pre-condyle L
horizontal sutures of malar bones R and L; M 3, R absent
losing; palate defective
eginning to close, sagittal obliterated, lambdoid closed; occipito-mastoid L ob-
; posterior half of temporal squama L fused to parietal; 2 molars lost before death

—

s closing; palate defective
bone R; palate missing

—

issing; small exostosis on supra-occipital
nd sagittal closing; palate defective

—

eginning to close, sagittal closing; palate defective
losed, sagittal obliterated; palate defective
ll; coronal and lambdoid nearly obliterated, sagittal obliterated; palate defective
lars lost before death; trace of horizontal suture of interparietal bones R and L
eginning to close, sagittal closing
d closing; occipito-mastoid obliterated R and L, parieto-mastoid L obliterated;
efective
ipteric bone R
eginning to close; palate defective

—

beginning to close; trace of horizontal suture on malar bone R; 1 molar lost
leath, M 3, R apparently absent
beginning to close
beginning to close, sagittal closing; palate defective; 2 wounds on frontal, pro-
word-cuts

---

completely obliterated, coronal and lambdoid open; large epipteric bone R;
defective
completely obliterated, coronal open, lambdoid nearly obliterated; large epipteric
; palate missing
completely obliterated, coronal and lambdoid beginning to close; articular facet
o occipital condyle L; wound external angular process of frontal L; all molars
remolar lost before death
completely obliterated, coronal and lambdoid open; 1 molar lost before death
completely obliterated, coronal open, lambdoid closing
completely obliterated, coronal and lambdoid open
completely obliterated, coronal and lambdoid closing, single epipteric bone L;
defective
completely obliterated, coronal and lambdoid open; occipito-mastoid obliterated
L
sagittal completely obliterated, metopic and coronal beginning to close, lambdoid

completely obliterated, coronal and lambdoid open
completely obliterated, coronal and lambdoid open

---

sing; all sutures beginning to close; pteria normal; 2 small pre-condyles R and L;
ion of frontal and occipital flattening due to artificial deformation; no post-
depression
beginning to close; pteria normal; 2 small pre-condyles R and L; suggestion of
and occipital flattening due to artificial deformation; slight post-coronal de-
n; no teeth lost before death
es open; single epipteric bones R and L; clear frontal flattening due to artificial
ation; no suggestion of occipital flattening; no post-coronal depression; 3 or 4
lost before death
sing; coronal open, sagittal closed, lambdoid beginning to close; fronto-temporal
tion R and L; suggestion of frontal flattening, and clear suggestion of occipital
ng due to artificial interference; no post-coronal depression
all sutures open; single epipteric bone L and 4 R; 2 incisors only, separated by
a, no teeth lost before death; slight suggestion of frontal flattening due to
l interference; no suggestion of occipital flattening; clear post-coronal depression
nearly obliterated, sagittal and lambdoid open; single epipteric bone L; no wor-
ones at pterion R, but several between temporal squama and parietal R; single
ars on either side, and diastemae between them and 1st molars, M 3's probably
ore death but possibly never erupted; oxycephalic form with clear flattening of
bone (but occipital bone normal) probably due to premature closing of coronal
but possibly (in part at least) due to artificial interference; no post-coronal
ion
sagittal beginning to close, coronal and lambdoid open; single epipteric bone
2 R; large ossicle between temporal squama and parietal R; 2 pre-condyles; 1
ost before death; marked artificial deformation, frontal and occipital bones clearly
d and post-coronal depression

---

res open; pteria normal; only L side of palate preserved, M 3, L absent; markedly
led calvaria, possibly due to hydrocephaly; post-coronal depression

No.	*L*	*B*	*B′*	*H′*	S_1'	S_2'	S_3'	S_1	S_2	S_3	*S*	*βQ′*	*U*	*fml*	*fmb*	*LB*	*SC*	*SS*	10
207	189	139·5	93·2	139·5	115·0?	117·3?	99·4?	128·5	130·5	120·5	380	320	528	37·2?	30·1	108·2	—	—	7
208	186·5	139·5	98·4?	128·5	113·4?	115·0	91·9	130	127	115·5	373	311	518?	34·8	26·1	102·2?	—	—	7
209	181	131·5	89·1?	135·5	110·2	110·6	—	125·5	138	—	—	311	503	—	—	99·0	9·5	2·9	7
210	195·5	135·5	97·2	140	121·5?	120·6	—	135·5	135·5	—	—	312	533	—	30·0	111·2?	—	—	6
211	188	136	86·0	—	—	116·1	88·8?	—	129	115?	—	301	516	—	—	—	—	—	7
212	188	147	95·9	130·5	118·1	114·0	97·8	136·5	126·5	124	387	315	536	37·1	31·0	97·0	—	—	7
213	—	135	91·0	128·5	—	117·0	102·2	—	130·5	122	—	294	519	40·8	33·9	—	—	—	
214	190	132·5	99·1	141·5	115·6?	126·0	100·0	132?	140·5	118	391	318	525	40·4	30·0	101·9?	—	—	6
215	186·5	142·5	99·1	138·5	116·2	125·1	95·1	134	142·5	110	387	323	535	41·1	30·4	94·9	—	—	7
216	186	135·5	92·2	131	—	113·1	—	—	127	—	—	297?	517?	—	30·8	—	—	—	
217	199	137·5	102·6	141	123·9	127·0	110·6	143	146	135	424	323	547	40·9	30·3	96·6	10·6	5·7	6
218	186	140·5	98·0	132	116·8?	117·1	91·7	133?	132	113	378	311	528	36·0	27·6	102·3?	—	—	
219	193	134·5	101·0	—	—	123·6	94·1	—	138·5	111	—	—	532?	—	—	—	—	—	
220	194·5	137·5	92·2	134	113·1	—	—	128	—	—	391	307	531	37·9	33·0	100·5	8·0	3·7	
221	—	129	—	—	—	120·7	100·2?	—	136	126	—	—	—	—	—	—	—	—	
222	183	139·5	102·8	127·5	103·1?	107·2	95·8	115?	119	121	355	302	519	35·0	30·8	102·3	—	—	
223	190	137	98·6	—	116·5	126·9	88·0?	131·5	142·5	106?	379	—	527?	—	—	—	11·3	5·3	
224	185·5	136·5	93·8	134·5	117·0	110·4	101·0	135·5	123	121	379	310	—	40·5	34·0	99·1	—	—	
225	184·5	135	98·0	—	113·8?	117·6	—	132·5	134	—	—	—	518	—	—	—	—	—	
226	184·5	138·5	100·2	—	119·9?	119·5	93·3	129?	133	119	380	316?	528?	—	30·2	—	—	—	
227	185	140·5	96·9	130	108·4?	111·2	94·9	120·5?	124	121	366	308	531	37·3	32·9	103·0	—	—	
228	185	135·5	91·4	140	112·9?	120·7	106·1	129?	134	127	390	311	512	35·7	29·9	98·1?	—	—	
229	192·5	—	—	139·5	—	118·2	106·0	—	128·5	130	—	317	—	38·4	30·2	—	—	—	
230	193	143·5	99·5	138	117·2	123·3	100·7	136	136	127·5	400	318?	540?	36·2	30·0	103·4	10·8	5·1	
231	190	136	101·0	138	112·8	120·6	101·1	131	131·5	126·5	389	325	534	36·5	32·8	104·9	11·9	5·1	
232	188·5	135·5	92·9	134·5	117·8?	121·1	97·0	135·5?	136·5	117	388	315	524	36·1	—	101·9?	—	—	
233	180·5	142·5	96·2	129·5	112·8?	115·2	98·3	129·5?	128	117·5	375	306	520	37·2	28·9	97·6?	—	—	
234	187·5?	136	89·7	135	—	122·5	97·7	—	139·5	117	—	302	525	39·2	—	—	—	—	
235	182	140	96·8	143	119·2	113·3	96·5	137	126	112·5	376	320	510	38·3	35·4	106·0	10·9	4·2	
236	—	150	97·8	127·5	—	112·9	97·1	—	124	118·5	—	—	530	36·8	33·2	—	—	—	
237	182	130·5	91·2	126	109·3	117·5	89·1?	121·5?	134	102·5?	359	293	505	39·1	—	100·1	—	—	
238	189·5	143	103·4	—	114·0	116·3	—	129·5	128·5	—	—	—	541	—	—	—	—	—	
239	187·5	141	99·9	139·5	110·9	120·0	98·0	125·5	136	120·5	383	315?	534?	37·2	31·0	103·0	—	—	
240	184·5	142·5	101·3	—	—	115·4	98·8	—	130	118·5	—	316	530	—	32·8	—	—	—	
241	189	142	95·0	140	117·0	119·2	100·9	135	129·5	122·5	387	327	533	39·1	28·8	100·6	—	—	
242	186	148	96·8	—	115·2?	106·6	103·3?	131?	118	130·5?	379	324	532	—	—	—	—	—	
243	189·5	138·5	92·1	131·5	110·1?	117·7	97·1	124?	131	122·5	378	313	524	38·9	28·0	103·1?	—	—	
244	190·5	140	98·3	133	108·0?	122·1	94·2	122?	136	117	375	306	535	37·1	31·0	106·9?	—	—	
245	199	142·5	99·8	136	115·3	125·7?	96·0	130	137·5?	119	387	319	551	37·0	34·7	109·0	12·5	4·2	
246	192?	128	97·9	—	—	120·5	89·9	—	135·5	110	—	297?	—	—	—	—	—	—	
247	193	140	97·7	138	113·8?	121·2	98·2	129·5?	135·5	121·5	386	310	530	38·9	—	105·6	—	—	
248	—	139	102·1	133	—	116·2	95·0	—	129	115·5	—	312	521	39·1	28·8	—	—	—	
249	181·5	137	105·0	145	—	127·1	97·0	—	142	109	391	328	522	34·3	31·0	99·4	—	—	
250	188	130·5	93·8?	—	108·2	117·7	—	123	137	—	—	292	512	—	—	—	—	—	
251	173	135	90·8	—	106·9?	105·8?	—	121·5?	117?	—	—	300	—	—	—	—	—	—	
252	194	147·5	102·1	136	116·2	110·3	103·2	131·5	124	124·5	380	326	548	39·1	32·9	108·1	—	—	
253	199	137·5	92·8	136	115·2	121·9	104·4	129	136	129	394	312	537	38·0	32·6	107·1	—	—	
254	181·5	141·5	105·4	—	111·3	106·0	96·5	129·5	122	111·5	362	317	524	—	—	—	—	—	
255	199?	136·5	102·2	130	119·1?	123·1?	93·1?	133?	138·5	119?	391	311?	542	37·8	29·3	107·5?	—	—	
256	195·5	139	102·2	—	—	121·1?	96·6?	—	134?	120?	—	311	538	—	—	—	—	—	
257	192	142	99·1	—	111·3	123·9	—	127	139·5	—	—	319	541	—	—	—	—	—	
258	191?	136·5	98·1	—	112·1?	124·9	—	125·5?	138·5	—	—	309	534	—	—	—	10·9	5·1	
259	—	142	—	137	—	131·3	97·0	—	144	121·5	—	326	—	38·4	32·4	—	—	—	
260	182·5	145·5	90·1	—	120·0	117·2	99·5	135·5	131·5	121	388	324?	529?	—	—	—	—	—	
261	189·5	142·5	99·9	140?	117·2?	119·5	93·4	131·5?	136·5?	116	384	317	538	41·2	32·3	100·1?	—	—	
262	195·5	143	97·0	132·5	109·8	119·4	105·9	126·5	135·5	134	395	301	542	38·7	32·0	105·3	—	—	
263	189·5	136·5	—	129?	111·2	119·9	89·6	126	136·5	110·5	373	303	—	36·8	30·0	104·2?	—	—	
264	182·5	135·5	91·3	138·5	107·0	122·0	96·7	122	137	118·5	378	309	511	35·1	30·2	101·9	10·0	3·7	
265	188	129	91·1	125	118·7	115·5	92·8	135·5	131·5	115	382	300	515?	36·3	29·8	97·6	—	—	
266	185·5	138·5	97·1	135·5	—	122·1	99·0	—	140	111·5	—	310	516	35·0	32·3	—	—	—	
267	182·5	135	99·0	126·5	106·1	108·1	95·0	122	124	114·5	361	302	509	38·8	29·7	100·2	—	—	
268	180·5	140	92·8	136	114·7	118·2	99·0	137	135	117	389	316	521	37·7	30·0	91·4	—	—	
269	193·5	136·5	94·0	—	120·8?	120·2	103·0	137	134·5	130·5	402	315	532	—	29·9	—	—	—	
270	—	144·5	100·6	137	—	119·7	103·3	—	133·5	128·5	—	318	517	37·8	34·9	—	—	—	
271	183	138·5	98·9	—	—	115·3	98·5	—	130·5	120·5	—	307	519?	—	30·7	—	—	—	
272	173	138	88·4	—	115·1	106·6	83·1	136	123	103?	361?	—	493?	—	—	—	—	—	
273	178	138·5	—	138·5	106·9	112·1	100·3	119·5	123·5	122	365	304?	—	33·1	31·4	102·1	—	—	
274	—	140	—	143·5	—	121·9	104·0	—	135	126·5	—	323	—	35·1	31·8	—	—	—	
275	185	141·5	98·5	—	117·5?	111·5	—	134	121·5	—	—	310	521	—	—	—	—	—	
276	—	140	—	135	—	121·8	103·5	—	137	122	—	317	—	37·7	33·5	—	—	—	
277	183	142·5	98·3	130·5	113·0?	109·0	—	133?	120	—	—	317	527	—	—	93·5?	—	—	
278	—	136	102·2	—	113·2	113·3	98·2?	129·5	123·5	124?	376	—	519?	—	—	—	—	—	

Measurements of Lachish Skulls. IV. *Male Calvariae from Tomb* 120

	SC	*SS*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	*Oc.I.*	$100\frac{fmb}{fml}$	$100\frac{SS}{SC}$	No.	Remarks
	—	—	73·8	73·8	100·0	0	59·2	80·9	—	207	Coronal and lambdoid beginning to close, sagittal nearly c
?	—	—	74·8	68·9	108·6	5·9	56·7	75·0	—	208	Coronal beginning to close, sagittal closed; trace of hor bones R
	9·5	2·9	72·7	74·9	97·0	−2·2	—	—	30·5	209	Coronal beginning to close, sagittal closed; healed injuries
?	—	—	69·3	71·6	96·8	−2·3	—	—	—	210	Coronal closed, sagittal nearly obliterated
	—	—	72·3	—	—	—	55·4	—	—	211	Coronal and lambdoid closing, sagittal nearly obliterated
	—	—	78·2	69·4	112·6	8·8	56·3	83·6	—	212	Coronal closing; 2 epipteric bones R continued by worm squama and parietal; pterion L normal but wormian bo and parietal
	—	—	—	—	105·1	—	60·5	83·1	—	213	Two or more epipteric bones L and 1 R; wormian bone parietal L, and 1 R?
?	—	—	69·7	74·5	93·6	−4·7	61·7	74·3	—	214	Wormian bone in coronal R
	—	—	76·4	74·3	102·9	2·1	64·1	74·0	—	215	Metopic; coronal beginning to close; large supernumerary b and parietal R, extending to pterion
	—	—	72·8	70·4	103·4	2·4	—	—	—	216	Incomplete fusion of basi-occipital and sphenoid, although
	10·6	5·7	69·1	70·9	97·5	−1·8	58·6	74·1	53·8	217	Three epipteric bones L
?	—	—	75·5	71·0	106·4	4·6	57·9	76·7	—	218	Coronal closing, sagittal nearly obliterated, lambdoid begin
	—	—	69·7	—	—	—	61·7	—	—	219	Base and L temporal missing
	8·0	3·7	70·7	68·9	102·6	1·8	—	87·1	46·3	220	Coronal beginning to close, sagittal obliterated, lambdoid n
	—	—	—	—	—	—	56·7	—	—	221	Base and greater part of temporals missing; part of L te fused to parietal
	—	—	76·2	69·7	109·4	6·6	56·5	88·0	—	222	Two small pre-condyles
	11·3	5·3	72·1	—	—	—	59·6	—	46·9	223	Base and L temporal missing; coronal and sagittal beginnir
	—	—	73·6	72·5	101·5	1·1	60·2	84·0	—	224	Coronal beginning to close, sagittal closing
	—	—	73·2	—	—	—	—	—	—	225	L temporal and part of base missing
	—	—	75·1	—	—	—	56·0	—	—	226	Sagittal and lambdoid closed
	—	—	75·9	70·3	108·1	5·7	56·0	88·2	—	227	Sagittal closing
?	—	—	73·2	75·7	96·8	−2·4	60·2	83·8	—	228	Coronal and lambdoid beginning to close, sagittal closed
	—	—	—	72·5	—	—	58·3	78·6	—	229	Single epipteric bone L
	10·8	5·1	74·4	71·5	104·0	2·8	56·4	82·9	47·2	230	Coronal closing; temporal squama R largely fused to pariet
	11·9	5·1	71·6	72·6	98·6	−1·1	57·0	89·9	42·9	231	Coronal beginning to close, sagittal closed; single epipteric suture of interparietal bones R
?	—	—	71·9	71·4	100·7	0·5	59·6	—	—	232	—
?	—	—	78·9	71·7	110·0	7·2	60·4	77·7	—	233	Metopic; trace of horizontal suture of interparietal bones R
	—	—	72·5	72·0	100·7	0·5	60·2	—	—	234	Coronal and sagittal beginning to close
	10·9	4·2	76·9	78·6	97·9	−1·6	63·1	92·4	38·5	235	Single epipteric bone R
	—	—	—	—	117·6	—	58·6	90·2	—	236	Coronal closing, sagittal nearly obliterated, lambdoid begin L obliterated; large post-mastoid foramen L
	—	—	71·7	69·2	103·6	2·5	65·0	—	—	237	Two epipteric bones L and 1 R
	—	—	75·5	—	—	—	—	—	—	238	Base missing; coronal and lambdoid closing, sagittal nearly
	—	—	75·2	74·4	101·1	0·8	58·1	83·3	—	239	Coronal and sagittal beginning to close
	—	—	77·2	—	—	—	60·1	—	—	240	Coronal closing, sagittal closed, lambdoid beginning to clos
	—	—	75·1	74·1	101·4	1·1	59·0	73·7	—	241	—
	—	—	79·6	—	—	—	56·5	—	—	242	Small epipteric bone R; 2 small wounds on frontal
?	—	—	73·1	69·4	105·3	3·7	56·5	72·0	—	243	—
?	—	—	73·5	69·8	105·3	3·7	57·4	83·6	—	244	—
	12·5	4·2	71·6	68·3	104·8	3·3	57·5	93·8	33·6	245	Trace of horizontal suture of interparietal bones R and L
	—	—	66·7	—	—	—	58·4	—	—	246	Sagittal beginning to close
	—	—	72·5	71·5	101·4	1·0	57·7	—	—	247	—
	—	—	—	—	104·5	—	58·9	73·7	—	248	Metopic
	—	—	75·5	79·9	94·5	−4·4	69·1	90·4	—	249	Sagittal closed
	—	—	69·4	—	—	—	—	—	—	250	Metopic; coronal beginning to close
	—	—	78·0	—	—	—	—	—	—	251	Coronal and sagittal closing
	—	—	76·0	70·1	108·5	5·9	59·6	84·1	—	252	Coronal closing; sagittal closed; exostosis parietal L
	—	—	69·1	68·3	101·1	0·8	57·7	85·8	—	253	Single epipteric bone L
	—	—	78·0	—	—	—	64·4	—	—	254	—
5?	—	—	68·6	65·3	105·0	3·3	55·9	77·5	—	255	Coronal and lambdoid closed, sagittal nearly obliterated
	—	—	71·1	—	—	—	57·4	—	—	256	Coronal and lambdoid beginning to close, sagittal closed; wo
	—	—	74·0	—	—	—	—	—	—	257	Coronal beginning to close, sagittal closing
	10·9	5·1	71·5	—	—	—	—	—	46·8	258	Metopic; metopic and sagittal beginning to close; fused nasa
	—	—	—	—	103·6	—	56·9	84·4	—	259	All sutures beginning to close
	—	—	79·7	—	—	—	58·9	—	—	260	Single epipteric bone R
1?	—	—	75·2	73·9	101·8	1·3	57·4	78·4	—	261	Sagittal closing
3	—	—	73·1	67·8	107·9	5·4	56·4	82·7	—	262	Coronal and lambdoid beginning to close, sagittal closing; occ parieto-mastoid L obliterated; wound supra-occipital
2?	—	—	72·0	68·1	105·8	4·0	57·9	81·5	—	263	Coronal and lambdoid beginning to close, sagittal closing
9	10·0	3·7	74·2	75·9	97·8	−1·6	58·4	86·0	37·0	264	All sutures beginning to close; nasal bones fused
6	—	—	68·6	66·5	103·2	2·1	57·6	82·1	—	265	Sagittal beginning to close, lambdoid nearly obliterated; occ and L; temporal squama completely fused to parietal R, par
	—	—	74·7	73·0	102·2	1·6	68·7	92·3	—	266	Anomalous foramen in temporal squama R
2	—	—	74·0	69·3	106·7	4·7	59·6	76·5	—	267	—
4	—	—	77·6	75·3	102·9	2·2	61·5	79·6	—	268	Coronal closing, sagittal closed, lambdoid nearly obliterated;
	—	—	70·5	—	—	—	56·3	—	—	269	Sagittal closing
	—	—	—	—	105·5	—	57·3	92·3	—	270	Sagittal nearly obliterated, lambdoid beginning to close
	—	—	75·7	—	—	—	58·5	—	—	271	Sagittal nearly obliterated, lambdoid beginning to close
	—	—	79·8	—	—	—	57·6	—	—	272	R temporal and most of base missing; coronal closing, sagittal close
1	—	—	77·8	77·8	100·0	0	58·9	94·9	—	273	Metopic; metopic closed, coronal closing, sagittal closed
	—	—	—	—	97·6	—	58·9	90·6	—	274	Coronal and lambdoid beginning to close, sagittal obliterated
	—	—	76·5	—	—	—	—	—	—	275	—
	—	—	—	—	103·7	—	61·8	88·9	—	276	Greater part of frontal bone missing; coronal closing, sagittal n closed; exostosis parietal L
5?	—	—	77·9	71·3	109·2	6·6	—	—	—	277	Metopic; 2 small epipteric bones L
	—	—	—	—	—	—	56·5	—	—	278	Temporal bones and base missing; injury to glabella region, a

S_2'	S_3'	S_1	S_2	S_3	S	$\beta Q'$	U	*fml*	*fmb*	*LB*	*SC*	*SS*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	*Oc.I.*
17·3?	99·4?	128·5	130·5	120·5	380	320	528	37·2?	30·1	108·2	—	—	73·8	73·8	100·0	0	59·2
15·0	91·9	130	127	115·5	373	311	518?	34·8	26·1	102·2?	—	—	74·8	68·9	108·6	5·9	56·7
19·6	—	125·5	138	—	—	311	503	—	—	99·0	9·5	2·9	72·7	74·9	97·0	−2·2	—
20·6	—	135·5	135·5	—	—	312	533	—	30·0	111·2?	—	—	69·3	71·6	96·8	−2·3	—
16·1	88·8?	—	129	115?	—	301	516	—	—	—	—	—	72·3	—	—	—	55·4
14·0	97·8	136·5	126·5	124	387	315	536	37·1	31·0	97·0	—	—	78·2	69·4	112·6	8·8	56·3
17·0	102·2	—	130·5	122	—	294	519	40·8	33·9	—	—	—	—	—	105·1	—	60·5
26·0	100·0	132?	140·5	118	391	318	525	40·4	30·0	101·9?	—	—	69·7	74·5	93·6	−4·7	61·7
25·1	95·1	134	142·5	110	387	323	535	41·1	30·4	94·9	—	—	76·4	74·3	102·9	2·1	64·1
13·1	—	—	127	—	—	297?	517?	—	30·8	—	—	—	72·8	70·4	103·4	2·4	—
27·0	110·6	143	146	135	424	323	547	40·9	30·3	96·6	10·6	5·7	69·1	70·9	97·5	−1·8	58·6
17·1	91·7	133?	132	113	378	311	528	36·0	27·6	102·3?	—	—	75·5	71·0	106·4	4·6	57·9
23·6	94·1	—	138·5	111	—	—	532?	—	—	—	—	—	69·7	—	—	—	61·7
—	—	128	—	—	391	307	531	37·9	33·0	100·5	8·0	3·7	70·7	68·9	102·6	1·8	—
20·7	100·2?	—	136	126	—	—	—	—	—	—	—	—	—	—	—	—	56·7
07·2	95·8	115?	119	121	355	302	519	35·0	30·8	102·3	—	—	76·2	69·7	109·4	6·6	56·5
26·9	88·0?	131·5	142·5	106?	379	—	527?	—	—	—	11·3	5·3	72·1	—	—	—	59·6
10·4	101·0	135·5	123	121	379	310	—	40·5	34·0	99·1	—	—	73·6	72·5	101·5	1·1	60·2
17·6	—	132·5	134	—	—	—	518	—	—	—	—	—	73·2	—	—	—	—
19·5	93·3	129?	133	119	380	316?	528?	—	30·2	—	—	—	75·1	—	—	—	56·0
11·2	94·9	120·5?	124	121	366	308	531	37·3	32·9	103·0	—	—	75·9	70·3	108·1	5·7	56·0
20·7	106·1	129?	134	127	390	311	512	35·7	29·9	98·1?	—	—	73·2	75·7	96·8	−2·4	60·2
18·2	106·0	—	128·5	130	—	317	—	38·4	30·2	—	—	—	—	72·5	—	—	58·3
23·3	100·7	136	136	127·5	400	318?	540?	36·2	30·0	103·4	10·8	5·1	74·4	71·5	104·0	2·8	56·4
20·6	101·1	131	131·5	126·5	389	325	534	36·5	32·8	104·9	11·9	5·1	71·6	72·6	98·6	−1·1	57·6
21·1	97·0	135·5?	136·5	117	388	315	524	36·1	—	101·9?	—	—	71·9	71·4	100·7	0·5	59·0
15·2	98·3	129·5?	128	117·5	375	306	520	37·2	28·9	97·6?	—	—	78·9	71·7	110·0	7·2	60·4
22·5	97·7	—	139·5	117	—	302	525	39·2	—	—	—	—	72·5	72·0	100·7	0·5	60·2
13·3	96·5	137	126	112·5	376	320	510	38·3	35·4	106·0	10·9	4·2	76·9	78·6	97·9	−1·6	63·1
12·9	97·1	—	124	118·5	—	—	530	36·8	33·2	—	—	—	—	—	117·6	—	58·6
17·5	89·1?	121·5?	134	102·5?	359	293	505	39·1	—	100·1	—	—	71·7	69·2	103·6	2·5	65·4
16·3	—	129·5	128·5	—	—	—	541	—	—	—	—	—	75·5	—	—	—	—
20·0	98·0	125·5	136	120·5	383	315?	534?	37·2	31·0	103·0	—	—	75·2	74·4	101·1	0·8	58·1
15·4	98·8	—	130	118·5	—	316	530	—	32·8	—	—	—	77·2	—	—	—	60·4
19·2	100·9	135	129·5	122·5	387	327	533	39·1	28·8	100·6	—	—	75·1	74·1	101·4	1·1	59·6
06·6	103·3?	131?	118	130·5?	379	324	532	—	—	—	—	—	79·6	—	—	—	56·3
17·7	97·1	124?	131	122·5	378	313	524	38·9	28·0	103·1?	—	—	73·1	69·4	105·3	3·7	56·4
22·1	94·2	122?	136	117	375	306	535	37·1	31·0	106·9?	—	—	73·5	69·8	105·3	3·7	57·1
25·7?	96·0	130	137·5?	119	387	319	551	37·0	34·7	109·0	12·5	4·2	71·6	68·3	104·8	3·3	57·1
20·5	89·9	—	135·5	110	—	297?	—	—	—	—	—	—	66·7	—	—	—	58·3
21·2	98·2	129·5?	135·5	121·5	386	310	530	38·9	—	105·6	—	—	72·5	71·5	101·4	1·0	57·1
16·2	95·0	—	129	115·5	—	312	521	39·1	28·8	—	—	—	—	—	104·5	—	58·6
27·1	97·0	—	142	109	391	328	522	34·3	31·0	99·4	—	—	75·5	79·9	94·5	−4·4	69·1
17·7	—	123	137	—	—	292	512	—	—	—	—	—	69·4	—	—	—	—
05·8?	—	121·5?	117?	—	—	300	—	—	—	—	—	—	78·0	—	—	—	—
10·3	103·2	131·5	124	124·5	380	326	548	39·1	32·9	108·1	—	—	76·0	70·1	108·5	5·9	59·1
21·9	104·4	129	136	129	394	312	537	38·0	32·6	107·1	—	—	69·1	68·3	101·1	0·8	57·1
06·0	96·5	129·5	122	111·5	362	317	524	—	—	—	—	—	78·0	—	—	—	64·1
23·1?	93·1?	133?	138·5	119?	391	311?	542	37·8	29·3	107·5?	—	—	68·6	65·3	105·0	3·3	55·1
21·1?	96·6?	—	134?	120?	—	311	538	—	—	—	—	—	71·1	—	—	—	57·1
23·9	—	127	139·5	—	—	319	541	—	—	—	—	—	74·0	—	—	—	—
24·9	—	125·5?	138·5	—	—	309	534	—	—	—	10·9	5·1	71·5	—	—	—	—
31·3	97·0	—	144	121·5	—	326	—	38·4	32·4	—	—	—	—	—	103·6	—	56·4
17·2	99·5	135·5	131·5	121	388	324?	529?	—	—	—	—	—	79·7	—	—	—	58·3
19·5	93·4	131·5?	136·5?	116	384	317	538	41·2	32·3	100·1?	—	—	75·2	73·9	101·8	1·3	57·1
19·4	105·9	126·5	135·5	134	395	301	542	38·7	32·0	105·3	—	—	73·1.	67·8	107·9	5·4	56·4
19·9	89·6	126	136·5	110·5	373	303	—	36·8	30·0	104·2?	—	—	72·0	68·1	105·8	4·0	57·1
22·0	96·7	122	137	118·5	378	309	511	35·1	30·2	101·9	10·0	3·7	74·2	75·9	97·8	−1·6	58·3
15·5	92·8	135·5	131·5	115	382	300	515?	36·3	29·8	97·6	—	—	68·6	66·5	103·2	2·1	57·1
22·1	99·0	—	140	111·5	—	310	516	35·0	32·3	—	—	—	74·7	73·0	102·2	1·6	68·1
08·1	95·0	122	124	114·5	361	302	509	38·8	29·7	100·2	—	—	74·0	69·3	106·7	4·7	59·1
18·2	99·0	137	135	117	389	316	521	37·7	30·0	91·4	—	—	77·6	75·3	102·9	2·2	61·1
20·2	103·0	137	134·5	130·5	402	315	532	—	29·9	—	—	—	70·5	—	—	—	56·4
19·7	103·3	—	133·5	128·5	—	318	517	37·8	34·9	—	—	—	—	—	105·5	—	57·1
15·3	98·5	—	130·5	120·5	—	307	519?	—	30·7	—	—	—	75·7	—	—	—	58·3
06·6	83·1	136	123	103?	361?	—	493?	—	—	—	—	—	79·8	—	—	—	57·1
12·1	100·3	119·5	123·5	122	365	304?	—	33·1	31·4	102·1	—	—	77·8	77·8	100·0	0	58·3
21·9	104·0	—	135	126·5	—	323	—	35·1	31·8	—	—	—	—	—	97·6	—	58·3
11·5	—	134	121·5	—	—	310	521	—	—	—	—	—	76·5	—	—	—	—
21·8	103·5	—	137	122	—	317	—	37·7	33·5	—	—	—	—	—	103·7	—	61·1
09·0	—	133?	120	—	—	317	527	—	—	93·5?	—	—	77·9	71·3	109·2	6·6	—
13·3	98·2?	129·5	123·5	124?	376	—	519?	—	—	—	—	—	—	—	—	—	56·4

IV. *Male Calvariae from Tomb* 120

$100\frac{B-H'}{L}$	*Oc.I.*	$100\frac{fmb}{fml}$	$100\frac{SS}{SC}$	No.	Remarks
0	59·2	80·9	—	207	Coronal and lambdoid beginning to close, sagittal nearly obliterated
5·9	56·7	75·0	—	208	Coronal beginning to close, sagittal closed; trace of horizontal suture of interparietal bones R
−2·2	—	—	30·5	209	Coronal beginning to close, sagittal closed; healed injuries on frontal R and parietal L
−2·3	—	—	—	210	Coronal closed, sagittal nearly obliterated
—	55·4	—	—	211	Coronal and lambdoid closing, sagittal nearly obliterated
8·8	56·3	83·6	—	212	Coronal closing; 2 epipteric bones R continued by wormian bones between temporal squama and parietal; pterion L normal but wormian bones between temporal squama and parietal
—	60·5	83·1	—	213	Two or more epipteric bones L and 1 R; wormian bone between temporal squama and parietal L, and 1 R?
−4·7	61·7	74·3	—	214	Wormian bone in coronal R
2·1	64·1	74·0	—	215	Metopic; coronal beginning to close; large supernumerary bone between temporal squama and parietal R, extending to pterion
2·4	—	—	—	216	Incomplete fusion of basi-occipital and sphenoid, although skull is almost certainly adult
−1·8	58·6	74·1	53·8	217	Three epipteric bones L
4·6	57·9	76·7	—	218	Coronal closing, sagittal nearly obliterated, lambdoid beginning to close
—	61·7	—	—	219	Base and L temporal missing
1·8	—	87·1	46·3	220	Coronal beginning to close, sagittal obliterated, lambdoid nearly obliterated
—	56·7	—	—	221	Base and greater part of temporals missing; part of L temporal remaining completely fused to parietal
6·6	56·5	88·0	—	222	Two small pre-condyles
—	59·6	—	46·9	223	Base and L temporal missing; coronal and sagittal beginning to close
1·1	60·2	84·0	—	224	Coronal beginning to close, sagittal closing
—	—	—	—	225	L temporal and part of base missing
—	56·0	—	—	226	Sagittal and lambdoid closed
5·7	56·0	88·2	—	227	Sagittal closing
−2·4	60·2	83·8	—	228	Coronal and lambdoid beginning to close, sagittal closed
—	58·3	78·6	—	229	Single epipteric bone L
2·8	56·4	82·9	47·2	230	Coronal closing; temporal squama R largely fused to parietal
−1·1	57·0	89·9	42·9	231	Coronal beginning to close, sagittal closed; single epipteric bone L; trace of horizontal suture of interparietal bones R
0·5	59·6	—	—	232	—
7·2	60·4	77·7	—	233	Metopic; trace of horizontal suture of interparietal bones R
0·5	60·2	—	—	234	Coronal and sagittal beginning to close
−1·6	63·1	92·4	38·5	235	Single epipteric bone R
—	58·6	90·2	—	236	Coronal closing, sagittal nearly obliterated, lambdoid beginning to close; occipito-mastoid L obliterated; large post-mastoid foramen L
2·5	65·0	—	—	237	Two epipteric bones L and 1 R
—	—	—	—	238	Base missing; coronal and lambdoid closing, sagittal nearly obliterated
0·8	58·1	83·3	—	239	Coronal and sagittal beginning to close
—	60·1	—	—	240	Coronal closing, sagittal closed, lambdoid beginning to close; single epipteric bone R
1·1	59·0	73·7	—	241	—
—	56·5	—	—	242	Small epipteric bone R; 2 small wounds on frontal
3·7	56·5	72·0	—	243	—
3·7	57·4	83·6	—	244	—
3·3	57·5	93·8	33·6	245	Trace of horizontal suture of interparietal bones R and L
—	58·4	—	—	246	Sagittal beginning to close
1·0	57·7	—	—	247	—
—	58·9	73·7	—	248	Metopic
−4·4	69·1	90·4	—	249	Sagittal closed
—	—	—	—	250	Metopic; coronal beginning to close
—	—	—	—	251	Coronal and sagittal closing
5·9	59·6	84·1	—	252	Coronal closing; sagittal closed; exostosis parietal L
0·8	57·7	85·8	—	253	Single epipteric bone L
—	64·4	—	—	254	—
3·3	55·9	77·5	—	255	Coronal and lambdoid closed, sagittal nearly obliterated
—	57·4	—	—	256	Coronal and lambdoid beginning to close, sagittal closed; wound parietal L
—	—	—	—	257	Coronal beginning to close, sagittal closing
—	—	—	46·8	258	Metopic; metopic and sagittal beginning to close; fused nasal bones
—	56·9	84·4	—	259	All sutures beginning to close
—	58·9	—	—	260	Single epipteric bone R
1·3	57·4	78·4	—	261	Sagittal closing
5·4	56·4	82·7	—	262	Coronal and lambdoid beginning to close, sagittal closing; occipito-mastoid L obliterated, parieto-mastoid L obliterated; wound supra-occipital
4·0	57·9	81·5	—	263	Coronal and lambdoid beginning to close, sagittal closing
−1·6	58·4	86·0	37·0	264	All sutures beginning to close; nasal bones fused
2·1	57·6	82·1	—	265	Sagittal beginning to close, lambdoid nearly obliterated; occipito-mastoid obliterated R and L; temporal squama completely fused to parietal R, partly fused L
1·6	68·7	92·3	—	266	Anomalous foramen in temporal squama R
4·7	59·6	76·5	—	267	—
2·2	61·5	79·6	—	268	Coronal closing, sagittal closed, lambdoid nearly obliterated; large epipteric bone L
—	56·3	—	—	269	Sagittal closing
—	57·3	92·3	—	270	Sagittal nearly obliterated, lambdoid beginning to close
—	58·5	—	—	271	Sagittal nearly obliterated, lambdoid beginning to close
—	57·6	—	—	272	R temporal and most of base missing; coronal closing, sagittal and lambdoid beginning to close
0	58·9	94·9	—	273	Metopic; metopic closed, coronal closing, sagittal closed
—	58·9	90·6	—	274	Coronal and lambdoid beginning to close, sagittal obliterated
—	—	—	—	275	—
—	61·8	88·9	—	276	Greater part of frontal bone missing; coronal closing, sagittal nearly obliterated, lambdoid closed; exostosis parietal L
6·6	—	—	—	277	Metopic; 2 small epipteric bones L
—	56·5	—	—	278	Temporal bones and base missing; injury to glabella region, apparently during life

Tomb no.	Sex	No.	*L*	*B*	*B′*	*H′*	S_1'	S_2'	S_3'	S_1	S_2	S_3	*S*	*βQ′*	*U*	*fml*	*fmb*	*LB*	*SC*	*S*
120	♂	279	179·5	133	89·8	—	108·3	113·9	100·8	122?	131	121	374	298	498	—	31·6?	—	—	
"	"	280	182·5	139·5	97·4	136	110·1	116·6	102·9	127	132·5	117·5	377	324	522	36·9	29·7	102·3	—	
"	"	281	194·5	137·5	95·1	139·5	117·4	124·6	104·8	140	141	129	410	316	538	32·0	24·8	97·7	—	
"	"	282	186·5	133	91·2	139	114·1	121·2	102·3	135·5	134·5	126·5	396	315	516	35·1	25·7	98·6	—	
"	"	283	179·5	136·5	100·5	133·5	104·7?	112·0	95·1	119?	125	116·5	360	303?	511?	32·1	26·1	102·9?	—	
"	"	284	182·5	141	96·2	130·5	111·0	122·1	88·9	126	137·5	107·5	370	307?	519?	38·1	33·0	96·3	—	
"	"	285	188·5	135·5	82·3	138	122·3	—	—	138·5	—	—	388	312	523	37·2	34·9	106·2	—	
"	"	286	189	127	94·6	133	109·1?	115·6	95·2	122·5	130	115	367	—	510	40·0	34·9	109·0?	—	
"	"	287	184	138	92·9	127	115·6	109·3	95·1	131·5	121·5	114	367	297	512	41·1	31·0	96·1	8·8	
"	"	288	181·5	128	88·2	132·5	105·2?	121·0	93·0	118·5	136·5	111	366	287	505	34·9	30·1	103·2?	—	
"	"	289	189·5	138·5	99·0	—	117·1	123·1	94·2	133·5	138	113·5	384	318	532	—	—	—	6·0	
"	"	290	—	143·5	91·9	126	—	108·6	94·1	—	122	119?	—	301	519?	31·1	—	—	—	
"	"	291	176	135·5	98·7	130·5	105·6	112·5	92·2	121·5	125	108·5	354	294	503	35·6	28·2	97·2	10·2	
"	"	292	—	130·5	—	123·5	—	109·3	—	—	122	—	—	—	494?	39·6	33·7	—	—	
"	"	293	—	150·5	105·0	—	—	115·1	116·6	—	125	146·5	—	335	563	—	—	—	—	
"	"	294	178	138·5	97·9	132·5	108·1	115·4	97·2	126	127	115	368	315	515	36·1	32·9	100·1	—	
"	"	295	184·5	130·5	94·0	132	115·3	113·1	91·2	133	125	111	369	302	508	37·1	28·0	98·9	—	
"	"	296	189·5	136	95·8	129	116·3	116·1	97·2	133	129	124·5	386	306	525	33·0	27·4	98·9	—	
"	"	297	188·5	135·5	—	—	113·7	106·8?	—	130	115	—	—	297	516?	—	27·2	—	—	
"	"	298	181·5	136	100·0	—	—	115·8	—	—	130	—	—	—	—	—	—	—	—	
"	"	299	190	135·5	88·1	138	117·0?	118·2	103·0	133	131·5	124	389	322?	—	36·8	31·0	104·1	—	
"	"	300	190·5	143·5	—	132·5	118·9	111·3	99·1?	139	124·5	120	383	318	—	41·5?	31·9	101·4	9·0?	
"	"	301	192	142	—	135·5	119·2	122·1	94·2	136·5	137·5	116	389	—	—	37·6	26·9	106·1	—	
"	"	302	194	133	94·0	134·5	110·9	127·7	97·9	125·5	144	122	391	307	530	37·6	—	101·0	11·2?	
"	"	303	180·5	130	90·4	135	115·3	107·9	94·5	130	119·5	112	361	298	—	40·2	32·0	98·2	—	
"	♂?	304	188	139·5	—	—	114·4	119·2	93·9?	132·5	134	119·5?	386	—	—	—	—	—	—	
"	"	305	192·5	140·5	103·6	—	111·1	122·3	—	129	134·5	—	—	309	540	—	32·2	—	10·6	
"	"	306	187·5	133·5	91·1	136·5	115·3?	121·6	94·8	129·5?	135·5	114	379	299	514	38·5	34·7	102·2	—	
"	"	307	—	141·5	—	—	—	112·3	—	—	125·5	—	—	—	—	—	—	—	—	
"	"	308	185·5	134·5	94·8	—	108·0	122·3	103·0?	123	136	121?	380?	—	519?	—	—	—	—	
"	"	309	182·5	142	98·2	130·5	110·9	118·3?	90·9	128	131·5	111	370?	312?	520	36·3	32·1	95·8	—	
"	"	310	177·5	135·5	93·0	127·5	105·7	108·3	92·5	116	121	111·5	349	289	496	36·1	29·5	100·5	—	
"	"	311	183·5	138	93·8	144·5	113·5	125·9	101·1	133·5	144	121·5	399	319	518	34·4	29·3	97·3?	—	
"	"	312	186	135·5	94·9?	137	114·2?	113·0	101·6	129·5?	125·5	123	377	308	523	40·0	32·0	101·8?	—	
"	"	313	175·5	130·5	82·9	126	109·3	100·3	98·0	129	108·5	124	361	286	503	32·4	31·0	94·2	11·3	
"	"	314	180·5	—	92·9	136·5	110·2	116·6	98·2?	129·5	129	120·5?	379?	313	517	34·0	29·0	94·7	6·1	
"	"	315	177	135	—	125	111·2	108·8	94·3	126	120	116·5	363	301	—	32·8	29·0	93·4	—	
"	"	316	—	133·5	—	—	—	115·2	87·0	—	127·5	108·5	—	—	—	—	—	—	—	
"	"	317	184·5	—	89·1	134·5	116·2	116·2	100·0	134·5	136	115·5	386	310	517?	44·9	35·7	98·4	—	
"	"	318	179·5	135	97·7	—	110·5	111·2	94·2?	126	124	116·5	366	308	506	—	—	—	—	
"	"	319	174·5	122·5	89·1	127·5	104·4	109·7?	90·0?	119	120·5	108?	347	—	—	35·9	—	98·3	—	
"	"	320	—	133	—	—	—	—	—	—	—	—	—	300	—	—	—	—	—	
"	"	321	180·5	133	92·3	128·5	107·1	116·1	97·9?	124	131·5	121	375	292	506	40·5	—	91·1	9·7	
"	"	322	171·5	135·5	89·6	—	100·5?	106·0	—	116	119·5	—	—	293	495	—	—	—	—	
"	"	323	172·5	139	97·0	138·5	111·8	115·2	94·9	127	133	107	366	310	501	35·8	31·0	97·6	9·3	
"	"	324	187	138	99·0	134	116·0	118·2	97·1	132	131·5	125·5	388	—	525	35·0	28·6	97·2	9·4	
"	"	325	188·5	126·5?	—	129·5	117·2	114·0	93·9	134·5	126·5	117	377	297	—	33·0	29·1	105·8	—	
"	"	326	186·5	137	96·9	134	111·9	118·3	96·6	128	133	116	378	306	524	36·8	31·8	100·1	—	
"	"	327	178·5	133·5	96·8	131·5	108·9	113·9	94·1	124·5	130	109	363	301	505	36·9	31·0	100·2	—	
"	"	328	182	128	90·6	—	107·2?	110·7	100·2	130?	123·5	124·5	378	288	508	35·4?	31·4	—	—	
"	"	329	186·5	132·5	99·1	—	117·8	110·9	—	134·5	123	—	—	298?	514	—	—	—	—	
"	"	330	193·5	137·5	95·7	128·5	122·1	123·7	88·1	143·5	140	108	391	313	526	36·0	26·0	101·2	—	
"	"	331	174·5	142	95·5	137	107·0	116·1	98·1	124	129	119·5	372	317	517	36·8	31·5	96·0	8·8	
"	"	332	178·5	132·5	96·9	133·5	115·1?	118·9	87·1	133·5?	134	107	374	306	503	33·8	29·9	98·0	—	
"	"	333	180	132·5	95·9	132	114·0	110·9	93·6	132	124·5	115·5	371	309	511	39·0	32·1	95·8	10·5	
"	"	334	185·5	135·5	99·6?	130	117·0?	101·4	—	134·5	110·5	—	—	309	523	—	31·2	107·5?	—	
"	"	335	187·5	135	96·2	133·5	115·0	125·7	96·9	132	139·5	118·5	389	312?	523?	33·1	30·8	99·0	—	
"	"	336	175	147	93·1	130·5	—	107·9	90·7	—	123	106	—	311	510	35·9	29·0	—	—	
"	"	337	192	146·5	101·5	—	—	129·0	97·8?	—	143	119·5?	—	—	—	—	—	—	—	
"	"	338	194·5	138	92·9	135·5	118·0	125·3	102·3	138	143	123	404	316	537	37·7	30·7	101·1	—	
"	"	339	178·5	139·5	93·8	—	110·5	115·8	98·1	126·5	126	116	368	31	512	—	31·8	—	—	
"	"	340	—	137?	98·4	—	—	121·8	—	—	139	—	—	—	—	—	—	—	—	
"	"	341	—	—	—	123?	—	121·7	93·2	—	139·5	114·5	—	295?	—	36·2	30·0	—	—	
"	"	342	185·5	140	99·1	125·5	—	115·3	91·2	—	129	117·5	—	301	526	35·0	31·0	—	—	
"	"	343	178·5	140·5	94·6	—	107·2	108·1	—	123·5	119·5	—	—	—	—	—	—	—	10·2?	
107	♂	344	194·5	121·5	94·0	—	115·7	119·0	—	129	136	—	—	295	516	—	—	—	—	
"	"	345	192·5	140	101·2	134	120·0	121·1	92·1	140	135·5	111	386	313	538	35·0	31·0	107·2	—	
"	"	346	190·5	137·5	90·1	—	115·0	118·2	—	127·5	131	—	—	315	525	—	—	—	—	
"	"	347	180	—	96·7	135	108·1	—	—	122·5	—	—	—	301	—	34·3	29·8	104·9	—	
"	♂?	348	187	140·5	97·9	137	—	113·8	101·7	—	125·5	122	—	311	536	38·0	27·8	—	—	
"	"	349	183·5	134·5	94·6	—	106·1	121·6	91·0	122·5	136·5	115·5	374	303	513	—	30·7	—	—	
"	"	350	175·5	135	99·5	135·5	108·7	110·0	92·2	126·5	121·5	109	357	310	503	35·1	27·0	99·4	—	
"	"	351	175	120	89·0	130·5	—	113·6	95·7	—	131	115	—	297	494?	30·5	27·0	—	—	
116	♂	352	181	135·5	97·2	128	115·2	111·4	88·9	129·5	122·5	108	359	305	514	35·0	28·1	101·2	12·5	
"	"	353	188·5	125·5	98·0	130·5	110·4	117·0	92·0	126	130	117·5	374	285	516	39·8	29·6	103·2	10·9	
"	"	354	190·5	133·5	93·0	—	—	—	—	—	—	—	—	295?	524	—	—	—	9·1	
"	"	355	182	126·5	103·3	—	—	108·9?	93·3?	—	122?	111?	—	307	509	—	—	—	—	
"	♂?	356	192	136·5	100·0?	144·5	—	127·0	—	—	141	—	—	319	535?	—	—	—	—	
"	"	357	175·5?	132	103·2	—	—	—	—	—	—	—	—	—	503	37·9	—	—	—	

LB	*SC*	*SS*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	*Oc.I.*	$100\frac{fmb}{fml}$	$100\frac{SS}{SC}$	No.	Remarks
—	—	—	74·1	—	—	—	60·0	—	—	279	Sagittal closed; single epipteric bone R; foramen mag
102·3	—	—	76·4	74·5	102·6	1·9	66·1	80·5	—	280	Trace of horizontal suture of interparietal bones R
97·7	—	—	70·7	71·7	98·6	−1·0	58·0	77·5	—	281	—
98·6	—	—	71·3	74·5	95·7	−3·2	57·7	73·2	—	282	Coronal and sagittal beginning to close
102·9?	—	—	76·0	74·4	102·2	1·7	58·4	81·3	—	283	Metopic; single small epipteric bone L
96·3	—	—	77·3	71·5	108·0	5·8	59·4	86·6	—	284	Sagittal closing
106·2	—	—	71·9	73·2	98·2	−1·3	—	93·8	—	285	Aged skull; coronal and lambdoid closed and partly ob
109·0?	—	—	67·2	70·4	95·5	−3·2	59·4	87·3	—	286	—
96·1	8·8	3·6	75·0	69·0	108·7	6·0	60·1	75·6	40·9	287	Single epipteric bone R and L
103·2?	—	—	70·5	73·0	96·6	−2·5	60·5	86·2	—	288	—
—	6·0	2·8	73·1	—	—	—	59·6	—	46·7	289	Single small epipteric bone R
—	—	—	—	—	113·9	—	56·4	—	—	290	—
97·2	10·2	4·3	77·0	74·1	103·8	2·8	62·0	79·2	42·2	291	Metopic; coronal beginning to close, sagittal closed
—	—	—	—	—	105·7	—	—	85·1	—	292	—
—	—	—	—	—	—	—	56·8	—	—	293	Coronal beginning to close; 1 or more epipteric bones
100·1	—	—	77·8	74·4	104·5	3·4	61·4	91·1	—	294	—
98·9	—	—	70·7	71·5	98·7	−0·8	58·8	75·5	—	295	—
98·9	—	—	71·8	68·1	105·4	3·7	55·8	83·0	—	296	Coronal closing, sagittal and lambdoid beginning to cl
—	—	—	71·9	—	—	—	—	—	—	297	Coronal and sagittal closing; occipito-mastoid L oblit
—	—	—	74·9	—	—	—	—	—	—	298	Base and temporal bones missing; sagittal and lambd
104·1	—	—	71·3	72·6	98·2	−1·3	59·7	84·2	—	299	Coronal closing, sagittal closed; apparently large worn
101·4	9·0?	5·6?	75·3	69·6	108·3	5·8	59·2	76·9	62·2	300	Sagittal beginning to close; single epipteric bone R
106·1	—	—	74·0	70·6	104·8	3·4	58·0	71·5	—	301	Apparent absence of jugular foramen R; wound paric
101·0	11·2?	4·7?	68·6	69·3	98·9	−0·8	57·2	—	42·0	302	Coronal beginning to close
98·2	—	—	72·0	74·8	96·3	−2·8	61·2	79·6	—	303	Sagittal and lambdoid beginning to close
—	—	—	74·2	—	—	—	56·1	—	—	304	Coronal closed, sagittal beginning to close
—	10·6	5·5	73·0	—	—	—	—	—	51·9	305	Metopic; metopic beginning to close, sagittal closed;
102·2	—	—	71·2	72·8	97·8	−1·6	59·8	90·1	—	306	Sagittal beginning to close; small epipteric bone L; j wounds on frontal
—	—	—	—	—	—	—	—	—	—	307	Base and temporal bones missing
—	—	—	72·5	—	—	—	62·1	—	—	308	Sagittal beginning to close
95·8	—	—	77·8	71·5	108·8	6·3	58·6	88·4	—	309	Coronal and sagittal closing
100·5	—	—	76·3	71·8	106·3	4·5	59·6	81·7	—	310	Single pre-condyle R
97·3?	—	—	75·2	78·7	95·5	−3·5	59·9	85·2	—	311	Sagittal and lambdoid beginning to close
101·8?	—	—	72·8	73·7	98·9	−0·8	59·2	80·0	—	312	Coronal beginning to close; single small epipteric bone temporal squama and parietal above mastoid L
94·2	11·3	4·9	74·4	71·8	103·6	2·6	56·4	95·7	43·4	313	Metopic; fronto-temporal articulation L 15 mm.; 3 ep bones in lambdoid suture round asteria and betwe jugular foramen R much larger than L
94·7	6·1	2·1	—	75·6	—	—	58·2	85·3	34·4	314	—
93·4	—	—	76·3	70·6	108·0	5·6	57·8	88·4	—	315	—
—	—	—	—	—	—	—	57·2	—	—	316	Base, L temporal, and part of frontal missing; coron
98·4	—	—	—	72·9	—	—	64·4	79·5	—	317	Wound parietal L
—	—	—	75·2	—	—	—	57·7	—	—	318	Coronal beginning to close
98·3	—	—	70·2	73·1	96·1	−2·9	60·0	—	—	319	Coronal beginning to close, sagittal nearly obliterated
—	—	—	—	—	—	—	—	—	—	320	Sagittal closing, lambdoid beginning to close
91·1	9·7	3·4	73·7	71·2	103·5	2·5	57·7	—	35·1	321	Large epipteric bone R
—	—	—	79·0	—	—	—	—	—	—	322	Coronal and sagittal closing, lambdoid beginning to c
97·6	9·3	5·1	80·6	80·3	100·4	0·3	68·4	86·6	54·8	323	Coronal and sagittal closing; single pre-condyle R
97·2	9·4	4·1	73·8	71·7	103·0	2·1	55·5	81·7	43·6	324	Sagittal closing, lambdoid beginning to close; single ep passage R
105·8	—	—	67·1	68·7	97·7	−1·6	57·2	88·2	—	325	Coronal and lambdoid beginning to close, sagittal clo
100·1	—	—	73·5	71·8	102·2	1·6	60·0	86·4	—	326	Sagittal closing
100·2	—	—	74·8	73·7	101·5	1·1	64·0	84·0	—	327	—
—	—	—	70·3	—	—	—	57·4	88·7	—	328	—
—	—	—	71·0	—	—	—	—	—	—	329	Base missing; fronto-temporal articulation L?
101·2	—	—	71·1	66·4	107·0	4·7	58·3	72·2	—	330	Parts of lambdoid near asteria, and parieto- and occ squamae largely fused to parietals
96·0	8·8	5·1	81·4	78·5	103·9	2·9	58·8	85·6	58·0	331	Two epipteric bones L (?) and 2 R; small exostoses on R; depression above pterion R, probably due to inju
98·0	—	—	74·2	74·8	99·3	−0·6	58·2	88·5	—	332	Sagittal partly obliterated
95·8	10·5	5·8	73·6	73·3	100·4	0·3	57·8	82·3	55·2	333	Coronal and lambdoid closed, sagittal nearly obliter
107·5?	—	—	73·0	70·1	104·2	3·0	—	—	—	334	Sagittal partly obliterated
99·0	—	—	72·0	71·2	101·1	0·8	58·5	93·1	—	335	—
—	—	—	84·0	74·6	112·6	9·4	62·8	80·8	—	336	Two epipteric bones L, and 1 R
—	—	—	76·3	—	—	—	58·5	—	—	337	Base missing; coronal and sagittal beginning to clo several wormian bones between temporal squamae a
101·1	—	—	71·0	69·7	101·8	1·3	59·9	81·4	—	338	Occipito-mastoid obliterated R and L; *os pentagonale*
—	—	—	78·2	—	—	—	61·5	—	—	339	Metopic; sagittal beginning to close
—	—	—	—	—	—	—	—	—	—	340	Base missing; sagittal and lambdoid beginning to close
—	—	—	—	—	—	—	58·1	82·9	—	341	17 mm. stretch of metopic suture anterior to bregma
—	—	—	75·5	67·7	111·6	7·8	55·6	88·6	—	342	*Os pentagonale* and *os triangulare* R separate; large si
—	10·2?	5·3?	78·7	—	—	—	—	—	52·0	343	Base and L temporal missing
—	—	—	62·5	—	—	—	—	—	—	344	—
107·2	—	—	72·7	69·6	104·5	3·1	59·6	88·6	—	345	Coronal beginning to close, sagittal closing; small h with 1 molar lost before death
—	—	—	72·2	—	—	—	—	—	—	346	All sutures beginning to close
104·9	—	—	—	75·0	—	—	—	86·9	—	347	All sutures beginning to close; marked superciliary de
—	—	—	75·1	73·3	102·6	1·9	60·0	73·2	—	348	Single epipteric bone L
—	—	—	73·3	—	—	—	56·3	—	—	349	Coronal and sagittal beginning to close; 2 small woun
99·4	—	—	76·9	77·2	99·6	−0·3	61·5	76·9	—	350	Metopic
—	—	—	68·6	74·6	92·0	−6·0	59·9	88·5	—	351	Trace of horizontal suture of interparietal bones R
101·2	12·5	5·1	74·9	70·7	105·9	4·1	59·0	80·3	40·8	352	—
103·2	10·9	6·7	66·6	69·2	96·2	−2·7	56·0	74·4	61·5	353	Two epipteric bones L
—	9·1	3·7	70·1	—	—	—	—	—	40·7	354	Aged skull; coronal and lambdoid nearly obliterated;
—	—	—	69·5	—	—	—	60·8	—	—	355	Metopic; lower half of metopic suture nearly oblite frontal
—	—	—	71·1	75·3	94·5	−4·2	—	—	—	356	Sagittal beginning to close
—	—	—	75·2	—	—	—	—	—	—	357	Aged skull; all sutures obliterated; temporals comple

	U	*fml*	*fmb*	*LB*	*SC*	*SS*	100 $\frac{B}{L}$	100 $\frac{H'}{L}$	100 $\frac{B}{H'}$	100 $\frac{B-H'}{L}$	*Oc.I.*	100 $\frac{fmb}{fml}$	100 $\frac{SS}{SC}$	No.	
	498	—	31·6?	—	—	—	74·1	—	—	—	60·0	—	—	279	Sagittal closed; single
	522	36·9	29·7	102·3	—	—	76·4	74·5	102·6	1·9	66·1	80·5	—	280	Trace of horizontal sut
	538	32·0	24·8	97·7	—	—	70·7	71·7	98·6	−1·0	58·0	77·5	—	281	
	516	35·1	25·7	98·6	—	—	71·3	74·5	95·7	−3·2	57·7	73·2	—	282	Coronal and sagittal b
?	511?	32·1	26·1	102·9?	—	—	76·0	74·4	102·2	1·7	58·4	81·3	—	283	Metopic; single small
?	519?	38·1	33·0	96·3	—	—	77·3	71·5	108·0	5·8	59·4	86·6	—	284	Sagittal closing
	523	37·2	34·9	106·2	—	—	71·9	73·2	98·2	−1·3	—	93·8	—	285	Aged skull; coronal an
	510	40·0	34·9	109·0?	—	—	67·2	70·4	95·5	−3·2	59·4	87·3	—	286	
	512	41·1	31·0	96·1	8·8	3·6	75·0	69·0	108·7	6·0	60·1	75·6	40·9	287	Single epipteric bone
	505	34·9	30·1	103·2?	—	—	70·5	73·0	96·6	−2·5	60·5	86·2	—	288	
	532	—	—	—	6·0	2·8	73·1	—	—	—	59·6	—	46·7	289	Single small epipteric
	519?	31·1	—	—	—	—	—	—	113·9	—	56·4	—	—	290	
	503	35·6	28·2	97·2	10·2	4·3	77·0	74·1	103·8	2·8	62·0	79·2	42·2	291	Metopic; coronal begi
	494?	39·6	33·7	—	—	—	—	—	105·7	—	—	85·1	—	292	
	563	—	—	—	—	—	—	—	—	—	56·8	—	—	293	Coronal beginning to
	515	36·1	32·9	100·1	—	—	77·8	74·4	104·5	3·4	61·4	91·1	—	294	
	508	37·1	28·0	98·9	—	—	70·7	71·5	98·7	−0·8	58·8	75·5	—	295	
	525	33·0	27·4	98·9	—	—	71·8	68·1	105·4	3·7	55·8	83·0	—	296	Coronal closing, sagitt
	516?	—	27·2	—	—	—	71·9	—	—	—	—	—	—	297	Coronal and sagittal c
	—	—	—	—	—	—	74·9	—	—	—	—	—	—	298	Base and temporal bo
?	—	36·8	31·0	104·1	—	—	71·3	72·6	98·2	−1·3	59·7	84·2	—	299	Coronal closing, sagitt
	—	41·5?	31·9	101·4	9·0?	5·6?	75·3	69·6	108·3	5·8	59·2	76·9	62·2	300	Sagittal beginning to
	—	37·6	26·9	106·1	—	—	74·0	70·6	104·8	3·4	58·0	71·5	—	301	Apparent absence of j
	530	37·6	—	101·0	11·2?	4·7?	68·6	69·3	98·9	−0·8	57·2	—	42·0	302	Coronal beginning to
	—	40·2	32·0	98·2	—	—	72·0	74·8	96·3	−2·8	61·2	79·6	—	303	Sagittal and lambdoic
	—	—	—	—	—	—	74·2	—	—	—	56·1	—	—	304	Coronal closed, sagitt
	540	—	32·2	—	10·6	5·5	73·0	—	—	—	—	—	51·9	305	Metopic; metopic beg
	514	38·5	34·7	102·2	—	—	71·2	72·8	97·8	−1·6	59·8	90·1	—	306	Sagittal beginning to wounds on frontal
	—	—	—	—	—	—	—	—	—	—	—	—	—	307	Base and temporal bo
	519?	—	—	—	—	—	72·5	—	—	—	62·1	—	—	308	Sagittal beginning to
?	520	36·3	32·1	95·8	—	—	77·8	71·5	108·8	6·3	58·6	88·4	—	309	Coronal and sagittal c
	496	36·1	29·5	100·5	—	—	76·3	71·8	106·3	4·5	59·6	81·7	—	310	Single pre-condyle R
	518	34·4	29·3	97·3?	—	—	75·2	78·7	95·5	−3·5	59·9	85·2	—	311	Sagittal and lambdoic
	523	40·0	32·0	101·8?	—	—	72·8	73·7	98·9	−0·8	59·2	80·0	—	312	Coronal beginning to temporal squama an
	503	32·4	31·0	94·2	11·3	4·9	74·4	71·8	103·6	2·6	56·4	95·7	43·4	313	Metopic; fronto-temp bones in lambdoid s jugular foramen R n
	517	34·0	29·0	94·7	6·1	2·1	—	75·6	—	—	58·2	85·3	34·4	314	
	—	32·8	29·0	93·4	—	—	76·3	70·6	108·0	5·6	57·8	88·4	—	315	
	—	—	—	—	—	—	—	—	—	—	57·2	—	—	316	Base, L temporal, an
	517?	44·9	35·7	98·4	—	—	—	72·9	—	—	64·4	79·5	—	317	Wound parietal L
	506	—	—	—	—	—	75·2	—	—	—	57·7	—	—	318	Coronal beginning to
	—	35·9	—	98·3	—	—	70·2	73·1	96·1	−2·9	60·0	—	—	319	Coronal beginning to
	—	—	—	—	—	—	—	—	—	—	—	—	—	320	Sagittal closing, lamb
	506	40·5	—	91·1	9·7	3·4	73·7	71·2	103·5	2·5	57·7	—	35·1	321	Large epipteric bone
	495	—	—	—	—	—	79·0	—	—	—	—	—	—	322	Coronal and sagittal c
	501	35·8	31·0	97·6	9·3	5·1	80·6	80·3	100·4	0·3	68·4	86·6	54·8	323	Coronal and sagittal c
	525	35·0	28·6	97·2	9·4	4·1	73·8	71·7	103·0	2·1	55·5	81·7	43·6	324	Sagittal closing, lamb passage R
	—	33·0	29·1	105·8	—	—	67·1	68·7	97·7	−1·6	57·2	88·2	—	325	Coronal and lambdoic
	524	36·8	31·8	100·1	—	—	73·5	71·8	102·2	1·6	60·0	86·4	—	326	Sagittal closing
	505	36·9	31·0	100·2	—	—	74·8	73·7	101·5	1·1	64·0	84·0	—	327	
	508	35·4?	31·4	—	—	—	70·3	—	—	—	57·4	88·7	—	328	
?	514	—	—	—	—	—	71·0	—	—	—	—	—	—	329	Base missing; fronto-
	526	36·0	26·0	101·2	—	—	71·1	66·4	107·0	4·7	58·3	72·2	—	330	Parts of lambdoid ne squamae largely fus
	517	36·8	31·5	96·0	8·8	5·1	81·4	78·5	103·9	2·9	58·8	85·6	58·0	331	Two epipteric bones R; depression above
	503	33·8	29·9	98·0	—	—	74·2	74·8	99·3	−0·6	58·2	88·5	—	332	Sagittal partly oblite
	511	39·0	32·1	95·8	10·5	5·8	73·6	73·3	100·4	0·3	57·8	82·3	55·2	333	Coronal and lambdoic
	523	—	31·2	107·5?	—	—	73·0	70·1	104·2	3·0	—	—	—	334	Sagittal partly oblite
?	523?	33·1	30·8	99·0	—	—	72·0	71·2	101·1	0·8	58·5	93·1	—	335	
	510	35·9	29·0	—	—	—	84·0	74·6	112·6	9·4	62·8	80·8	—	336	Two epipteric bones
	—	—	—	—	—	—	76·3	—	—	—	58·5	—	—	337	Base missing; coron several wormian bo
	537	37·7	30·7	101·1	—	—	71·0	69·7	101·8	1·3	59·9	81·4	—	338	Occipito-mastoid obl
	512	—	31·8	—	—	—	78·2	—	—	—	61·5	—	—	339	Metopic; sagittal beg
	—	—	—	—	—	—	—	—	—	—	—	—	—	340	Base missing; sagitta
?	—	36·2	30·0	—	—	—	—	—	—	—	58·1	82·9	—	341	17 mm. stretch of m
	526	35·0	31·0	—	—	—	75·5	67·7	111·6	7·8	55·6	88·6	—	342	*Os pentagonale* and c
	—	—	—	—	10·2?	5·3?	78·7	—	—	—	—	—	52·0	343	Base and L temporal
	516	—	—	—	—	—	62·5	—	—	—	—	—	—	344	
	538	35·0	31·0	107·2	—	—	72·7	69·6	104·5	3·1	59·6	88·6	—	345	Coronal beginning t with 1 molar lost b
	525	—	—	—	—	—	72·2	—	—	—	—	—	—	346	All sutures beginning
	—	34·3	29·8	104·9	—	—	—	75·0	—	—	—	86·9	—	347	All sutures beginning
	536	38·0	27·8	—	—	—	75·1	73·3	102·6	1·9	60·0	73·2	—	348	Single epipteric bone
	513	—	30·7	—	—	—	73·3	—	—	—	56·3	—	—	349	Coronal and sagittal
	503	35·1	27·0	99·4	—	—	76·9	77·2	99·6	−0·3	61·5	76·9	—	350	Metopic
	494?	30·5	27·0	—	—	—	68·6	74·6	92·0	−6·0	59·9	88·5	—	351	Trace of horizontal s
	514	35·0	28·1	101·2	12·5	5·1	74·9	70·7	105·9	4·1	59·0	80·3	40·8	352	
	516	39·8	29·6	103·2	10·9	6·7	66·6	69·2	96·2	−2·7	56·0	74·4	61·5	353	Two epipteric bones
?	524	—	—	—	9·1	3·7	70·1	—	—	—	—	—	40·7	354	Aged skull; coronal
	509	—	—	—	—	—	69·5	—	—	—	60·8	—	—	355	Metopic; lower half frontal
	535?	—	—	—	—	—	71·1	75·3	94·5	−4·2	—	—	—	356	Sagittal beginning t
	503	37·9	—	—	—	—	75·2	—	—	—	—	—	—	357	Aged skull; all sutur

	Remarks
	tal closed; single epipteric bone R; foramen magnum markedly asymmetrical
	e of horizontal suture of interparietal bones R
	—
	nal and sagittal beginning to close
	pic; single small epipteric bone L
	tal closing
	l skull; coronal and lambdoid closed and partly obliterated, sagittal nearly obliterated
	—
	le epipteric bone R and L
	—
	le small epipteric bone R
	—
	pic; coronal beginning to close, sagittal closed
	—
	nal beginning to close; 1 or more epipteric bones R
	—
	—
	nal closing, sagittal and lambdoid beginning to close
	nal and sagittal closing; occipito-mastoid L obliterated; jugular foramen L divided
	and temporal bones missing; sagittal and lambdoid closing
	nal closing, sagittal closed; apparently large wormian bone above pterion R
	ttal beginning to close; single epipteric bone R
	arent absence of jugular foramen R; wound parietal R
	nal beginning to close
	ttal and lambdoid beginning to close
	nal closed, sagittal beginning to close
	pic; metopic beginning to close, sagittal closed; single epipteric bone L
	ttal beginning to close; small epipteric bone L; jugular foramen L divided; 2 slight
	unds on frontal
	and temporal bones missing
	ttal beginning to close
	nal and sagittal closing
	le pre-condyle R
	ttal and lambdoid beginning to close
	nal beginning to close; single small epipteric bones R and L; wormian bone between
	mporal squama and parietal above mastoid L
	pic; fronto-temporal articulation L 15 mm.; 3 epipteric bones R; numerous wormian
	nes in lambdoid suture round asteria and between temporal squama and parietals;
	ular foramen R much larger than L
	—
	—
	, L temporal, and part of frontal missing; coronal beginning to close, sagittal closed
	nd parietal L
	nal beginning to close
	nal beginning to close, sagittal nearly obliterated
	ttal closing, lambdoid beginning to close
	ge epipteric bone R
	nal and sagittal closing, lambdoid beginning to close
	nal and sagittal closing; single pre-condyle R
	ttal closing, lambdoid beginning to close; single epipteric bone L; absence of auricular
	sage R
	nal and lambdoid beginning to close, sagittal closed
	ttal closing
	—
	—
	missing; fronto-temporal articulation L?
	s of lambdoid near asteria, and parieto- and occipito-mastoid obliterated; temporal
	amae largely fused to parietals
	epipteric bones L (?) and 2 R; small exostoses on parietal R, temporal R and occipital
	depression above pterion R, probably due to injury
	ttal partly obliterated
	nal and lambdoid closed, sagittal nearly obliterated
	ttal partly obliterated
	—
	epipteric bones L, and 1 R
	e missing; coronal and sagittal beginning to close; occipito-mastoid L obliterated;
	veral wormian bones between temporal squamae and parietals
	ipito-mastoid obliterated R and L; *os pentagonale* and *os triangulare* R separate
	opic; sagittal beginning to close
	e missing; sagittal and lambdoid beginning to close; large trepanned opening parietal R
	am. stretch of metopic suture anterior to bregma
	pentagonale and *os triangulare* R separate; large single tympanic perforations R and L
	e and L temporal missing
	—
	nal beginning to close, sagittal closing; small healed wound on frontal; mandible
	th 1 molar lost before death
	sutures beginning to close
	sutures beginning to close; marked superciliary development
	gle epipteric bone L
	nal and sagittal beginning to close; 2 small wounds on frontal
	opic
	ce of horizontal suture of interparietal bones R
	—
	o epipteric bones L
	d skull; coronal and lambdoid nearly obliterated; sagittal obliterated
	topic; lower half of metopic suture nearly obliterated, possibly owing to injury on
	ntal
	ittal beginning to close
	d skull; all sutures obliterated; temporals completely fused to occipital and parietals

No.	L	B	B'	H'	S_1'	S_2'	S_3'	S_1	S_2	S_3	S	$\beta Q'$	U	fml	fmb	LB	GL	$G'H$	J	GB
383	180	132	89·1	123	107·2	119·0	92·6	121·5	134·5	113	369	299	504	30·8	22·2	96·2	93·2	63·2	—	91·2
384	174	—	86·3	126	110·0	102·4?	99·4?	124·5	110·5	124·5	359	—	490	35·0	28·9	90·7	88·1	61·9	—	—
385	182·5	126	92·6	128·5	114·7	117·0	96·3	131	131·5	116·5	379	288	509	39·0	30·0	93·6	91·0	66·9	—	89·9
386	175	134·5	84·3	133·5	106·3	112·9	101·1	119	128·5	119	366	298	493	37·6	30·0	97·1	93·2	73·3	123	90·2
387	183·	128·5	89·4	126·5	107·6	112·8	97·5	122·5	123	121	366	292	507	37·9	30·0	96·1	—	—	117	94·0
388	180·5	131	88·1	128·5	107·4	121·3	96·0	122·5	138	114	375	295	505	34·6	26·8	96·0	92·0	63·0	117·5	91·3
389	179·5	128·5	97·2	126·5	110·0	103·7	88·7	128	113	109·5	350	299	503	37·6	29·2	103·0	95·1	61·0?	—	94·3
390	180	135	90·8	140	112·6	120·3	94·0	126·5	135	108	370	307	508	36·0	29·0	102·1	—	—	—	—
391	182	128·5	95·8	122·5	107·2	116·2?	—	122	128?	—	—	290	510	—	26·1	93·0	—	—	—	—
392	181·5	141·5	92·0	125	115·2	117·2	—	131	131·5	—	—	302?	515	—	29·8?	90·1	—	—	—	—
393	177	128·5	91·9	130·5	109·3	113·1	87·2	126	127	101·5	355	299	500	37·9	30·8	101·9	87·9	66·1	124·5	85·1
394	174·5	137	100·8	—	110·0	105·9	—	126·5	117·5	—	—	312	506	—	—	—	—	—	—	—
395	171·5	128·5	87·1	121	99·9	111·0	92·1	113	121·5	112·5	347	288	488	33·1	—	94·4	88·9	68·5	—	89·4
396	177	127	92·7	128·5	109·0	104·1	92·3	126	113·5	114·5	354	291	496	38·6	—	99·6	92·7?	70·9	—	102·2
397	173·5	136·5	92·5	127	99·9	—	—	114·5	—	—	354	294	496	36·8	30·9	94·6	87·1	67·0	—	96·2
398	171	129	90·0	120·5	106·2	105·1	—	123	114·5	—	—	283	486	—	—	89·2	85·1	69·7	—	—
399	174·5?	127·5	90·1	139·5	117·1?	116·4?	93·1	137·5?	132·5?	107	377?	309?	495	35·0	28·9	95·0	85·4	73·3	—	—
400	171	133·5	97·0	119	105·2	107·4	87·2	117	122	105	344	285	488	33·8	24·8	94·9	—	—	—	98·8
401	172·5	133	85·2?	124	105·1	112·0	84·2	119·5	129	99·5	347	290	488	35·2	28·6	96·2	—	—	—	86·2
402	169·5	124·5	91·3	126·5	103·4	110·5	95·6	117·5	123	110	351	285	479	36·1	29·2	94·0	89·2	67·7	—	90·4
403	169·5	132·5	87·1	127·5	104·0	110·1	96·1	115	127	109·5	351	285	482	34·0	29·1	95·0	87·0	63·1	123	94·3
404	179	124	89·0	126	103·2	111·0	89·1	117·5	123	110	350	287	491	40·1	28·2	99·8	—	—	—	—
405	174·5	130	99·9?	127	103·6	102·0	95·2	114·5	111·5	118	344	295	489	37·0	27·0	99·2	89·3	72·3	124	96·0
406	182	140	93·0	135·5	117·9	117·4	97·4	136	129·5	116·5	382	314	515	35·5	28·9	96·7	95·0	66·7	—	88·3
407	180	138·5	90·1?	118	109·4	102·1	88·1	130	113	114·5	357	302	511?	35·0	29·1	92·6	—	—	—	90·7
408	178·5	132·5	99·9	124·5	109·9	109·1	91·0	131	124·5	116	371	300	503	34·0	29·1	95·4	—	—	—	90·2
409	170	136·5	87·6	126·5	104·2	115·4	—	119	129·5	—	—	299	494	—	—	90·0	81·4	60·3	—	88·0
410	—	119·5	91·1	127	—	108·0	86·1	—	122	97·5	—	284	470	33·5	—	—	—	—	111	87·2
411	179·5	131·5	98·3	131	110·8	109·2	98·0	126	120·5	113·5	360	291	504	40·3	—	101·0	87·4?	72·0?	126	97·3
412	174·5	136	92·2	127·5	105·5	110·2	89·9	120	123	107	350	301	497	34·9	28·8	99·7	—	—	—	—
413	169	134·5	86·0	123·5	107·0	110·2	91·3	121·5	122	113	357	291	484	30·7	27·0	88·1	77·1	71·1	118	85·1
414	171	131·5	98·7	—	102·3	117·8	89·6	118·5	128·5	109	355	294	500	—	30·3	—	—	62·2	—	97·1
415	181·5	133·5	97·7	131·5	116·0	99·4	—	130	106	—	—	302	501	—	—	102·0	95·0	66·2	—	89·2
416	172·5	137·5	93·0	—	110·9	114·0	85·2	134·5	126	105·5	367	314	503	—	26·8?	—	—	—	—	87·8
417	180·5	126·5	85·9	133	111·6	106·2	95·6	130·5	118	115	363	296	502	38·8	30·1	101·1	91·1	69·0	—	—
418	180	136?	94·3	140·5	116·9	—	—	132·5	—	—	—	304	—	35·1	—	95·2	89·0	67·6	—	—
419	175·5	133	94·5	132·5	110·9	112·1	96·1	130	126	115·5	371	302	497	36·0	28·2	96·0	88·1	66·3	—	—
420	173	128	83·1	129	105·3	110·8	92·4	118·5	121	106	345	285	480	37·9	29·9	97·7	89·3	67·6	—	92·5
421	171·5	129·5	87·3	—	110·0	106·1	—	130	117·5	—	—	294	489	—	—	—	—	—	116·5	90·2
422	178	126	93·2	126·5	112·1	113·2	85·3	130·5	123·5	104·5	358	292	496	33·2	27·2	98·4	—	—	—	93·8?
423	166·5	132·5	86·9	124·5	106·8	100·8	89·0	119	111·5	107	337	291	479	32·1	28·8	96·2	—	—	119	90·2
424	181·5	131	96·1	129	111·0	116·1	98·2	127·5	128	116	371	301	512	38·9	—	99·9	90·2	66·1	—	—
425	171	127·5	90·3	129	111·1	107·3	—	129	115	—	—	296	489?	—	—	92·3	—	—	—	—
426	170	129	88·3	125	105·7	112·8	90·9	121	130	105·5	356	281	480	35·0	28·9	94·0	94·0	70·5	114·5?	89·3
427	—	129·5	89·3	—	104·9	—	—	117	—	—	—	—	—	—	—	—	—	66·1	—	—
428	173	130·5	91·8	138	111·0	117·3	93·0	132	132·5	111	375	303	494?	33·7	28·0	92·2	—	—	—	87·1
429	178·5	131	90·1	126	106·3	107·1	104·0	122	118	129·5	370	290	495	37·0	27·1	93·4	95·9	63·2	116	95·2
430	170	130	89·0?	129·5	112·9	113·3	92·1	125·5	127	109·5	361	286	485	34·0	29·0	93·2	94·9	67·8	—	94·6
431	170·5	131·5	89·3	121·5	110·1	105·3	87·2	126	115	104·5	345	289	483	36·0	28·5	95·4	96·2	63·2	—	97·3
432	166·5	131	91·0?	130·5	107·1	113·0	90·0	122·5	130	106·5	359	297	483	34·9	27·0	91·2	83·4	65·2	—	—
433	172·5	137	91·2	133	110·2	114·1	95·1	130·5	126	114	370	311	498	33·2	31·1	93·1	87·0	61·8	—	—
434	175·5	134·5	95·0	127	109·2	112·1	94·1	125	129	111	364	289	503	37·9	—	95·1	97·9	71·8	—	95·6
435	175	125·5	90·2	121·5	105·5	111·4	86·9	121·5	126	106	353	282	492	38·1	25·6	93·1	—	—	—	—
436	176·5	135	89·9	127	109·8	107·7	92·2	125	118·5	108	351	296	502	39·3	27·8	98·5	93·0	63·9	127·5?	95·3
437	175	133	—	125·5	105·0	105·2	94·0	116·5	118	111	345	287	—	33·2	26·2	102·2	104·4	72·3	121	91·1
438	—	128	—	—	—	113·3	86·0	—	128	103	—	292?	—	—	26·8	—	—	—	119	—
439	167·5	130	90·5	—	107·0	98·0	93·0	121	107·5	110·5	339	273	476	—	26·1	—	—	—	—	—
440	172	122	83·9	131	103·3	112·2	92·2	114·5	125	105	345	283	472	40·0	29·8	97·7	—	—	—	93·0
441	172	133·5	92·0	—	—	—	90·0	—	—	104·5	—	—	493	33·0	26·8	90·0	—	—	—	—
442	185	129·5	—	129·5	112·3	124·5	97·0	134·5	139	121	395	300	—	—	—	92·4	84·7	59·5	—	92·8
443	180·5	132·5	92·0	—	115·8	117·0	—	135·5	131·5	—	—	—	—	—	—	—	—	—	—	—
444	169	126	88·5	118	101·1	100·7	93·0	115	111·5	114·5	340	277	482	32·6	27·8	90·9	—	—	113	86·4
445	173	132·5	89·6	123	110·0	109·0	88·0	125	120	107	352	290	494	32·9	27·0	93·9	90·0	63·9	116	88·2
446	177	128	87·1	127	111·3	112·3	93·0	127	126·5	116	370	282	490	35·2	29·0	91·1	87·9	71·2	—	—
447	172	129·5	89·9	—	106·2	101·9	—	124	111	—	—	296	490	—	—	—	—	—	—	—
448	173·5	133·5	85·1	119·5	104·3	112·0	93·9	116·5	126·5	111·5	354	285	487	34·1	27·0	92·0	—	—	118	89·1
449	183	137·5	98·0	—	—	—	—	—	—	—	—	—	514	—	—	—	—	66·9	—	92·0
450	177	132·5	97·2	131·5	111·0	115·4	99·0	125	127	119	371	308	506	35·0	28·1	98·6	95·9	71·0	124·5	88·0
451	180·5	136·5	95·1	133·5	102·1	110·1	—	112·5	123·5	—	—	297?	516?	—	—	109·1	—	—	—	—
452	179	132·5	98·4	130	108·0	115·0	97·2	122	130	117	368	305	504	38·1	31·0	101·2	97·1	63·1	—	93·2
453	184·5	—	100·0	135	109·0	111·8	—	124	122·5	—	—	—	—	—	—	104	—	—	—	—
454	185·5	139	97·7	129	113·9	110·3	100·0	130·5	120	118·5	369	307	525	36·9	33·2	105·7	103·2	73·2	—	99·0

J	*GB*	*NB*	*NH,L*	*SC*	*SS*	O_2L	O_1L	G_1'	G_2	Ml_3	Ml_1	*C(ml)*	*S(ml)*	*OH*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	
—	91·2	—	47·1	—	—	31·5	38·7	—	—	44·5	60·5	53·0	13·0	110·7	73·3	68·3	107·3	5·1	5
—	—	20·9	42·2	9·0	4·3	33·1	38·0	42·8	—	—	—	—	—	117·2	—	72·4	—	—	5
—	89·9	24·0	47·5	6·9	2·4	33·3?	42·0	45·8	—	46·5	—	—	—	—	69·0	70·4	98·1	−1·4	5
123	90·2	22·1?	50·0?	9·2	5·3	33·0	38·9	48·3	36·0?	46·0	59·5	51·9	12·3	114·2	76·9	76·3	100·7	0·5	6
117	94·0	23·0	50·8	10·0	5·6	31·7	39·6	51·4?	39·8	45·0	56·5	48·3	10·2	107·6	70·2	69·1	101·6	1·1	5
117·5	91·3	23·8	48·2	12·0	4·9	31·4	39·1	43·1	—	48·0	62·0	54·1	12·2	110·3	72·6	71·2	101·9	1·4	6
—	94·3	25·8	48·1	11·2	4·3	31·1	36·4	44·0	39·0	41·0	—	—	—	108·5	71·6	70·5	101·6	1·1	5
—	—	—	—	7·0	—	33·3	—	—	—	—	—	—	—	—	75·0	77·8	96·4	−2·8	6
—	—	—	—	—	—	34·0	42·1	—	—	49·5	—	—	—	—	70·6	67·3	104·9	3·3	
—	—	—	53·0	—	—	34·4	42·1	—	41·1	—	—	—	—	109·8	78·0	68·9	113·2	9·1	
124·5	85·1	23·2	50·0	10·0	4·5	32·7	42·7	41·4	39·8	48·5	60·0	53·9	10·9	109·8	72·6	73·7	98·5	−1·1	6
—	—	—	49·0?	—	—	—	—	45·3?	—	—	—	—	—	115·6	78·5	—	—	—	
—	89·4	27·0?	50·2	11·1	3·9	33·0	41·7?	—	—	46·5?	55·5	48·7	11·4	106·5	74·9	70·6	106·2	4·4	5
—	102·2	22·8?	52·6	8·2	2·9	36·5	39·1	44·5	36·5	49·5	60·0	52·2	12·0	106·5	71·8	72·6	98·8	−0·8	5
—	96·2	22·8	50·9	10·7	3·4	36·7	42·0	42·0?	39·7	45·5	—	—	—	—	78·7	73·2	107·5	5·5	
—	—	23·4	50·1	9·9	2·8	34·2	—	44·0	35·9	—	—	—	—	103·2	75·4	70·5	107·1	5·0	
—	—	—	—	—	—	34·1	40·1	—	—	53·0	—	—	—	—	73·1	79·9	91·4	−6·9	6
—	98·8	25·2	52·0	9·8	3·2	33·2	41·3	45·0	—	48·0	—	—	—	101·8	78·1	69·6	111·2	8·5	5
—	86·2	21·7?	53·1	11·5	6·0	34·0	41·1?	—	34·8	53·5	62·0	54·3	12·1	104·7	77·1	71·9	107·3	5·2	6
—	90·4	—	48·1	8·1	5·1	35·0	40·1	44·0?	40·0	46·0	—	—	—	—	73·5	74·6	98·4	−1·2	6
123	94·3	25·8	47·9	5·8	3·1	32·2	40·0	41·1	41·1?	45·5	57·0	49·2	11·9	105·8	78·2	75·2	103·9	2·9	6
—	—	—	—	—	—	29·1	40·0	—	—	—	56·0	49·0	12·8	—	69·3	70·4	98·4	−1·1	5
124	96·0	24·0	52·2	11·4	4·2	34·1	41·1?	44·7	38·1	53·0	58·0?	53·0	11·3	109·2	74·5	72·8	102·4	1·7	6
—	88·3	23·0	45·6	—	—	30·7	40·8	—	40·2	—	—	—	—	116·0	76·9	74·5	103·3	2·5	6
—	90·7	27·0	48·1	10·8	5·6	32·9	41·0	42·7?	35·9	47·0	50·0	46·0	8·6	—	76·9	65·6	117·4	11·4	5
—	90·2	24·0	47·0	11·5	3·3	30·5	39·0	42·0	36·1	45·0	—	—	—	106·8	74·2	69·7	106·4	4·5	5
—	88·0	—	44·0	9·0	3·5	—	40·9	40·0	40·0	43·5	—	—	—	109·3	80·3	74·4	107·9	5·9	
111	87·2	—	—	—	—	—	—	—	—	46·5?	—	—	—	—	—	—	94·1	—	6
126	97·3	27·7	54·2	11·9	4·9	35·0	43·1	43·5	—	49·0	59·5	51·2	11·7	107·0	73·3	73·0	100·4	0·3	6
—	—	—	—	13·0	7·0	—	38·3	—	—	—	—	—	—	—	77·9	73·1	106·7	4·9	6
118	85·1	—	—	12·1	5·1	—	40·3?	40·8	—	—	—	—	—	108·8	79·6	73·1	108·9	6·5	5
—	97·1	24·8?	46·4	11·5	5·6	33·5	41·3	38·0	—	49·5	—	—	—	108·3	76·9	—	—	—	5
—	89·2	26·8	47·0	14·2	5·6	—	44·1	48·2	—	48·5	—	—	—	112·9	73·6	72·5	101·5	1·1	
—	87·8	26·2	49·6	9·0	4·1	31·2	39·8	47·0	—	46·0	—	—	—	111·2	79·7	—	—	—	5
—	—	—	49·3	10·1	4·1	—	—	43·9	38·0	—	—	—	—	109·3	70·1	73·7	95·1	−3·6	5
—	—	23·2	45·0	8·0	2·3	—	40·4	44·4	—	—	—	—	—	—	75·6	78·1	96·8	−2·5	
—	—	29·0	49·6	14·2	5·1	31·3	38·0	42·5	40·0	—	—	—	—	110·8	75·8	75·5	100·4	0·3	5
—	92·5	—	50·1	—	—	32·5	40·1	45·1	—	45·5	—	—	—	105·8	74·0	74·6	99·2	−0·6	6
116·5	90·2	—	50·0	—	—	—	40·0	45·1	40·0	52·0	—	—	—	109·3	75·5	—	—	—	
—	93·8?	22·9	51·0	7·2	3·3	33·0	40·1	—	—	44·0	—	—	—	110·1	70·8	71·1	99·6	−0·3	5
119	90·2	25·1	46·0	7·6	4·7	32·9	41·9	—	—	49·0	51·5	46·1	8·9	106·3	79·6	74·8	106·4	4·8	5
—	—	27·0	51·0	13·3	4·5	33·5	—	—	—	—	—	—	—	112·2	72·2	71·1	101·6	1·1	6
—	—	23·3	46·0	6·7	—	33·9	40·0	44·7?	35·8	—	—	—	—	—	74·6	75·4	98·8	−0·9	
114·5?	89·3	24·0	49·0	9·2	3·6	34·0	39·7	45·1	—	46·5	48·0	43·2	8·9	104·3	75·9	73·5	103·2	2·4	6
—	—	25·0	48·2	8·4	3·7	33·5	40·5	46·0	—	—	—	—	—	—	—	—	—	—	
—	87·1	24·8	47·0	8·8	—	30·0	39·0	45·0	39·0	43·0	47·5?	43·1?	7·7?	114·2	75·4	79·8	94·6	−4·5	6
116	95·2	24·2	49·0	9·9	3·7	31·5	39·4	47·9	36·1?	44·0	54·5	47·9	9·7	107·7	73·4	70·6	104·0	2·8	5
—	94·6	25·1	48·9	10·0	4·3	32·8	40·8	51·0	38·5	48·0	—	—	—	106·8	76·5	76·2	100·4	0·3	6
—	97·3	29·3?	45·8	6·3	2·5	31·1	42·0?	49·2	—	45·5	55·0?	48·6	10·3	106·3	77·1	71·3	108·2	5·9	6
—	—	—	49·0	9·1	3·7	—	—	39·3	38·3	—	—	—	—	114·2	78·7	78·4	100·4	0·3	6
—	—	—	48·3	9·9	5·4	32·3	40·8	—	—	45·5	—	—	—	104·8	79·4	77·1	103·0	2·3	6
—	95·6	24·0	49·3	13·2	4·9	32·2	41·6?	—	44·0	46·5	—	—	—	107·5	76·6	72·4	105·9	4·3	6
—	—	23·3	—	11·1	4·6	33·2	39·0	—	—	49·5	—	—	—	103·4	71·7	69·4	103·3	2·3	5
127·5?	95·3	25·2	48·0	—	—	29·3	—	47·1	41·1	48·0	55·5	50·0	10·4	106·8	76·5	72·0	106·3	4·5	6
121	91·1	28·1	55·3	10·3	4·3	34·9	40·2	55·8	38·1	46·5	55·0	49·3	9·6	108·6	76·0	71·7	106·0	4·3	6
119	—	25·9	—	—	—	31·1	—	—	41·2?	44·0	56·0	52·2	8·9	—	—	—	—	—	6
—	—	24·6	47·8	14·6	6·0	32·2	41·0	—	—	48·5?	—	—	—	97·9	77·6	—	—	—	6
—	93·0	25·3	48·5	10·3	7·8	32·1	40·2	—	44·3	43·5	—	—	—	109·4	70·9	76·2	93·1	−5·2	6
—	—	—	—	—	—	34·6	—	—	—	—	—	—	—	—	77·6	—	—	—	6
—	92·8	—	42·9	9·9	—	—	—	41·2	35·2	—	—	—	—	—	70·0	70·0	100·0	0	5
—	—	—	—	9·0	4·2	35·0	43·9	—	—	51·0	—	—	—	—	73·4	—	—	—	
113	86·4	23·6	47·2	9·2	4·1	31·8	39·0	44·0	36·1	43·0	58·5	52·3	10·6	99·8	74·6	69·8	106·8	4·7	5
116	88·2	25·0?	47·0	9·0	2·2	34·0	41·0	44·5	38·0	47·0	52·5	47·1	9·1	105·0	76·6	71·1	107·7	5·5	5
—	—	23·9	51·1	9·9	—	33·5	—	43·9	41·0?	—	—	—	—	104·6	72·3	71·8	100·8	0·6	5
—	—	24·2	46·1	9·0	3·5	31·1	38·2	—	37·9	—	—	—	—	109·7	75·3	—	—	—	
118	89·1	23·2	49·0	8·9	3·5	36·0	41·5	40·1?	—	48·5	—	—	—	104·0	76·9	68·9	111·7	8·1	6
—	92·0	—	48·9	9·0	3·7	37·0?	42·3	47·4	42·2	47·0	—	—	—	—	75·1	—	—	—	
124·5	88·0	25·9	51·1	12·2	5·6	36·0	42·8	51·2	37·3	47·5	53·0	48·5	8·7	114·0	74·9	74·3	100·8	0·6	5
—	—	—	—	10·6	5·9	—	42·0	—	—	—	—	—	—	—	75·6	74·0	102·2	1·7	
—	93·2	26·7	49·9	10·0	4·6	—	42·8	—	38·2	45·5	—	—	—	115·6	74·0	72·6	101·9	1·4	5
—	—	24·2	49·3	12·2	6·1	35·3	42·8	—	37·9	50·5	—	—	—	—	—	73·2	—	—	
—	99·0	—	52·0	12·3	4·9	33·0	41·7	49·9	—	51·5	—	—	—	115·1	74·9	69·5	107·8	5·4	6

nd Queried Female (Nos. 450–454) Crania from Tomb 120

$100\frac{B-H'}{L}$	*Oc.I.*	$100\frac{fmb}{fml}$	$100\frac{G'H}{GB}$	$100\frac{NB}{NH'}, L$	$100\frac{O_2}{O_1}, L$	$100\frac{G_2}{G_1}$	$100\frac{SS}{SC}$	$100\frac{Ml_2}{Ml_1}$	$100\frac{S}{C}(ml)$	*P∠*	*N∠*	*A∠*	*B∠*	No.	
5·1	58·6	72·1	69·1	—	81·4	—	—	73·6	24·5	—	67°·9	73°·3	38°·9	383	Occipito-mastoi L; denticle em
—	56·9	82·6	—	49·5	87·1	—	47·7	—	—	83°·5?	67°·3	72°·4	40°·3	384	Fronto-tempora
−1·4	59·3	76·9	74·4	50·5	79·3	—	34·8	—	—	84°·0	66°·7	71°·0	42°·4	385	Four molars los
0·5	62·0	79·8	81·3	44·2	84·8	74·5	57·6	77·3	23·7	83°·5	64°·6	70°·3	45°·1	386	Trace of suture
1·1	57·5	79·2	—	45·3	80·1	77·4	56·0	79·6	21·1	—	—	—	—	387	
1·4	61·0	77·5	69·0	49·4	80·3	—	40·8	77·4	22·6	86°·0	66°·9	74°·0	39°·1	388	Coronal beginni
1·1	57·8	77·7	64·7	53·6	85·4	88·6	38·4	—	—	90°·0?	65°·0	79°·6	35°·5	389	
−2·8	65·1	80·6	—	—	—	—	—	—	—	—	—	—	—	390	Palate missing
3·3	—	—	—	—	80·8	—	—	—	—	—	—	—	—	391	Palate missing
9·1	—	—	—	—	81·7	—	—	—	—	—	—	—	—	392	Coronal nearly
−1·1	63·3	81·3	77·7	46·4	76·6	96·1	45·0	80·8	20·2	90°·0	58°·4	81°·7	39°·9	393	M 3's absent
—	—	—	—	—	—	—	—	—	—	—	—	—	—	394	Four molars los
4·4	58·6	—	76·6	53·8	79·1	—	35·1	98·2	23·4	87°·5?	63°·6	72°·8	43°·7	395	Coronal closing
−0·8	57·5	—	69·4	43·3	93·4	82·0	35·4	82·5	23·0	85°·5	63°·1	74°·0	42°·9	396	M 3's absent
5·5	—	84·0	69·6	44·8	87·4	94·5	31·8	—	—	85°·5	62°·4	74°·7	42°·9	397	Metopic; large
5·0	—	—	—	46·7	—	81·6	28·3	—	—	84°·5	63°·2	69°·8	47°·0	398	
−6·9	65·1	82·6	—	—	85·0	—	—	—	—	—	58°·4	73°·9	47°·7	399	Palate defectiv
8·5	59·7	73·4	—	48·5	80·4	—	32·7	—	—	—	—	—	—	400	
5·2	61·5	81·3	—	40·9	82·7	—	52·2	86·3	22·3	—	—	—	—	401	Trace of horizo flected and ur
−1·2	64·9	80·9	74·9	—	87·3	90·9	63·0	—	—	81°·0	64°·6	72°·4	43°·0	402	Additional cusp
2·9	66·6	85·6	66·9	53·9	80·5	100·0	53·4	79·8	24·2	83°·0?	62°·9	76°·8	40°·2	403	Single epipteric
−1·1	57·8	70·3	—	—	72·7	—	—	—	26·1	—	—	—	—	404	Palate missing
1·7	62·2	73·0	75·3	46·0	83·0	85·2	36·8	91·4	21·3	89°·5	60°·1	75°·1	44°·8	405	Coronal and sa
2·5	60·3	81·4	75·5	50·4	75·2	—	—	—	—	83°·0	68°·1	71°·2	40°·6	406	
11·4	55·2	83·1	—	56·1	80·2	84·1	51·9	94·0	18·7	—	—	—	—	407	
4·5	56·0	85·6	—	51·1	78·2	86·9	28·7	—	—	—	—	—	—	408	Metopic; front
5·9	—	—	68·5	—	—	100·0	38·9	—	—	86°·5	61°·8	77°·5	40°·8	409	Young adult; bones at aste
—	67·7	—	—	—	—	—	—	—	—	—	—	—	—	410	Occipito-masto
0·3	64·0	—	74·0	51·1	81·2	—	41·2	82·4	22·9	88°·0	57°·8	78°·1	44°·1	411	Coronal and sa
4·9	60·8	82·5	—	—	—	—	53·8	—	—	—	—	—	—	412	Metopic; palat
6·5	57·6	87·9	83·5	—	—	—	42·1	—	—	89°·0?	56°·6	73°·0	50°·5	413	Contracted pal
—	58·8	—	64·1	53·4	81·1	—	48·7	—	—	—	—	—	—	414	One incisor an
1·1	—	—	74·2	57·0	—	—	39·4	—	—	86°·5	64°·7	76°·4	39°·0	415	One molar lost
—	57·6	—	—	52·8	78·4	—	45·6	—	—	—	—	—	—	416	
−3·6	59·8	77·6	—	—	—	86·6	40·6	—	—	85°·5	61°·1	77°·1	41°·7	417	
−2·5	—	—	—	51·6	—	—	28·7	—	—	83°·0	63°·6	73°·7	42°·8	418	Single epipteri
0·3	59·9	78·3	—	58·5	82·4	94·1	35·9	—	—	85°·5	62°·4	75°·8	41°·9	419	Metopic; trace
−0·6	65·4	78·9	73·1	—	81·0	—	—	—	—	86°·0	62°·2	75°·8	42°·0	420	Coronal begin
—	—	—	—	—	—	88·7	—	—	—	—	—	—	—	421	
−0·3	58·4	81·9	—	44·9	82·3	—	45·8	—	—	—	—	—	—	422	Palate defecti
4·8	59·9	89·7	—	54·6	78·5	—	61·8	95·1	19·3	—	—	—	—	423	Single epipteri
1·1	61·6	—	—	52·9	—	—	33·8	—	—	90°·0	61°·8	78°·0	40°·2	424	
−0·9	—	—	—	50·7	84·7	80·1	—	—	—	—	—	—	—	425	Young adult;
2·4	63·6	82·6	78·9	49·0	85·6	—	39·1	96·9	20·6	—	68°·0	68°·0	43°·9	426	One molar los
—	—	—	—	51·9	82·7	—	44·0	—	—	—	—	—	—	427	Occipital bone
−4·5	60·5	83·1	—	52·8	76·9	86·7	—	90·5	17·9	—	—	—	—	428	
2·8	57·3	73·2	66·4	49·4	79·9	75·4	37·4	80·7	20·3	78°·5	72°·4	68°·7	39°·0	429	Coronal begin
0·3	60·9	85·3	71·7	51·3	80·4	75·5	43·0	—	—	77°·0	70°·0	67°·9	42°·1	430	Pterion in *K*,
5·9	60·2	79·2	65·0	64·0	74·0	—	39·7	82·7	21·2	—	71°·2	70°·2	38°·6	431	Ons molar los
0·3	61·4	77·4	—	—	—	97·5	40·7	—	—	84°·5?	61°·8	74°·8	43°·5	432	
2·3	60·1	93·7	—	—	79·2	—	54·5	—	—	86°·5	64°·6	75°·7	39°·8	433	Single epipter
4·3	61·7	—	75·1	48·7	77·4	—	37·1	—	—	78°·5	70°·1	66°·3	43°·6	434	Coronal closin
2·3	58·6	67·2	—	—	85·1	—	41·4	—	—	—	—	—	—	435	Sagittal begi defective
4·5	62·6	70·7	67·1	52·5	—	87·3	—	86·5	20·8	—	65°·8	75°·5	38°·7	436	Two small pr
4·3	61·6	78·9	79·4	50·8	86·8	68·3	41·7	84·5	19·5	78°·5	71°·0	68°·0	41°·0	437	One molar los nasal bones, alveolar mar
—	60·2	—	—	—	—	—	—	78·6	17·0	—	—	—	—	438	Two incisors
—	61·0	—	—	51·5	78·5	—	41·1	—	—	—	—	—	—	439	Palate defect
−5·2	66·6	74·5	—	52·2	79·9	—	75·7	—	—	—	—	—	—	440	
—	63·6	81·2	—	—	—	—	—	—	—	—	—	—	—	441	Palate defect
0	57·2	—	64·1	—	—	85·4	—	—	—	—	63°·2	77°·9	38°·9	442	Wound parie
—	—	—	—	—	79·7	—	46·7	—	—	—	—	—	—	443	Palate defect
4·7	58·0	85·3	—	50·0	81·5	82·0	44·6	73·5	20·3	—	—	—	—	444	
5·5	58·9	82·1	72·4	53·2	82·9	85·4	24·4	89·5	19·3	87°·0	66°·2	73°·2	40°·6	445	Incisor, cani flected to R
0·6	57·2	82·4	—	46·8	—	93·4	—	—	—	—	64°·0	69°·2	46°·8	446	
—	—	—	—	52·5	81·4	—	38·9	—	—	—	—	—	—	447	Several large
8·1	61·0	79·2	—	47·3	86·7	—	39·3	—	—	—	—	—	—	448	Sagittal begi
—	—	—	72·7	—	87·5	89·0	41·1	—	—	83°·5	—	—	—	449	
0·6	59·9	80·3	80·7	50·7	84·1	72·9	45·9	89·6	17·9	—	66°·4	70°·9	42°·8	450	
1·7	—	—	—	—	—	—	55·7	—	—	—	—	—	—	451	Palate missi frontal bon
1·4	59·7	81·4	67·7	53·5	—	—	46·0	—	—	86°·5	62°·8	75°·2	37°·0	452	
—	—	—	—	49·1	84·8	—	50·0	—	—	—	—	—	—	453	Young adult
5·4	61·2	90·0	73·9	—	79·1	—	39·8	—	—	83°·5	67°·7	71°·3	41°·0	454	Single epipte

	Remarks
	Occipito-mastoid obliterated; parieto-mastoid obliterated L, but parieto-squamous open L; denticle embedded in palate
	Fronto-temporal articulation L
	Four molars lost before death
	Trace of suture between ex- and supra-occipital L
	—
	Coronal beginning to close
	—
	Palate missing
	Palate missing
	Coronal nearly obliterated, sagittal closed
	M 3's absent
	Four molars lost before death; fused nasal bones
	Coronal closing, sagittal beginning to close; palate defective
	M 3's absent
	Metopic; large wormian bone at asterion L; 1 premolar lost before death
	—
	Palate defective; epipteric bone L; additional cusp on canine L
	—
	Trace of horizontal suture of interparietal bones at asteria; small palate; canine L deflected and unerupted, obstructing incisors; pre-condyle R
	Additional cusp on M 3, R
	Single epipteric bone L, and 2 R
	Palate missing
	Coronal and sagittal closed; additional cusp on M 3, R
	—
	—
	Metopic; front teeth crowded; M 3's probably unerupted, but possibly lost before death
	Young adult; M 3's not completely erupted; trace of horizontal suture of interparietal bones at asteria; 2 epipteric bones R
	Occipito-mastoid L obliterated; palate missing
	Coronal and sagittal closed; fused nasal bones
	Metopic; palate missing
	Contracted palate; 1 molar lost before death
	One incisor and 4 molars lost before death
	One molar lost before death; M 3's probably absent
	—
	—
	Single epipteric bone R and L; palate defective
	Metopic; trace of sutures between ex- and supra-occipitals; healed injury frontal R
	Coronal beginning to close; 4 molars lost before death
	—
	Palate defective
	Single epipteric bones R and L; 1 molar lost before death
	—
	Young adult; single epipteric bone R; M 3's not fully erupted
	One molar lost before death
	Occipital bone missing; 1 premolar and 1 molar lost before death
	—
	Coronal beginning to close
	Pterion in $\frac{K}{}$, R and L
	One molar lost before death
	—
	Single epipteric bone R; palate defective
	Coronal closing, sagittal closed
	Sagittal beginning to close; trace of sutures between ex- and supra-occipitals; palate defective
	Two small pre-condyles
	One molar lost before death, diastema between lateral incisor and canine R and L; fused nasal bones. Mandible: no teeth lost before death, 5 dental rudiments round outer alveolar margin, malocclusion of jaws, teeth markedly worn
	Two incisors lost before death
	Palate defective, nearly all teeth lost before death
	—
	Palate defective
	Wound parietal R
	Palate defective
	—
	Incisor, canine, or premolar missing R, and no space for it; intermaxillary region deflected to R
	—
	Several large wormian bones in lambdoid
	Sagittal beginning to close; 1 incisor lost before death
	—
	—
	Palate missing; possibly artificially deformed to some extent, suggestion of flattening of frontal bone, occipital normal, no post-coronal depression
	—
	Young adult; M 3's absent
	Single epipteric bone L; 1 molar lost before death; severe healed wound on frontal bone

Tomb no.	Sex	No.	*L*	*B*	*B′*	*H′*	S_1'	S_2'	S_3'	S_1	S_2	S_3	*S*	*βQ′*	*U*	*fml*	*fmb*
120	♀?	455	182·5	130	94·2	132·5	120·1	118·3	89·1	139	—	108	—	301	507	38·9	32·3
,,	,,	456	184·5	144	96·6	130·5	115·7	118·8	96·0	133·5	135	117·5	386	317	536	34·2	27·9
,,	,,	457	177·5	144·5	83·0	—	113·1	115·1	92·3	128·5	130	111	370	311	512	—	29·1
,,	,,	458	174·5	141·5	91·3	126	106·1	113·1	98·8	120	129	118·5	368	297	502	36·1	32·2
,,	,,	459	188	139	97·6	129	107·9	109·4	108·6	123	117	136	376	320	532	38·0	35·0
,,	,,	460	—	128·5	98·0	114·5·	—	105·1	89·2	—	118	108·5	—	291	498	31·1	26·9
,,	,,	461	174·5	133·5	101·1	134	109·1	117·3	91·1	129	133·5	107	370	—	508?	36·8	31·1
,,	,,	462	178·5	132	91·1	130	108·0	122·2?	94·9	126·5	139·5?	117	384?	294	506	32·6	27·0
,,	,,	463	178	125	96·1	132·5?	105·0	112·9	95·0	120·5	125·5	112·5	359	299	496	35·9?	25·0
,,	,,	464	180	141	92·6	131·5	113·1	120·9	90·6	131·5	138·5	110	380	312	517	38·2	32·0
,,	,,	465	180	137·5	87·8	127·5	108·5	115·7	97·0	121	131	119	371	—	506	34·1	29·8
,,	,,	466	183·5	134·5	96·2	135·5	113·6	116·2	—	130·5	129	—	—	304	515	—	30·3
,,	,,	467	183·5	134·5	96·9	138·5	110·2	115·3	102·9	131	125·5	127	383?	316	522	39·1	30·9
,,	,,	468	177	133·5	83·2	—	107·0	111·5	107·9	123	123·5	136	383	304	502	—	32·5
,,	,,	469	182·5	—	—	136·5	110·0	115·0	103·0	123	128·5	120·5	372	—	—	38·9	29·7
,,	,,	470	186	142·5	96·1	127·5	110·9	117·0	99·6	126	130·5	119	375	300	523	36·2	31·1
,,	,,	471	185·5	134	99·5	131	112·1	115·9	92·0	128·5	129	114	371	304	515	36·0	28·2
,,	,,	472	185	134·5	95·8	131	110·0	114·0	95·0	124·5	127	120	371	297	517	36·1	29·0
,,	,,	473	174	138·5	88·5	126	107·2	111·1	97·9	120	125	113·5	358	292	499	35·1	26·8
,,	,,	474	169·5	136·5	94·3	131	109·0	113·6	94·1	123	130·5	110·5?	364?	311	491	35·1	29·1
,,	,,	475	173	141	92·9	—	105·9	104·1	95·7	117	114·5	117·5	350	300	501	—	30·0
,,	,,	476	180	140·5	92·8	133·5	111·8	111·1	96·1	127	121·5	121	369	316	510	34·6	29·0
107	♀	477	171	137	92·0	—	106·8	114·9?	82·0?	125·5	131·5?	95·5	353	308	491	—	—
,,	,,	478	169·5	130	—	127	105·9	—	—	117·5	—	—	355	297	—	36·1	—
,,	,,	479	179·5	131	89·2	—	112·3	116·2	95·8	128	129·5	118	376	294	503	—	29·2
,,	,,	480	180	—	95·3	133	112·0	109·6?	104·9	126	120·5?	132	378?	—	—	35·0	31·0
,,	,,	481	176·5	133	96·4	—	104·0	102·4	95·9	120·5	111	117·5	349	293	500?	—	—
,,	,,	482	176·5	129·5	94·2	125·5?	105·4	111·9	89·9	121	124·5	110·5	355	290	495	—	29·3
,,	,,	483	179·5	—	94·0	132	108·9	115·3	94·2	127	129·5	114·5	371	—	—	38·0	31·6
,,	,,	484	180	131·5	87·2	—	—	—	93·2	—	—	115·5	—	—	499	34·0	25·2
,,	,,	485	177	130	88·0	126	106·0	111·8	96·9	117·5	126	113·5	357	295	493	38·1	30·9
,,	,,	486	175	135	—	128·5	104·5?	119·1	94·9	119?	134	112	365	293	—	37·7	27·3
,,	,,	487	167·5	135	85·3	126·5	107·9	99·4	100·0	123	115·5	118	356	290	481	33·9	30·2
,,	,,	488	181·5	132·5	89·6	—	116·6	115·1	95·0	135	130	111·5	376	300	507	—	—
,,	,,	489	172	—	89·9	129·5	108·9	111·9	—	123·5	122	—	—	—	—	—	29·9
,,	,,	490	188·5	136	97·2	—	114·2	109·0	—	131·5	117·5	—	—	301	524	—	—
,,	,,	491	186	133	99·1	136·5	111·0	125·1	100·8	127·5	137	127	392	309	518	32·0	26·6
,,	♀?	492	178	129	91·1	—	109·1	114·3?	—	123	127·5?	—	—	—	—	—	29·0
,,	,,	493	177·5	135	—	132·5	106·3	117·1	98·0	122·5	130·5	120	374	301	—	32·8	29·0
,,	,,	494	186·5	131	93·3	—	109·5	116·0?	101·0?	124	127·5?	123?	375	296	516	—	—
,,	,,	495	179	135	101·0?	126·5	109·6?	111·3?	92·2	125?	124?	116·5	366	302	506	36·1	—
,,	,,	496	177·5	138·5	—	130·5	105·2	115·9	93·0	120·5	131	109	361	314?	—	34·6	29·0
,,	,,	497	176	133	89·0	134·5	107·9	114·9	96·3	119	—	114·5	—	295?	493	36·0	31·0
,,	,,	498	183·5	131	90·0	134·5	113·0	—	—	130	—	—	361	300	503	37·0	30·6
116	♀	499	179·5	140·5	96·6	117·5	107·3	109·0	94·0	121·5	119·5	117	358	294	518	37·1	27·5
,,	,,	500	176·5	138·5	84·0	120·5	108·2	111·0	95·2	124·5	125·5	124·5	374	299	503	36·1	27·1
,,	,,	501	180	132·5	87·7	128·5	110·1	121·1	98·0	125·5	135·5	115·5	376	296	504	36·1	26·6
,,	,,	502	176·5	124·5	84·9	134·5	101·9	117·2	—	116	133·5	—	—	289	480	—	—
,,	,,	503	177	131·5	95·9	130	109·5	117·3	90·2	127	132	108	367	312	500	39·2	29·0
,,	,,	504	172	136	95·2	130	107·2	104·4	94·4	124·5	115·5	113·5	353	301	490	36·3	27·8
,,	,,	505	171·5	135	89·5	120	104·9	106·0	91·2	120	118	116·5	354	285	496	34·5	30·0
,,	,,	506	178	140·5	88·0	129·5	105·1	119·6	96·0	121	134	116	371	306	504	35·2	31·1
,,	,,	507	171·5	133	95·8	117·5	99·9	108·2	89·2	114·5	122·5	103·5	341	284	493	38·2	30·6
,,	,,	508	173	127	96·0	129	103·0	109·1	97·1	116·5	120	117·5	355	287	485	36·9	26·8
,,	,,	509	179	132	94·0	133·5	109·1	114·3	96·9	127	128	115·5	371	306	501	36·0	31·2
,,	,,	510	174	136·5	90·0	126·5?	112·0?	102·7?	98·4	128	114	118	360	293	—	36·0	—
,,	♀?	511	179·5	130·5	87·2	129	107·0	115·2	94·8	121·5	133	115·5	370	292	499	36·5	30·0
,,	,,	512	181·5	141·5	94·1	130·5	114·8	117·0	—	131	133	—	—	308	521	—	—
,,	,,	513	168·5	131	87·0	126	107·1	104·0	86·9	121·5	114·5	102·5	338	284	474	32·9	27·7
,,	,,	514	181	145	92·2	—	117·0	112·2	102·3	135	124·5	128	388	324	518	—	—
,,	,,	515	170·5	129·5	92·6	126	105·6	107·1	93·2	123	117·5	115	355	—	—	32·2	25·2

I. *Individual Measurements of Lachish Skulls.* VII. *Queried Female Crania (cont.) from Tomb* 120*; Female a*

fml	*fmb*	*LB*	*GL*	*G'H*	*J*	*GB*	*NB*	*NH,L*	*SC*	*SS*	O_2L	O_1L	G_1'	G_2	Ml_2	Ml_1	*C(ml)*	*S(ml)*	*OH*	IC
38·9	32·3	96·2	95·1	62·2	119	90·5	23·0	46·9	8·9	3·7	30·3	40·3	45·3	—	46·0	60·0	53·0	11·3	115·3	7
34·2	27·9	97·6	88·2	68·9	—	—	24·0	51·7	10·9	5·4?	35·4?	—	43·2	—	—	—	—	—	112·6	7
—	29·1	—	—	—	—	—	—	49·2	8·0	4·4	35·1	37·7	—	—	47·5	—	—	—	114·9	8
36·1	32·2	94·9	83·6	65·1	126	94·9	24·6	49·0	10·5	5·1	34·0	41·6	40·6	41·0	44·5	60·5	52·9	11·0	109·5	8
38·0	35·0	103·8	94·0	68·0	—	—	25·1?	52·1	8·3?	—	33·8?	—	—	—	—	—	—	—	118·4	7
31·1	26·9	—	92·1?	—	119·5	99·0	—	—	12·3	—	34·2	41·2	43·0	—	46·0	59·5	51·1	13·1	—	
36·8	31·1	98·0	93·1	73·0	—	97·8	—	53·6	10·1	3·8	33·0?	42·5	49·0	39·6	50·5	—	—	—	—	7
32·6	27·0	92·8	93·1	67·4	121·5	95·3	24·6	50·0	9·0	4·0	32·9	38·9	44·0	38·0	44·0	53·0	45·6	11·0	109·0	7
35·9?	25·0	102·2?	—	—	117	90·1	24·5	48·1	10·2	4·8	30·4	39·9	46·0	41·2	45·0	56·5	50·0	10·4	110·8	7
38·2	32·0	97·2	91·0	63·0?	123	86·2	—	50·0	11·8	4·3	34·1	40·1	48·4	—	49·5	56·5	50·6	10·3	116·6	7
34·1	29·8	94·4	91·9	64·2?	—	—	—	44·1	10·5	5·9	—	41·3?	44·2	39·6	—	—	—	—	—	7
—	30·3	98·2	—	—	—	—	—	—	11·6	6·9	34·5	39·1	—	—	53·5	62·0	53·9	11·1	—	7
39·1	30·9	98·0	85·8	67·4	—	—	—	51·6	13·9	4·1	36·1	41·0	—	—	51·5	54·0	48·2	10·1	118·3	7
—	32·5	—	—	71·0	124·5	89·0	22·0	49·9	9·9	5·1	29·8	36·5	48·2	—	45·0	51·5	47·0	10·9	114·7	7
38·9	29·7	99·0	88·9	69·2	—	—	24·1	—	11·9	5·9	33·1	42·1	43·5	—	52·5?	—	—	—	—	
36·2	31·1	101·8	98·6	69·9	—	—	25·3	49·9	9·0	3·0	—	—	48·0	—	—	—	—	—	107·3	7
36·0	28·2	104·0	90·2	65·2	122	99·0	27·8	47·1	12·1	5·1	33·9	43·8	45·1	44·1	51·5	60·0	51·3	13·3	112·6	7
36·1	29·0	103·1	93·3	74·8?	126·5	100·0	—	54·0	11·2	4·8	34·8	42·8	49·0?	42·9	47·0	56·0	50·7	9·1	109·9	7
35·1	26·8	90·6	89·2	64·7	—	—	23·2	47·1	8·0	2·6	31·0	40·7	45·1	40·1	45·0	—	—	—	109·6	7
35·1	29·1	92·2	—	—	123·5	91·0	—	—	—	—	32·0	40·7	—	—	46·5	64·0?	56·4?	12·0?	—	8
—	30·0	—	—	74·0	—	—	24·2	53·3	11·0	5·1	—	—	42·1	41·6	—	—	—	—	107·3	8
34·6	29·0	98·3	—	—	—	—	24·2	49·7	—	—	35·8	42·9	48·0?	40·0?	51·5	—	—	—	116·8	7
—	—	—	—	62·0	—	—	27·7?	—	9·9	—	—	—	40·4?	—	—	—	—	—	—	8
36·1	—	90·9	—	—	—	—	—	—	—	—	—	—	—	—	43·5	—	—	—	—	7
—	29·3	—	—	64·0	—	94·9	—	49·0	—	—	34·1	42·5	—	39·2	46·0	57·0	51·2	10·4	113·1	7
35·0	31·0	96·3	—	—	—	95·9	23·0	49·4	8·9	4·7	30·1	40·0	—	41·8	49·0	57·0	49·2	11·1	—	
—	—	—	—	—	125·5	94·2	24·9	49·3	8·7	—	31·8	40·9	46·0	39·0	49·0	56·5	51·1	8·7	105·1	7
—	29·3	—	—	64·3	—	96·9	—	50·0	11·2	5·2	32·1	40·0	48·1	—	45·0	—	—	—	107·8	
38·0	31·6	97·1	92·2	68·1	—	91·4	25·8	49·0	8·5	3·8	36·0	43·9	45·0	—	46·5	61·0	57·2	10·0	—	
34·0	25·2	92·9	88·0	64·9	—	—	—	46·1	—	—	—	—	43·1	42·5	43·5	—	—	—	—	
38·1	30·9	97·1	94·4	72·4	124·5	96·5	25·2	54·6	11·0	5·5	34·0	41·7	47·3?	40·0	45·5	56·5	49·1	11·1	109·1	
37·7	27·3	94·9	—	—	—	—	25·1	47·6	11·1	4·1	—	—	—	—	—	—	—	—	—	
33·9	30·2	89·1	82·1	61·2	—	87·2	—	45·0	8·2	3·2	34·0	39·5	40·1	39·9	46·5	58·0	50·9	11·0	104·2	
—	—	—	—	68·7	—	93·2	24·8?	47·9	9·1	4·6	37·0	41·5	44·2	42·1	52·5	63·0	53·9	14·0	109·7	
—	29·9	97·1	—	—	—	—	—	—	8·2	6·2	35·2	40·3	—	—	46·5	54·0	46·2	11·2	—	
—	—	—	—	73·0	—	88·6	25·1	48·6	10·1	3·5	31·5	41·2	45·0	—	47·5	—	—	—	113·6	
32·0	26·0	102·2	—	—	—	—	—	—	—	—	33·0	41·9	—	—	—	—	—	—	—	
—	29·0	—	—	—	—	—	—	48·6	10·2	6·0	—	43·8?	—	—	—	—	—	—	—	
32·8	29·0	97·1	93·0	69·7?	—	—	—	53·3?	—	—	36·9?	—	—	41·2	—	—	—	—	—	
—	—	—	—	61·9	—	—	25·0	48·0	7·5	3·5	—	—	—	—	—	—	—	—	—	
36·1	—	95·2	91·2	64·0	125	89·9	25·1	47·1	—	—	31·6	42·6	—	—	46·0	—	—	—	—	
34·6	29·0	101·7	—	—	—	—	25·8	48·0	13·7	6·4	—	—	45·6	38·7	—	—	—	—	—	
36·0	31·0	98·5	89·1?	68·2?	125·5	95·1	23·2	48·0	5·6?	—	32·5	41·0	45·7?	39·1	51·5	65·5	56·2	12·6	—	
37·0	30·6	104·7	91·1	74·1	127·5	90·2	22·2?	53·6	11·1	4·3	35·8	41·2	43·4	42·1	47·0	55·0	50·6	8·1	—	
37·1	27·5	91·9	89·1	66·9	—	—	24·2	50·9	12·1	5·8	36·8	41·6	43·2	—	53·5	53·5?	48·2?	10·1?	104·3	
36·1	27·1	82·1	76·7	71·2	—	90·2	—	51·6	7·1	3·5	33·0	36·7	41·1	37·0	45·5	50·5?	44·2?	9·8?	108·9	
36·1	26·6	91·7	92·0	61·4	—	—	—	44·2	7·0	2·1	29·0	38·2	45·7?	37·1	46·5	43·5	39·9	7·3	111·9	
—	—	99·2	91·3	60·0	—	84·3	21·5	43·8	9·1	4·4	31·0	38·8	41·0	—	45·0	—	—	—	—	
39·2	29·0	97·2	93·2	63·0	117	90·2	22·0	48·3	8·4	3·0	33·0	38·1	48·8	40·8	44·5	54·5	48·1	11·8	114·4	
36·3	27·8	94·0	—	—	—	—	22·3	43·8	9·3	4·0	—	—	43·1	40·0	—	—	—	—	109·3	
34·5	30·0	89·1	—	—	—	91·3	26·1	50·1	—	—	32·1	—	—	—	—	—	—	—	—	
35·2	31·1	94·2	90·0	67·0	—	94·2	25·2	46·2	10·3	5·2	30·3	38·1	42·2	—	45·0	59·0	53·3	10·8	—	
38·2	30·6	93·3	—	—	—	—	—	51·2	10·0	4·9	33·0	41·2	—	—	48·5	62·0	54·4	13·3	—	
36·9	26·8	95·3	93·3	64·9	123·5	89·4	23·2	47·8	9·9	3·1	35·0	40·2	47·1?	—	45·0	52·5	46·9	9·2	107·2	
36·0	31·2	97·1	89·8	61·2	—	84·2	21·0	43·0	10·3	5·2	29·1	39·8	41·0	34·1	43·0	52·5	46·5	10·1	116·2	
36·0	—	90·3?	94·7?	64·1?	—	—	—	44·9?	—	—	30·1?	—	—	40·0?	—	—	—	—	—	
36·5	30·0	95·3	—	—	124·5?	92·3	—	45·7	11·1	5·1	35·1	40·2	46·0	—	53·0	54·0	48·9	9·4	107·3	
—	—	93·1	85·9	73·0	—	—	—	52·1	8·9	4·4	35·1	40·1	41·7	39·7	45·5	—	—	—	112·5	
32·9	27·7	96·1	—	—	—	—	—	49·1?	8·8	5·9	30·1	39·0	—	—	45·0	—	—	—	102·6	
—	—	—	—	74·0	—	87·0	24·2	52·8	8·2	4·6	35·1	40·4	44·0	—	51·0	—	—	—	—	
32·2	25·2	94·1	88·3	64·9	—	—	26·9	49·8	11·9	5·0	32·9	38·0	—	—	41·5	51·5	45·0	10·8	107·2	

20; *Female and Queried Female from Tombs* 107, 116 *and* 108; *and Anomalous Crania and Calvariae from Tomb* 12

$\delta(ml)$	OH	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	$Oc.I.$	$100\frac{fmb}{fml}$	$100\frac{G'H}{GB}$	$100\frac{NB}{NH}, L$	$100\frac{O_2}{O_1}, L$	$100\frac{G_2}{G_1'}$	$100\frac{SS}{SC}$	$100\frac{Ml_2}{Ml_1}$	$100\frac{S}{C}(ml)$
11·3	115·3	71·2	72·6	98·1	−1·4	59·2	83·0	68·7	49·0	75·2	—	41·6	76·7	21·3
—	112·6	78·0	70·7	110·3	7·3	58·4	81·6	—	46·4	—	—	49·5	—	—
—	114·9	81·4	—	—	—	59·8	—	—	—	93·1	—	55·0	—	—
11·0	109·5	81·1	72·2	112·3	8·9	60·1	89·2	68·6	50·2	81·7	101·0	48·6	73·6	20·8
—	118·4	73·9	68·6	107·8	5·3	57·0	92·1	—	48·2	—	—	—	—	—
13·1	—	—	—	112·2	—	58·9	86·5	—	—	83·0	—	—	77·3	25·6
—	—	76·5	76·8	99·6	−0·3	62·1	84·5	74·6	—	77·6	80·8	37·6	—	—
11·0	109·0	73·9	72·8	101·5	1·1	57·9	82·8	70·7	49·2	84·6	86·4	44·4	83·0	24·1
10·4	110·8	70·2	74·4	94·3	−4·2	61·3	69·6	—	50·9	76·2	89·6	47·1	79·6	20·8
10·3	116·6	78·3	73·1	107·2	5·3	59·0	83·8	73·1	—	85·0	—	36·4	87·6	20·4
—	—	76·4	70·8	107·8	5·6	58·2	87·4	—	—	—	89·6	56·2	—	—
11·1	—	73·3	73·8	99·3	−0·5	—	—	—	—	88·2	—	59·5	86·3	20·6
10·1	118·3	73·3	75·5	97·1	−2·2	57·8	79·0	—	—	88·0	—	29·5	95·4	21·0
10·9	114·7	75·4	—	—	—	56·6	—	79·8	44·1	81·6	—	51·5	87·4	23·2
—	—	—	74·8	—	—	62·7	76·3	—	—	78·6	—	49·6	—	—
—	107·3	76·6	68·5	111·8	8·1	60·4	85·9	—	50·7	—	—	33·3	—	—
13·3	112·6	72·2	70·6	102·3	1·6	57·6	78·3	65·9	59·0	77·4	97·8	42·1	85·8	25·9
9·1	109·9	72·7	70·8	102·7	1·9	56·5	80·3	74·8	—	81·3	87·6	42·9	83·9	17·9
—	109·6	79·6	72·4	109·9	7·2	63·9	76·4	—	49·3	76·2	88·9	32·5	—	—
12·0?	—	80·5	77·3	104·2	3·2	62·2	82·9	—	—	78·6	—	—	72·7	21·3
—	107·3	81·5	—	—	—	58·2	—	—	45·4	—	98·8	46·4	—	—
—	116·8	78·1	74·2	105·2	3·9	56·7	83·8	—	48·7	83·4	83·3	—	—	—
—	—	80·1	—	—	—	63·2	—	—	—	—	—	—	—	—
—	—	76·7	74·9	102·4	1·8	—	—	—	—	—	—	—	—	—
10·4	113·1	73·0	—	—	—	57·9	—	67·4	—	80·2	—	—	80·7	20·3
11·1	—	—	73·9	—	—	56·7	88·6	—	46·6	75·3	—	52·8	86·0	22·6
8·7	105·1	75·4	—	—	—	58·4	—	—	50·5	77·8	84·8	—	86·7	17·0
—	107·8	73·4	71·1	103·2	2·3	58·1	—	66·4	—	80·3	—	46·4	—	—
10·0	—	—	73·5	—	—	59·0	83·2	74·5	52·7	82·0	—	44·7	76·2	17·5
—	—	73·1	—	—	—	57·6	74·1	—	—	—	98·6	—	—	—
11·1	109·1	73·4	71·2	103·2	2·3	62·6	81·1	75·0	46·2	81·5	84·6	50·0	80·5	22·6
—	—	77·1	73·4	105·1	3·7	61·7	72·4	—	52·7	—	—	36·9	—	—
11·0	104·2	80·6	75·5	106·7	5·1	61·7	89·1	70·2	—	86·1	99·5	39·0	80·2	21·6
14·0	109·7	73·0	—	—	—	62·2	—	73·7	51·8	89·2	95·2	50·5	83·3	26·0
11·2	—	—	75·3	—	—	—	—	—	—	87·3	—	75·6	86·1	24·2
—	113·6	72·1	—	—	—	—	—	82·4	51·6	76·5	—	34·7	—	—
—	—	71·5	73·4	97·4	−1·9	56·6	81·3	—	—	78·8	—	—	—	—
—	—	72·5	—	—	—	—	—	—	—	—	—	58·8	—	—
—	—	76·1	74·6	101·9	1·4	58·4	88·4	—	—	—	—	—	—	—
—	—	70·2	—	—	—	58·8	—	—	52·1	—	—	46·7	—	—
—	—	75·4	70·7	106·7	4·7	56·5	—	71·2	53·3	74·2	—	—	—	—
—	—	78·0	73·5	106·1	4·5	62·4	83·8	—	53·7	—	84·9	46·7	—	—
12·6	—	75·6	76·4	98·9	−0·9	60·9	86·1	71·7	48·3	79·3	85·6	—	78·6	22·4
8·1	—	71·4	73·3	97·4	−1·9	—	82·7	82·2	41·4	86·9	97·0	38·7	85·5	16·0
10·1?	104·3	78·3	65·5	119·6	12·8	57·3	74·1	—	47·5	88·5	—	47·9	100·0	21·0
9·8?	108·9	78·5	68·3	114·9	10·2	55·0	75·1	78·9	—	89·9	90·0	49·3	90·1	22·2
7·3	111·9	73·6	71·4	103·1	2·2	61·8	73·7	—	—	75·9	81·2	30·0	106·9	18·3
—	—	70·5	76·2	92·6	−5·7	—	—	71·2	49·1	79·9	—	48·4	—	—
11·8	114·4	74·3	73·4	101·2	0·8	60·2	74·0	69·8	45·5	86·6	83·6	35·7	81·7	24·5
—	109·3	79·0	75·6	104·6	3·4	59·9	76·6	—	50·9	—	92·8	43·0	—	—
—	—	78·7	70·0	112·5	8·7	56·0	87·0	—	52·1	—	—	—	—	—
10·8	—	78·9	72·8	108·5	6·2	59·4	88·4	71·1	54·5	79·5	—	50·5	76·3	20·3
13·3	—	77·6	68·5	113·2	9·0	63·8	80·1	—	—	80·1	—	49·0	78·2	24·4
9·2	107·2	73·4	74·6	98·4	−1·2	59·3	72·6	72·6	48·5	87·1	—	31·3	85·7	19·6
10·1	116·2	73·7	74·6	98·9	−0·8	60·6	86·7	72·7	48·8	73·1	83·2	50·5	81·9	21·7
—	—	78·4	72·7	107·9	5·7	60·1	—	—	—	—	—	—	—	—
9·4	107·3	72·7	71·9	101·2	0·8	58·8	82·2	—	—	87·3	—	45·9	98·1	19·2
—	112·5	78·0	71·9	108·4	6·1	—	—	—	—	87·5	95·2	49·4	—	—
—	102·6	77·7	74·8	104·0	3·0	61·7	84·2	—	—	77·2	—	67·0	—	—
—	—	80·1	—	—	—	57·1	—	85·1	45·8	86·9	—	56·1	—	—
10·8	107·2	76·0	73·9	102·8	2·1	57·8	78·3	—	54·0	86·6	—	42·0	80·6	24·0

$100\frac{SS}{SC}$	$100\frac{Ml_2}{Ml_1}$	$100\frac{S}{O}(ml)$	$P\angle$	$N\angle$	$A\angle$	$B\angle$	No.	Remarks
41·6	76·7	21·3	82°·5	70°·0	72°·2	37°·8	455	—
49·5	—	—	88°·0	61°·1	75°·8	43°·2	456	Coronal and sagittal beginning to close; 2 molars and 1 premolar lost before dea markedly worn
55·0	—	—	—	—	—	—	457	Coronal and lambdoid beginning to close, sagittal closing; palate defective
48·6	73·6	20·8	89°·0	59°·6	78°·3	42°·1	458	Exostosis parietal R
—	—	—	91°·0	62°·1	78°·1	39°·8	459	All sutures beginning to close; several teeth lost before death
—	77·3	25·6	—	—	—	—	460	Coronal partly obliterated, sagittal obliterated; several teeth lost before dea condyle L; small exostosis frontal R
37·6	—	—	—	63°·9	71°·3	44°·8	461	Metopic; *os triangulare* R separate
44·4	83·0	24·1	79°·0	68°·9	68°·7	42°·5	462	Metopic; single epipteric bone L; M 3's absent; several wormian bones in sagitt lambda
47·1	79·6	20·8	—	—	—	—	463	Metopic; M 3's absent
36·4	87·6	20·4	87°·5?	65°·1	76°·0	38°·9	464	Coronal closing, sagittal beginning to close; 1 incisor and M 3, L (?) lost before
56·2	—	—	—	67°·7	72°·1	40°·2	465	Trace of horizontal suture of interparietal bones R; single epipteric bones R and
59·5	86·3	20·6	—	—	—	—	466	Sagittal closed, lambdoid closing; 2 epipteric bones R; palate missing
29·5	95·4	21·0	—	58°·9	78°·8	42°·3	467	Metopic; single large epipteric bones R and L; nearly all teeth lost before dea cavity in anterior palate, exposing nasal aperture
51·5	87·4	23·2	85°·5	—	—	—	468	Sagittal beginning to close; 4 molars lost before death; traumatic depression pa
49·6	—	—	85°·0	60°·6	76°·7	42°·8	469	Sagittal beginning to close; 2 epipteric bones R; all molars lost before death, larg cavity at site of M 3, R
33·3	—	—	84°·5	67°·1	72°·1	40°·8	470	Coronal closed and obliterated at pterion R, sagittal nearly obliterated; fronto- articulation L
42·1	85·8	25·9	88°·0	59°·0	82°·6	38°·4	471	—
42·9	83·9	17·9	—	60°·8	75°·0	44°·2	472	One incisor and 1 canine lost before death, M 3's absent
32·5	—	—	83°·0	68°·7	69°·0	42°·4	473	Metopic
—	72·7	21·3	—	—	—	—	474	Coronal and lambdoid beginning to close, sagittal closing; palate missing
46·4	—	—	83°·5	—	—	—	475	—
—	—	—	—	—	—	—	476	Coronal and sagittal beginning to close
—	—	—	—	—	—	—	477	Coronal closed, sagittal and lambdoid closing; occipito-mastoid obliterated R 2 or more molars lost before death
—	—	—	—	—	—	—	478	Coronal and lambdoid beginning to close, sagittal closing; occipito-mastoid clos L; temporal squamae fused to parietals; palate missing
—	80·7	20·3	—	—	—	—	479	Single epipteric bone L
52·8	86·0	22·6	—	—	—	—	480	Large epipteric bone L
—	86·7	17·0	—	—	—	—	481	Mandible, with no teeth lost before death
46·4	—	—	81°·0	—	—	—	482	Metopic; 2 epipteric bones R and 1 L; palate defective
44·7	76·2	17·5	82°·5	65°·0	73°·0	42°·0	483	Coronal beginning to close; single epipteric bone L; 1 molar lost before dea roughened area on frontal, due to disease?
—	—	—	—	64°·9	73°·2	41°·9	484	Roughened area on parietal L, due to disease? Mandible; M 3's absent from bot
50·0	80·5	22·6	—	65°·8	69°·9	44°·4	485	Coronal and sagittal beginning to close; large burnt patch parietal and occ Mandible, with 1 premolar lost before death
36·9	—	—	—	—	—	—	486	Sagittal beginning to close; 1 incisor and 1 molar lost before death. Mandible teeth lost before death
39·0	80·2	21·6	80°·5	62°·8	75°·6	41°·7	487	—
50·5	83·3	26·0	—	—	—	—	488	Coronal closing
75·6	86·1	24·2	—	—	—	—	489	Palate missing
34·7	—	—	89°·5?	—	—	—	490	Coronal and sagittal beginning to close; 2 or more molars lost before death; sing pre-condyle
—	—	—	—	—	—	—	491	Fronto-sphenoid sutures obliterated; palate missing
58·8	—	—	—	—	—	—	492	Sagittal closed; palate defective, teeth markedly worn
—	—	—	—	65°·1	72°·0	42°·9	493	—
46·7	—	—	—	—	—	—	494	*Os pentagonale* separate; numerous large wormian bones in lambdoid; palate de
—	—	—	84°·5?	66°·4	73°·6	40°·0	495	Metopic
46·7	—	—	—	—	—	—	496	M 3, L below alveolar margin and apparently impacted
—	78·6	22·4	—	61°·2	76°·6	42°·2	497	Right side of *os épactal* separate; trace of horizontal suture of interparietal bones single epipteric bone L
38·7	85·5	16·0	90°·0	58°·1	78°·0	43°·9	498	—
47·9	100·0	21·0	85°·0	66°·0	70°·9	43°·1	499	—
49·3	90·1	22·2	—	59°·4	67°·7	53°·0	500	Three wormian bones in coronal L
30·0	106·9	18·3	80°·5	70°·9	70°·3	39°·0	501	Large epipteric bone R
48·4	—	—	—	64°·7	79°·0	36°·3	502	Single epipteric bone R; palate defective
35·7	81·7	24·5	85°·0	67°·1	74°·3	38°·5	503	—
43·0	—	—	—	—	—	—	504	Metopic; M 3, L lost before death
—	—	—	—	—	—	—	505	Palate defective
50·5	76·3	20·3	83°·5	65°·1	72°·2	42°·7	506	M 2, R and M 3, R below alveolar margin but not impacted, M 3, L not fully
49·0	78·2	24·4	—	—	—	—	507	Trace of horizontal suture of interparietal bones R; palate defective
31·3	85·7	19·6	78°·0	68°·1	71°·8	40°·1	508	Palate defective; teeth markedly worn; small exostosis base of mastoid R
50·5	81·9	21·7	87°·0	64°·2	77°·9	37°·9	509	M 3's absent
—	—	—	—	73°·1	66°·3	40°·5	510	Ossicle of bregma; M 3, R absent. Mandible, with 2 molars lost before death
45·9	98·1	19·2	—	—	—	—	511	Trace of sutures between ex- and supra-occipital; single epipteric bones R and
49·4	—	—	85°·5	60°·6	71°·5	48°·0	512	—
67·0	—	—	—	—	—	—	513	Coronal and sagittal beginning to close; palate defective; depression parietal L
56·1	—	—	90°·5	—	—	—	514	Occipito-mastoid obliterated; 5 molars lost before death; teeth markedly worn. with no teeth lost before death
42·0	80·6	24·0	85°·0	64°·2	74°·2	41°·5	515	Metopic; M 3's absent

ar lost before death, teeth
te defective
n lost before death; pre-
on bones in sagittal above
L (?) lost before death teric bones R and L nissing h lost before death; large
tic depression parietal L before death, large abscess
literated; fronto-temporal
te missing
oid obliterated R and L; pito-mastoid closed R and
r lost before death; small
s absent from both jaws parietal and occipital R.
death. Mandible, with no
efore death; single median
abdoid; palate defective
terparietal bones R and L;
M 3, L not fully erupted fective f mastoid R
st before death eric bones R and L
ression parietal L n markedly worn. Mandible

„	„	470	186	142·5	96·1	127·5	110·9	117·0	99·6	126	130·5	119	375	300	523	36·2	31·1	101·0
„	„	471	185·5	134	99·5	131	112·1	115·9	92·0	128·5	129	114	371	304	515	36·0	28·2	104·0
„	„	472	185	134·5	95·8	131	110·0	114·0	95·0	124·5	127	120	371	297	517	36·1	29·0	103·1
„	„	473	174	138·5	88·5	126	107·2	111·1	97·9	120	125	113·5	358	292	499	35·1	26·8	90·6
„	„	474	169·5	136·5	94·3	131	109·0	113·6	94·1	123	130·5	110·5?	364?	311	491	35·1	29·1	93·2
„	„	475	173	141	92·9	—	105·9	104·1	95·7	117	114·5	117·5	350	300	501	—	30·0	—
„	„	476	180	140·5	92·8	133·5	111·8	111·1	96·1	127	121·5	121	369	316	510	34·6	29·0	98·3
107	♀	477	171	137	92·0	—	106·8	114·9?	82·0?	125·5	131·5?	95·5	353	308	491	—	—	—
„	„	478	169·5	130	—	127	105·9	—	—	117·5	—	—	355	297	—	36·1	—	92·9
„	„	479	179·5	131	89·2	—	112·3	116·2	95·8	128	129·5	118	376	294	503	—	29·3	—
„	„	480	180	—	95·3	133	112·0	109·6?	104·9	126	120·5?	132	378?	—	—	35·0	31·0	96·3
„	„	481	176·5	133	96·4	—	104·0	102·4	95·9	120·5	111	117·5	349	293	500?	—	—	—
„	„	482	176·5	129·5	94·2	125·5?	105·4	111·9	89·9	121	124·5	110·5	355	290	495	—	29·3	—
„	„	483	179·5	—	94·0	132	108·9	115·3	94·2	127	129·5	114·5	371	—	—	38·0	31·6	97·1
„	„	484	180	131·5	87·2	—	—	—	93·2	—	—	115·5	—	—	499	34·0	25·2	92·9
„	„	485	177	130	88·0	126	106·0	111·8	96·9	117·5	126	113·5	357	295	493	38·1	30·9	97·1
„	„	486	175	135	—	128·5	104·5?	119·1	94·9	119?	134	112	365	293	—	37·7	27·3	94·9
„	„	487	167·5	135	85·3	126·5	107·9	99·4	100·0	123	115·5	118	356	290	481	33·9	30·2	89·1
„	„	488	181·5	132·5	89·6	—	116·6	115·1	95·0	135	130	111·5	376	300	507	—	—	—
„	„	489	172	—	89·9	129·5	108·9	111·9	—	123·5	122	—	—	—	—	—	29·9	97·1
„	„	490	188·5	136	97·2	—	114·2	109·0	—	131·5	117·5	—	—	301	524	—	—	—
„	„	491	186	133	99·1	136·5	111·0	125·1	100·8	127·5	137	127	392	309	518	32·0	26·0	102·2
„	♀?	492	178	129	91·1	—	109·1	114·3?	—	123	127·5?	—	—	—	—	—	29·0	—
„	„	493	177·5	135	—	132·5	106·3	117·1	98·0	122·5	130·5	120	374	301	—	32·8	29·0	97·1
„	„	494	186·5	131	93·3	—	109·5	116·0?	101·0?	124	127·5?	123?	375	296	516	—	—	—
„	„	495	179	135	101·0?	126·5	109·6?	111·3?	92·2	125?	124?	116·5	366	302	506	36·1	—	95·2
„	„	496	177·5	138·5	—	130·5	105·2	115·9	93·0	120·5	131	109	361	314?	—	34·6	29·0	101·7
„	„	497	176	133	89·0	134·5	107·9	114·9	96·3	119	—	114·5	—	295?	493	36·0	31·0	98·5
„	„	498	183·5	131	90·0	134·5	113·0	—	—	130	—	—	361	300	503	37·0	30·6	104·7
116	♀	499	179·5	140·5	96·6	117·5	107·3	109·0	94·0	121·5	119·5	117	358	294	518	37·1	27·5	91·9
„	„	500	176·5	138·5	84·0	120·5	108·2	111·0	95·2	124·5	125·5	124·5	374	299	503	36·1	27·1	82·1
„	„	501	180	132·5	87·7	128·5	110·1	121·1	98·0	125·5	135·5	115·5	376	296	504	36·1	26·6	91·7
„	„	502	176·5	124·5	84·9	134·5	101·9	117·2	—	116	133·5	—	—	289	480	—	—	99·2
„	„	503	177	131·5	95·9	130	109·5	117·3	90·2	127	132	108	367	312	500	39·2	29·0	97·2
„	„	504	172	136	95·2	130	107·2	104·4	94·4	124·5	115·5	113·5	353	301	490	36·3	27·8	94·0
„	„	505	171·5	135	89·5	120	104·9	106·0	91·2	120	118	116·5	354	285	496	34·5	30·0	89·1
„	„	506	178	140·5	88·0	129·5	105·1	119·6	96·0	121	134	116	371	306	504	35·2	31·1	94·2
„	„	507	171·5	133	95·8	117·5	99·9	108·2	89·2	114·5	122·5	103·5	341	284	493	38·2	30·6	93·3
„	„	508	173	127	96·0	129	103·0	109·1	97·1	116·5	120	117·5	355	287	485	36·9	26·8	95·3
„	„	509	179	132	94·0	133·5	109·1	114·3	96·9	127	128	115·5	371	306	501	36·0	31·2	97·1
„	„	510	174	136·5	90·0	126·5?	112·0?	102·7?	98·4	128	114	118	360	293	—	36·0	—	90·3?
„	♀?	511	179·5	130·5	87·2	129	107·0	115·2	94·8	121·5	133	115·5	370	292	499	36·5	30·0	95·3
„	„	512	181·5	141·5	94·1	130·5	114·8	117·0	—	131	133	—	—	308	521	—	—	93·1
„	„	513	168·5	131	87·0	126	107·1	104·0	86·9	121·5	114·5	102·5	338	284	474	32·9	27·7	96·1
„	„	514	181	145	92·2	—	117·0	112·2	102·3	135	124·5	128	388	324	518	—	—	—
„	„	515	170·5	129·5	92·6	126	105·6	107·1	93·2	123	117·5	115	355	—	—	32·2	25·2	94·1
„	„	516	183	140	90·3	136	112·3	113·2	93·0	130·5	122	111·5	364	—	—	38·5	—	105·0
108	♀	517	174	129·5	89·2	123	105·3	103·2	92·0	118	116	109·5	343	288	485	35·6	27·9	97·9
„	„	518	181	132	91·8	130·5	116·9	115·3	86·3	136	133	101	370	295	506	36·4	31·1	97·0
„	♀?	519	186·5	129	94·1	—	111·1	118·0	93·0	125·5	131·5	113	370	292	512	—	—	—
120	♀	667	178·5	121	95·4	128·5	109·4	123·4?	—	127	136·5?	—	—	293	498	—	—	96·4
„	„	668	179	131	98·0	129	105·3?	107·2?	—	122?	118?	—	—	293	505	—	28·7	99·0
„	„	669	187	134·5	95·5	128	110·5?	122?	100·3?	130?	135·5?	119	385	295	525	34·9	29·2	97·4
„	„	670	176	138·5	—	127·5?	102·1?	104·2?	95·3?	115·5?	114?	113·5?	343	298	—	36·7	29·8	106·1
„	♀?	671	190·5	138	103·5	130·5	110·3	122·1?	98·3?	129	138?	116?	383	313	538	38·6	31·0	104·0
„	„	672	196?	122	98·1	—	—	119·2?	—	—	131·5?	—	—	—	531	—	—	—
120	♀?	673	184·5	127·5?	98·3	135·5	113·4	109·4	109·0	123	131·5	115	370	—	500?	38·9	32·4	99·3

38·9	29·7	99·0	88·9	69·2	—	—	24·1	—	11·9	5·9	33·1	42·1	43·5	—	52·5?	—	—	—	—	
36·2	31·1	101·8	98·6	69·9	—	—	25·3	49·9	9·0	3·0	—	—	48·0	—	—	—	—	—	107·3	7
36·0	28·2	104·0	90·2	65·2	122	99·0	27·8	47·1	12·1	5·1	33·9	43·8	45·1	44·1	51·5	60·0	51·3	13·3	112·6	7
36·1	29·0	103·1	93·3	74·8?	126·5	100·0	—	54·0	11·2	4·8	34·8	42·8	49·0?	42·9	47·0	56·0	50·7	9·1	109·9	7
35·1	26·8	90·6	89·2	64·7	—	—	23·2	47·1	8·0	2·6	31·0	40·7	45·1	40·1	45·0	—	—	—	109·6	7
35·1	29·1	92·2	—	—	123·5	91·0	—	—	—	—	32·0	40·7	—	—	46·5	64·0?	56·4?	12·0?	—	8
—	30·0	—	—	74·0	—	—	24·2	53·3	11·0	5·1	—	—	42·1	41·6	—	—	—	—	107·3	8
34·6	29·0	98·3	—	—	—	—	24·2	49·7	—	—	35·8	42·9	48·0?	40·0?	51·5	—	—	—	116·8	7
—	—	—	—	62·0	—	—	27·7?	—	9·9	—	—	—	40·4?	—	—	—	—	—	—	8
36·1	—	90·9	—	—	—	—	—	—	—	—	—	—	—	—	43·5	—	—	—	—	7
—	29·3	—	—	64·0	—	94·9	—	49·0	—	—	34·1	42·5	—	39·2	46·0	57·0	51·2	10·4	113·1	7
35·0	31·0	96·3	—	—	—	95·9	23·0	49·4	8·9	4·7	30·1	40·0	—	41·8	49·0	57·0	49·2	11·1	—	
—	—	—	—	—	125·5	94·2	24·9	49·3	8·7	—	31·8	40·9	46·0	39·0	49·0	56·5	51·1	8·7	105·1	7
—	29·3	—	—	64·3	—	96·9	—	50·0	11·2	5·2	32·1	40·0	48·1	—	45·0	—	—	—	107·8	7
38·0	31·6	97·1	92·2	68·1	—	91·4	25·8	49·0	8·5	3·8	36·0	43·9	45·0	—	46·5	61·0	57·2	10·0	—	
34·0	25·2	92·9	88·0	64·9	—	—	—	46·1	—	—	—	—	43·1	42·5	43·5	—	—	—	—	7
38·1	30·9	97·1	94·4	72·4	124·5	96·5	25·2	54·6	11·0	5·5	34·0	41·7	47·3?	40·0	45·5	56·5	49·1	11·1	109·1	7
37·7	27·3	94·9	—	—	—	—	25·1	47·6	11·1	4·1	—	—	—	—	—	—	—	—	—	7
33·9	30·2	89·1	82·1	61·2	—	87·2	—	45·0	8·2	3·2	34·0	39·5	40·1	39·9	46·5	58·0	50·9	11·0	104·2	8
—	—	—	—	68·7	—	93·2	24·8?	47·9	9·1	4·6	37·0	41·5	44·2	42·1	52·5	63·0	53·9	14·0	109·7	7
—	29·9	97·1	—	—	—	—	—	—	8·2	6·2	35·2	40·3	—	—	46·5	54·0	46·2	11·2	—	
—	—	—	—	73·0	—	88·6	25·1	48·6	10·1	3·5	31·5	41·2	45·0	—	47·5	—	—	—	113·6	7
32·0	26·0	102·2	—	—	—	—	—	—	—	—	33·0	41·9	—	—	—	—	—	—	—	7
—	29·0	—	—	—	—	—	—	48·6	10·2	6·0	—	43·8?	—	—	—	—	—	—	—	7
32·8	29·0	97·1	93·0	69·7?	—	—	—	53·3?	—	—	36·9?	—	—	41·2	—	—	—	—	—	7
—	—	—	—	61·9	—	—	25·0	48·0	7·5	3·5	—	—	—	—	—	—	—	—	—	7
36·1	—	95·2	91·2	64·0	125	89·9	25·1	47·1	—	—	31·6	42·6	—	—	46·0	—	—	—	—	7
34·6	29·0	101·7	—	—	—	—	25·8	48·0	13·7	6·4	—	—	45·6	38·7	—	—	—	—	—	7
36·0	31·0	98·5	89·1?	68·2?	125·5	95·1	23·2	48·0	5·6?	—	32·5	41·0	45·7?	39·1	51·5	65·5	56·2	12·6	—	7
37·0	30·6	104·7	91·1	74·1	127·5	90·2	22·2?	53·6	11·1	4·3	35·8	41·2	43·4	42·1	47·0	55·0	50·6	8·1	—	7
37·1	27·5	91·9	89·1	66·9	—	—	24·2	50·9	12·1	5·8	36·8	41·6	43·2	—	53·5	53·5?	48·2?	10·1?	104·3	7
36·1	27·1	82·1	76·7	71·2	—	90·2	—	51·6	7·1	3·5	33·0	36·7	41·1	37·0	45·5	50·5?	44·2?	9·8?	108·9	7
36·1	26·6	91·7	92·0	61·4	—	—	—	44·2	7·0	2·1	29·0	38·2	45·7?	37·1	46·5	43·5	39·9	7·3	111·9	7
—	—	99·2	91·3	60·0	—	84·3	21·5	43·8	9·1	4·4	31·0	38·8	41·0	—	45·0	—	—	—	—	7
39·2	29·0	97·2	93·2	63·0	117	90·2	22·0	48·3	8·4	3·0	33·0	38·1	48·8	40·8	44·5	54·5	48·1	11·8	114·4	7
36·3	27·8	94·0	—	—	—	—	22·3	43·8	9·3	4·0	—	—	43·1	40·0	—	—	—	—	109·3	7
34·5	30·0	89·1	—	—	—	91·3	26·1	50·1	—	—	32·1	—	—	—	—	—	—	—	—	7
35·2	31·1	94·2	90·0	67·0	—	94·2	25·2	46·2	10·3	5·2	30·3	38·1	42·2	—	45·0	59·0	53·3	10·8	—	7
38·2	30·6	93·3	—	—	—	—	—	51·2	10·0	4·9	33·0	41·2	—	—	48·5	62·0	54·4	13·3	—	7
36·9	26·8	95·3	93·3	64·9	123·5	89·4	23·2	47·8	9·9	3·1	35·0	40·2	47·1?	—	45·0	52·5	46·9	9·2	107·2	7
36·0	31·2	97·1	89·8	61·2	—	84·2	21·0	43·0	10·3	5·2	29·1	39·8	41·0	34·1	43·0	52·5	46·5	10·1	116·2	7
36·0	—	90·3?	94·7?	64·1?	—	—	—	44·9?	—	—	30·1?	—	—	40·0?	—	—	—	—	—	7
36·5	30·0	95·3	—	—	124·5?	92·3	—	45·7	11·1	5·1	35·1	40·2	46·0	—	53·0	54·0	48·9	9·4	107·3	7
—	—	93·1	85·9	73·0	—	—	—	52·1	8·9	4·4	35·1	40·1	41·7	39·7	45·5	—	—	—	112·5	7
32·9	27·7	96·1	—	—	—	—	—	49·1?	8·8	5·9	30·1	39·0	—	—	45·0	—	—	—	102·6	7
—	—	—	—	74·0	—	87·0	24·2	52·8	8·2	4·6	35·1	40·4	44·0	—	51·0	—	—	—	—	7
32·2	25·2	94·1	88·3	64·9	—	—	26·9	49·8	11·9	5·0	32·9	38·0	—	—	41·5	51·5	45·0	10·8	107·2	7
38·5	—	105·0	—	—	—	—	22·0	51·5	11·4	—	35·2	40·0	47·2	—	46·0	—	—	—	—	7
35·6	27·9	97·9	95·0	64·9	125?	89·2	25·6	47·7	13·1	6·2	—	38·1	47·0	37·3	47·5	—	—	—	105·8	7
36·4	31·1	97·0	85·0	67·1	—	83·2	25·1	51·0	7·9	2·6	35·0	41·1	44·8	36·1	50·5	57·5	51·0	9·9	107·8	7
—	—	—	—	—	124·5	92·1	—	—	9·4	7·1	39·0	41·7	—	—	47·0	56·0	51·1	8·1	108·3	7

A cranium and calvariae showing premature closing of the sagittal suture.

—	—	96·4	—	—	—	—	24·0	47·2	11·2	5·7	33·0	39·0	—	—	46·0	—	—	—	—	
—	28·7	99·0	—	—	—	—	—	—	11·2	4·0	—	—	—	—	—	—	—	—	—	
34·9	29·2	97·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
36·7	29·8	106·1	—	—	—	—	—	—	8·0	3·6	—	—	—	—	—	—	—	—	—	
38·6	31·0	104·0	—	—	—	—	—	—	9·1	3·8?	—	—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	

An artifically deformed calvaria.

38·9	32·4	99·3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	

—	—	—	74·8	—	—	62·7	76·3	—	—	78·6	—	49·5	—	—
—	107·3	76·6	68·5	111·8	8·1	60·4	85·9	—	50·7	—	—	33·3	—	—
3·3	112·6	72·2	70·6	102·3	1·6	57·6	78·3	65·9	59·0	77·4	97·8	42·1	85·8	25·9
9·1	109·9	72·7	70·8	102·7	1·9	56·5	80·3	74·8	—	81·3	87·6	42·9	83·9	17·9
—	109·6	79·6	72·4	109·9	7·2	63·9	76·4	—	49·3	76·2	88·9	32·5	—	—
2·0?	—	80·5	77·3	104·2	3·2	62·2	82·9	—	—	78·6	—	—	72·7	21·3
—	107·3	81·5	—	—	—	58·2	—	—	45·4	—	98·8	46·4	—	—
—	116·8	78·1	74·2	105·2	3·9	56·7	83·8	—	48·7	83·4	83·3	—	—	—
—	—	80·1	—	—	—	63·2	—	—	—	—	—	—	—	—
—	—	76·7	74·9	102·4	1·8	—	—	—	—	—	—	—	—	—
0·4	113·1	73·0	—	—	—	57·9	—	67·4	—	80·2	—	—	80·7	20·3
1·1	—	—	73·9	—	—	56·7	88·6	—	46·6	75·3	—	52·8	86·0	22·6
8·7	105·1	75·4	—	—	—	58·4	—	—	50·5	77·8	84·8	—	86·7	17·0
—	107·8	73·4	71·1	103·2	2·3	58·1	—	66·4	—	80·3	—	46·4	—	—
0·0	—	—	73·5	—	—	59·0	83·2	74·5	52·7	82·0	—	44·7	76·2	17·5
—	—	73·1	—	—	—	57·6	74·1	—	—	—	98·6	—	—	—
1·1	109·1	73·4	71·2	103·2	2·3	62·6	81·1	75·0	46·2	81·5	84·6	50·0	80·5	22·6
—	—	77·1	73·4	105·1	3·7	61·7	72·4	—	52·7	—	—	36·9	—	—
1·0	104·2	80·6	75·5	106·7	5·1	61·7	89·1	70·2	—	86·1	99·5	39·0	80·2	21·6
4·0	109·7	73·0	—	—	—	62·2	—	73·7	51·8	89·2	95·2	50·5	83·3	26·0
1·2	—	—	75·3	—	—	—	—	—	—	87·3	—	75·6	86·1	24·2
—	113·6	72·1	—	—	—	—	—	82·4	51·6	76·5	—	34·7	—	—
—	—	71·5	73·4	97·4	−1·9	56·6	81·3	—	—	78·8	—	—	—	—
—	—	72·5	—	—	—	—	—	—	—	—	—	58·8	—	—
—	—	76·1	74·6	101·9	1·4	58·4	88·4	—	—	—	—	—	—	—
—	—	70·2	—	—	—	58·8	—	—	52·1	—	—	46·7	—	—
—	—	75·4	70·7	106·7	4·7	56·5	—	71·2	53·3	74·2	—	—	—	—
—	—	78·0	73·5	106·1	4·5	62·4	83·8	—	53·7	—	84·9	46·7	—	—
12·6	—	75·6	76·4	98·9	−0·9	60·9	86·1	71·7	48·3	79·3	85·6	—	78·6	22·4
8·1	—	71·4	73·3	97·4	−1·9	—	82·7	82·2	41·4	86·9	97·0	38·7	85·5	16·0
0·1?	104·3	78·3	65·5	119·6	12·8	57·3	74·1	—	47·5	88·5	—	47·9	100·0	21·0
9·8?	108·9	78·5	68·3	114·9	10·2	55·0	75·1	78·9	—	89·9	90·0	49·3	90·1	22·2
7·3	111·9	73·6	71·4	103·1	2·2	61·8	73·7	—	—	75·9	81·2	30·0	106·9	18·3
—	—	70·5	76·2	92·6	−5·7	—	—	71·2	49·1	79·9	—	48·4	—	—
11·8	114·4	74·3	73·4	101·2	0·8	60·2	74·0	69·8	45·5	86·6	83·6	35·7	81·7	24·5
—	109·3	79·0	75·6	104·6	3·4	59·9	76·6	—	50·9	—	92·8	43·0	—	—
—	—	78·7	70·0	112·5	8·7	56·0	87·0	—	52·1	—	—	—	—	—
10·8	—	78·9	72·8	108·5	6·2	59·4	88·4	71·1	54·5	79·5	—	50·5	76·3	20·3
13·3	—	77·6	68·5	113·2	9·0	63·8	80·1	—	—	80·1	—	49·0	78·2	24·4
9·2	107·2	73·4	74·6	98·4	−1·2	59·3	72·6	72·6	48·5	87·1	—	31·3	85·7	19·6
10·1	116·2	73·7	74·6	98·9	−0·8	60·6	86·7	72·7	48·8	73·1	83·2	50·5	81·9	21·7
—	—	78·4	72·7	107·9	5·7	60·1	—	—	—	—	—	—	—	—
9·4	107·3	72·7	71·9	101·2	0·8	58·8	82·2	—	—	87·3	—	45·9	98·1	19·2
—	112·5	78·0	71·9	108·4	6·1	—	—	—	—	87·5	95·2	49·4	—	—
—	102·6	77·7	74·8	104·0	3·0	61·7	84·2	—	—	77·2	—	67·0	—	—
—	—	80·1	—	—	—	57·1	—	85·1	45·8	86·9	—	56·1	—	—
10·8	107·2	76·0	73·9	102·8	2·1	57·8	78·3	—	54·0	86·6	—	42·0	80·6	24·0
—	—	76·5	74·3	102·9	2·2	60·1	—	—	42·7	88·0	—	—	—	—
—	105·8	74·4	70·7	105·3	3·7	60·8	78·4	72·8	53·7	—	79·4	47·3	—	—
9·9	107·8	72·9	72·1	101·1	0·8	62·7	85·4	80·6	49·2	85·2	80·6	32·9	87·8	19·4
8·1	108·3	69·2	—	—	—	59·0	—	—	—	93·5	—	75·5	83·9	15·9

ure.

—	—	67·8	72·0	94·2	−4·2	—	—	—	50·8	84·6	—	50·9	—	—
—	—	73·2	72·1	101·6	1·1	—	—	—	—	—	—	35·7	—	—
—	—	71·9	68·4	105·1	3·5	61·1	83·7	—	—	—	—	—	—	—
—	—	78·7	72·4	108·6	6·3	60·7	81·2	—	—	—	—	45·0	—	—
—	—	72·4	68·5	105·7	3·9	61·7	80·3	—	—	—	—	41·8	—	—
—	—	62·2	—	—	—	—	—	—	—	—	—	—	—	—
—	—	69·1	73·4	94·1	−4·3	94·3	83·3	—	—	—	—	—	—	—

51·5	87·4	23·2	85·5	—	—	—	[illegible]	Sagittal beginning to close; 4 molars lost before death; all molars lost before death,
49·6	—	—	85°·0	60°·6	76°·7	42°·8	469	Sagittal beginning to close; 2 epipteric bones R; all molars lost before death, cavity at site of M 3, R
33·3	—	—	84°·5	67°·1	72°·1	40°·8	470	Coronal closed and obliterated at pterion R, sagittal nearly obliterated; fro articulation L
42·1	85·8	25·9	88°·0	59°·0	82°·6	38°·4	471	—
42·9	83·9	17·9	—	60°·8	75°·0	44°·2	472	One incisor and 1 canine lost before death, M 3's absent
32·5	—	—	83°·0	68°·7	69°·0	42°·4	473	Metopic
—	72·7	21·3	—	—	—	—	474	Coronal and lambdoid beginning to close, sagittal closing; palate missing
46·4	—	—	83°·5	—	—	—	475	—
—	—	—	—	—	—	—	476	Coronal and sagittal beginning to close
—	—	—	—	—	—	—	477	Coronal closed, sagittal and lambdoid closing; occipito-mastoid obliterat 2 or more molars lost before death
—	—	—	—	—	—	—	478	Coronal and lambdoid beginning to close, sagittal closing; occipito-mastoid L; temporal squamae fused to parietals; palate missing
—	80·7	20·3	—	—	—	—	479	Single epipteric bone L
52·8	86·0	22·6	—	—	—	—	480	Large epipteric bone L
—	86·7	17·0	—	—	—	—	481	Mandible, with no teeth lost before death
46·4	—	—	81°·0	—	—	—	482	Metopic; 2 epipteric bones R and 1 L; palate defective
44·7	76·2	17·5	82°·5	65°·0	73°·0	42°·0	483	Coronal beginning to close; single epipteric bone L; 1 molar lost before roughened area on frontal, due to disease?
—	—	—	—	64°·9	73°·2	41°·9	484	Roughened area on parietal L, due to disease? Mandible; M 3's absent fro
50·0	80·5	22·6	—	65°·8	69°·9	44°·4	485	Coronal and sagittal beginning to close; large burnt patch parietal and Mandible, with 1 premolar lost before death
36·9	—	—	—	—	—	—	486	Sagittal beginning to close; 1 incisor and 1 molar lost before death. Man teeth lost before death
39·0	80·2	21·6	80°·5	62°·8	75°·6	41°·7	487	—
50·5	83·3	26·0	—	—	—	—	488	Coronal closing
75·6	86·1	24·2	—	—	—	—	489	Palate missing
34·7	—	—	89°·5?	—	—	—	490	Coronal and sagittal beginning to close; 2 or more molars lost before death; pre-condyle
—	—	—	—	—	—	—	491	Fronto-sphenoid sutures obliterated; palate missing
58·8	—	—	—	—	—	—	492	Sagittal closed; palate defective, teeth markedly worn
—	—	—	—	65°·1	72°·0	42°·9	493	—
46·7	—	—	—	—	—	—	494	*Os pentagonale* separate; numerous large wormian bones in lambdoid; pala
—	—	—	84°·5?	66°·4	73°·6	40°·0	495	Metopic
46·7	—	—	—	—	—	—	496	M 3, L below alveolar margin and apparently impacted
—	78·6	22·4	—	61°·2	76°·6	42°·2	497	Right side of *os épactal* separate; trace of horizontal suture of interparietal b single epipteric bone L
38·7	85·5	16·0	90°·0	58°·1	78°·0	43°·9	498	—
47·9	100·0	21·0	85°·0	66°·0	70°·9	43°·1	499	—
49·3	90·1	22·2	—	59°·4	67°·7	53°·0	500	Three wormian bones in coronal L
30·0	106·9	18·3	80°·5	70°·9	70°·3	39°·0	501	Large epipteric bone R
48·4	—	—	—	64°·7	79°·0	36°·3	502	Single epipteric bone R; palate defective
35·7	81·7	24·5	85°·0	67°·1	74°·3	38°·5	503	—
43·0	—	—	—	—	—	—	504	Metopic; M 3, L lost before death
—	—	—	—	—	—	—	505	Palate defective
50·5	76·3	20·3	83°·5	65°·1	72°·2	42°·7	506	M 2, R and M 3, R below alveolar margin but not impacted, M 3, L not fu
49·0	78·2	24·4	—	—	—	—	507	Trace of horizontal suture of interparietal bones R; palate defective
31·3	85·7	19·6	78°·0	68°·1	71°·8	40°·1	508	Palate defective; teeth markedly worn; small exostosis base of mastoid R
50·5	81·9	21·7	87°·0	64°·2	77°·9	37°·9	509	M 3's absent
—	—	—	—	73°·1	66°·3	40°·5	510	Ossicle of bregma; M 3, R absent. Mandible, with 2 molars lost before de
45·9	98·1	19·2	—	—	—	—	511	Trace of sutures between ex- and supra-occipital; single epipteric bones R
49·4	—	—	85°·5	60°·6	71°·5	48°·0	512	—
67·0	—	—	—	—	—	—	513	Coronal and sagittal beginning to close; palate defective; depression parie
56·1	—	—	90°·5	—	—	—	514	Occipito-mastoid obliterated; 5 molars lost before death; teeth markedly w with no teeth lost before death
42·0	80·6	24·0	85°·0	64°·2	74°·2	41°·5	515	Metopic; M 3's absent
—	—	—	—	—	—	—	516	Palate defective
47·3	—	—	83°·5	67°·8	73°·0	39°·2	517	—
32·9	87·8	19·4	86°·5?	58°·9	78°·6	42°·6	518	Large epipteric bone R; 1 molar lost before death; filling in M 2, R: dimens max. height 1·0, max. length 3·0, max. breadth 1·0 mm.
75·5	83·9	15·9	—	—	—	—	519	Several teeth lost before death; small pre-condyle L. Mandible, with 3 molars l
50·9	—	—	—	—	—	—	667	Sagittal completely obliterated, coronal and lambdoid open; palate com lost before death
35·7	—	—	—	—	—	—	668	Metopic; sagittal completely obliterated, metopic, coronal, and lambdo missing. Mandible, with 2 molars lost before death
—	—	—	—	—	—	—	669	Sagittal completely obliterated, coronal and lambdoid open; palate missi
45·0	—	—	—	—	—	—	670	Metopic; sagittal completely obliterated, coronal and lambdoid begi metopic closing; palate defective, 1 molar lost before death; deep healed w
41·8	—	—	—	—	—	—	671	Sagittal completely obliterated, coronal and lambdoid open; palate missi
—	—	—	—	—	—	—	672	Sagittal completely obliterated, coronal closed, lambdoid closing; butt supra-occipital R
—	—	—	—	—	—	—	673	Metopic; all sutures open; R pterion normal in form but narrow, L defecti marked artificial deformation, frontal and occipital bones clearly flatten depression

lost before death, large abscess

y obliterated; fronto-temporal

palate missing

nastoid obliterated R and L;

occipito-mastoid closed R and

nolar lost before death; small

M 3's absent from both jaws
atch parietal and occipital R.

fore death. Mandible, with no

ost before death; single median

n lambdoid; palate defective

of interparietal bones R and L;

ed, M 3, L not fully erupted
e defective
ase of mastoid R

ars lost before death
epipteric bones R and L

depression parietal L
teeth markedly worn. Mandible

n M 2, R: dimensions of filling—
ole, with 3 molars lost before death

---

open; palate complete, no teeth

nal, and lambdoid open; palate

pen; palate missing
lambdoid beginning to close,
th; deep healed wound parietal L
pen; palate missing
oid closing; button exostosis on

---

narrow, L defective; face missing;
nes clearly flattened, post-coronal

No.	L	B	B'	H'	S_1'	S_2'	S_3'	S_1	S_2	S_3	S	$\beta Q'$	U
532	164·5	130·5	85·8	—	103·8?	95·4	—	118·5?	102	—	—	—	—
533	177	137·5	103·9	—	104·0	113·3	—	121·5	127	—	—	—	—
534	170	130·5	—	122	99·1	108·8	87·4	114·5	122	107	344	286	—
535	176	131·5	93·7	128·5	108·2	110·0	94·4	122	124·5	112	359	287	498
536	175	134	91·4	129	109·0	114·0	94·7	129	126·5	111	366	305	500
537	175	127	87·8	119·5	109·0	—	—	125	—	—	348	288	—
538	177	134·5	97·0	120	105·1	107·2	93·3	120·5	116·5	116	353	290	505
539	183·5	136·5	93·0	122·5	112·0?	121·0	90·2	129·5?	136·5	112	378	306	518
540	176·5	130·5	84·4	133	111·8	121·0	89·9	129	135	104·5	369	294	495
541	182	135	94·2	136	110·3	114·0	98·4	127	126·5	119	373	304	516
542	168·5	132	95·0	123	—	109·9	88·2	—	122·5	107·5	—	284	482
543	—	134	95·8	—	—	109·7	—	—	122	—	—	—	—
544	170	134·5?	88·0	128·5	111·2	107·0	—	132	119	—	—	308?	—
545	181	142·5	98·1	—	107·0	116·4	96·8	124·5	130·5	112·5	367	—	511
546	180·5	133·5	95·1	130	110·4	113·6	93·1	127	127·5	115·5	369	297	512
547	177	138·5	91·1	126·5	113·0	105·1	100·1	130·5	113·5	118	362	295	502
548	183·5	133	98·4	133·5	107·2	116·1	92·1	124	128·5	110·5	363	300	513
549	186?	135	90·6	125	108·3	120·4	94·0	127·5	134	115·5	378	298	516
550	177·5	139·5	89·9	129	111·1	114·9	96·2	127	128	113	368	302?	—
551	181·5?	131·5	92·0	—	—	117·9	—	—	131	—	—	299	—
552	171	137	88·2	125	102·3	114·8	89·3	117	131	110	358	294	489
553	177·5	128·5	86·1	125·5	108·0	113·9	96·2	128	126	118	372	293	492
554	174	135	90·0	134	107·9	109·9	91·9	122·5	124	105	351	299	492
555	173·5	128	95·3?	122	100·9	112·9	88·3	117	124	104·5	346	250	492
556	180·5	—	95·0	123?	—	—	—	—	—	—	359	295?	—
557	168·5	134·5	92·1	129·5	109·8	112·1	88·3	127·5	125	104	357	299	488
558	179	130·5	84·6	128	110·0	109·0?	95·2?	128·5	120?	115·5?	364	293	492
559	170	129	90·0	119·5	101·1	105·2	92·4	113·5	121	111	346	283?	482
560	171·5	136	88·1	120·5	106·0	111·2	90·7	122·5	127	108·5	358	296	492
561	179	123	88·3	128	112·0	109·7	—	129	123	—	—	283	482
562	170	135	77·9	—	113·9	108·4	93·2	130	122	108	360	—	492
563	171·5	136·5	91·9	126	107·7	107·9	87·2	125	120	103	348	298	492
564	170·5	134	91·8	117·5	104·1	106·0	94·2	119·5	116	114·5	350	294	492
565	174·5	132·5	90·0	—	—	122·2	—	—	138·5	—	—	—	—
566	171	127·5	91·2	133·5	105·0	114·9	94·3	121	124·5	112·5	358	296	488
567	175	130·5	88·5	123	109·8	97·3	100·2	126	104·5	123·5	355	295	502
568	179·5	132·5	92·3	125·5	114·2	—	—	134	—	—	365	—	—
569	166·5	133·5	—	118	102·3	102·1	87·1	117·5	113	102	332	281	—
570	174·5	130	93·0	131	—	110·8	93·7	—	124·5	109	—	292	492
571	176	132·5	92·3	122	109	107·2	86·0	123	120	109	352	297	502
572	169	145	89·5	129	110·5	110·1	90·8	126·5	122	109·5	358	308	492
573	177·5	132	86·8	124	109·1	110·8	96·0	127	124	117	367	289	482
574	179·5	134	91·1	131·5	117·9	117·5	87·1	140·5	134	104	378	308	492
575	172	134	91·6	—	104·3?	111·9	—	117?	125?	—	—	—	—
576	—	137·5	—	126·5	—	107·0	97·2	—	117·5	—	—	—	—
577	178	127	96·0	128·5	109·3	—	—	128	—	—	366	289	492
578	172·5	134·5	95·7	132·5	106·9	104·3	95·2	122	114·5	114·5	351	301	492
579	182	134·5?	99·9	130	102·9	114·1	—	117	121·5	—	—	303	502
580	182·5?	138	90·0	—	—	113·0	—	—	125	—	—	300	502
581	182	133·5	96·0	—	109·2	119·2	100·1	126	133	118	377	309	502
582	178·5	124	—	134·5	104·0	120·4	91·1	118·5	134·5	107	360	291	—
583	—	137·5	—	128	—	109·1	94·0	—	122·5	114	—	299	—
584	180·5	134	91·2	—	116·9	110·4	92·9	132·5	121	113·5	367	—	502
585	166·5	124	83·3	122·5	100·1	96·9	94·0	112·5	105	115	332	271	482
586	174·5	136	96·0	124·5	102·9	117·2	94·1	117	132·5	112·5	363	297	502
587	173·5	126·5	—	—	105·2	109·0	99·0	118	120	118·5	356	291	—
588	177	129	92·0	121	103·1	108·3	95·1	120	119	119	358	295	492
589	177·5	131·5	87·0	125·5	109·0	115·0	92·3	127·5	129	110·5	367	295?	502
590	177	130·5	97·3	—	112·4	107·8	—	130	120	—	—	303	492
591	168·5	128	88·0	124	103	106·3	96·0	116	119	121	356	287	492
592	179·5	135·5	97·2	131	101·9	110·0	101·9	117	120	127·5	364	306	502
593	176	138	99·3	130	111·4	112·3	97·2	127·5	124·5	121	373	307	502
594	175·5	127·5	89·6	—	116·0	115·5	90·0	134	133·5	102·5	370	297	492
595	—	132·5	—	—	—	117·6	86·4	—	132·5	105	—	—	—
596	179	132·5	95·2	137·5	116·0?	108·6	93·6	134·5	121·5	111	367	309	502
597	180	125	92·0	127	108·1	113·3	97·1	121·5	125·5	122	369	291	492
598	174	138	93·1	—	112·0	113·0	93·1	130	127·5	110	367	309	502
599	172	137·5	—	—	115·2	111·2	—	130·5	124	—	—	—	—
600	174·5	138·5	87·8	—	115·0	110·2	96·8	135	121·5	114·5	371	312	502
601	171	126	90·0	130·5	109·1	110·0	92·2	128	121·5	110	359	288	492
602	174	128·5	94·2	129·5	113·2	112·4	—	132	127·5	—	—	296	492
603	180·5	136·5	—	128·0	111·0	109·0	90·3	127·5	122·5	108·5	358	—	—

APPENDIX II. *Individual Measurements of Lachish Skulls.* VIII. *Female Calvariae from Tomb* 120

S	*βQ′*	*U*	*fml*	*fmb*	*LB*	*SC*	*SS*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	*Oc.I.*	$100\frac{fmb}{fml}$	$100\frac{SS}{SC}$	No.	
—	—	—	—	—	—	—	—	79·3	—	—	—	—	—	—	532	Temporal bones and
—	—	—	—	—	—	11·4	5·8	77·7	—	—	—	—	—	50·9	533	Metopic; single epipte
344	286	—	34·2	30·0	94·5	8·2	2·9	76·8	71·8	107·0	5·0	58·4	87·7	35·4	534	Metopic; single epipte
359	287	498	36·5	29·1	99·0	9·0	3·5	74·7	73·0	102·3	1·7	61·1	79·7	38·9	535	
366	305	500	35·7	28·1	97·0	—	—	76·6	73·7	103·9	2·9	62·4	78·7	—	536	Coronal obliterated, several small button
348	288	—	33·0	29·0	98·2	9·8	6·1	72·6	68·3	106·3	4·3	—	87·9	62·2	537	Fused nasal bones; bu
353	290	505	36·0	34·0	96·3	10·1	5·3	76·0	67·8	112·1	8·2	57·4	94·4	52·5	538	Coronal and lambdoid
378	306	518	36·1	25·9	94·0	—	—	74·4	66·8	111·4	7·6	57·4	71·7	—	539	Coronal beginning to
369	294	495	37·0	28·8	94·1	—	—	73·9	75·4	98·1	−1·4	63·5	77·8	—	540	
373	304	510	41·0	34·6	97·2	—	—	74·2	74·7	99·3	−0·5	59·4	84·4	—	541	Single epipteric bone
—	284	484	37·1	29·8	—	—	—	78·3	73·0	107·3	5·3	58·7	80·3	—	542	Single epipteric bone
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	543	Temporal bones and
—	308?	—	—	27·1	89·0	—	—	79·1	75·6	104·7	3·5	—	—	—	544	Trace of horizontal su
367	—	517?	—	—	—	—	—	78·7	—	—	—	63·5	—	—	545	
369	297	514	34·8	31·7	102·1	—	—	74·0	72·0	102·7	1·9	57·5	91·1	—	546	Coronal closing, sagitt
362	295	504	36·1	29·1	90·0	—	—	78·2	71·5	109·5	6·8	61·8	80·6	—	547	Single epipteric bone
363	300	512	40·2	31·2	104·1	9·3	5·7	72·5	72·8	99·6	−0·3	60·0	77·6	61·3	548	Metopic; single epipte
378	298	516	33·0	26·5	97·3	—	—	72·6	67·2	108·0	5·4	58·1	80·3	—	549	Occipito-mastoid oblit
368	302?	—	37·4	28·1	92·9	12·2	4·0	78·6	72·7	108·1	5·9	62·1	75·1	32·8	550	Occipito-mastoid R n
—	299	—	—	—	—	—	—	72·5	—	—	—	—	—	—	551	Coronal closing
358	294	487	34·1	27·1	89·0	—	—	80·1	73·1	109·6	7·0	57·9	79·5	—	552	Sagittal beginning to
372	293	490	32·6	27·1	94·2	—	—	72·4	70·7	102·4	1·7	58·2	83·1	—	553	Sagittal closing
351	299	498	40·8	34·2	100·3	9·2	4·0	77·6	77·0	100·7	0·6	66·0	83·8	43·5	554	
346	250	494	36·2	28·2	98·1	—	—	73·8	70·3	104·9	−3·5	61·4	77·9	—	555	Coronal beginning to
359	295?	—	34·0	28·0	101·0	—	—	—	68·1	—	—	—	82·4	—	556	Coronal and lambdoid
357	299	488	34·0	28·8	93·2	—	—	79·8	76·9	103·9	3·0	61·9	84·7	—	557	Sagittal closing, lamb
364	293	497	34·2	28·0	100·2	8·2	2·3	72·9	71·5	102·0	1·4	59·1	81·9	28·0	558	Coronal closing
346	283?	480?	34·7	—	92·0	—	—	75·9	70·3	107·9	5·6	59·9	—	—	559	
358	296	492	33·7	29·1	91·8	—	—	79·3	70·3	112·9	9·0	60·3	86·4	—	560	
—	283	488	—	—	100·0	—	—	68·7	71·5	96·1	−2·8	—	—	—	561	
360	—	493	—	—	—	8·6	5·7	79·4	—	—	—	62·8	—	66·3	562	
348	298	491	36·0	29·8	98·2	—	—	79·6	73·5	108·3	6·1	61·6	82·8	—	563	
350	294	495	33·0	28·7	95·0	8·3	2·6	78·6	68·9	114·0	9·7	59·0	87·0	31·3	564	Suture between ex- bones R and L
—	—	—	—	—	—	—	—	75·9	—	—	—	—	—	—	565	
358	296	488	35·0	28·3	94·6	—	—	74·6	78·1	95·5	−3·5	60·5	80·9	—	566	Single (?) epipteric b
355	295	500	35·2	30·3	94·3	—	—	74·6	70·3	106·1	4·3	57·9	86·1	—	567	Single epipteric bone
365	—	—	37·9	—	95·2	—	—	73·8	69·9	105·6	3·9	—	—	—	568	
332	281	—	36·0	29·8	93·5	—	—	80·2	70·9	113·1	9·3	62·6	82·8	—	569	
—	292	496	36·3	29·1	—	—	—	74·5	75·1	99·2	−0·6	63·4	80·2	—	570	
352	297	500?	39·0	—	97·0	11·2	4·1	75·3	69·3	108·6	6·0	56·3	—	36·6	571	Sagittal beginning to
358	308	491	33·0	27·8	88·4	—	—	85·8	76·3	112·4	9·5	59·6	84·2	—	572	Slight trace of metop
367	289	489	35·6	29·2	90·3	—	—	74·4	69·9	106·5	4·5	58·7	82·0	—	573	Lateral half of sutur R and L
378	308	497	38·0	28·0	97·2	—	—	74·7	73·3	101·9	1·4	60·5	73·7	—	574	Exostosis frontal R, exostosis found in
—	—	—	—	—	—	—	—	77·9	—	—	—	—	—	—	575	Base missing; single
—	—	—	37·0	30·0	—	—	—	—	—	108·7	—	—	81·1	—	576	Coronal and sagittal
366	289	499	32·6	26·0	98·2	—	—	71·3	72·2	98·8	−0·8	—	79·8	—	577	Sagittal suture amen small perforation of
351	301	497	36·0	28·1	101·1	10·7	4·4	78·0	76·8	101·5	1·2	59·8	78·1	41·1	578	
—	303	514	—	30·0	104·0	—	—	73·9	71·4	103·5	2·5	—	—	—	579	Single large epipteric
—	300	510	—	—	—	—	—	75·6	—	—	—	—	—	—	580	Small epipteric bone
377	309	518	—	—	—	—	—	73·4	—	—	—	61·8	—	—	581	Epipteric bone L
360	291	—	34·2	30·9	104·9	—	—	69·5	75·4	92·2	−5·9	62·1	90·4	—	582	Slight traces of sutur
—	299	—	35·4	32·9	—	—	—	—	—	107·4	—	59·1	92·9	—	583	Sagittal closing
367	—	504?	—	—	—	—	—	74·2	—	—	—	58·5	—	—	584	Coronal closed R, cle
332	271	465	33·9	29·0	96·0	—	—	74·5	73·6	101·2	0·9	58·5	85·5	—	585	
363	297	501	35·2	30·6	93·0	—	—	77·9	71·3	109·2	6·6	60·3	86·9	—	586	
356	291	—	—	—	—	—	—	72·9	—	—	—	60·2	—	—	587	
358	295	495	36·3	29·0	97·0	—	—	72·9	68·4	106·6	4·5	57·1	79·9	—	588	Two small epipteric
367	295?	503?	37·8	26·0	91·1	—	—	74·1	70·7	104·8	3·4	60·2	68·8	—	589	Single epipteric bone and continued by c
—	303	497	—	28·0	—	—	—	73·7	—	—	—	—	—	—	590	Coronal beginning to
356	287	481	33·1	27·7	87·0	—	—	76·0	73·6	103·2	2·4	56·6	83·7	—	591	Single epipteric bones
364	306	509	36·7	30·1	103·2	—	—	75·5	73·0	103·4	2·5	57·0	82·0	—	592	Single epipteric bone
373	307	507	33·9	30·0	92·9	—	—	78·4	73·9	106·2	4·5	57·3	88·5	—	593	Single epipteric bones
370	297	489	—	—	—	—	—	72·6	—	—	—	66·5	—	—	594	
—	—	—	—	—	—	—	—	—	—	—	—	59·0	—	—	595	Suture between ex- a
367	309	503	37·2	29·0	103·0	—	—	74·0	76·8	96·4	−2·8	61·1	78·0	—	596	
369	291	496	34·8	29·0	96·7	—	—	69·4	70·6	98·4	−1·1	56·8	83·3	—	597	Fronto-temporal arti
367	309	500	—	—	—	10·3	3·9	79·3	—	—	—	61·5	—	37·9	598	Two epipteric bones
—	—	—	—	—	—	—	—	79·9	—	—	—	—	—	—	599	
371	312	503	—	32·0	—	—	—	79·4	—	—	—	61·4	—	—	600	
359	288	488	32·7	26·1	95·1	—	—	73·7	76·3	96·6	−2·6	60·5	79·8	—	601	Depression on pariet
—	296	495	—	—	96·1	—	—	73·9	74·4	99·2	−0·5	—	—	—	602	Single epipteric bone
358	—	—	38·9	28·2	100·5	—	—	75·6	70·9	106·6	4·7	59·9	72·5	—	603	

S_1	S_2	S_3	S	$\beta Q'$	U	*fml*	*fmb*	*LB*	*SC*	*SS*	$100\frac{B}{L}$	$100\frac{H'}{L}$	$100\frac{B}{H'}$	$100\frac{B-H'}{L}$	*Oc.I.*	$100\frac{fmb}{fml}$	$100\frac{SS}{SC}$
118·5?	102	—	—	—	—	—	—	—	—	—	79·3	—	—	—	—	—	—
121·5	127	—	—	—	—	—	—	—	11·4	5·8	77·7	—	—	—	—	—	50·9
114·5	122	107	344	286	—	24·2	30·0	94·5	8·2	2·9	76·8	71·8	107·0	5·0	58·4	87·7	35·4
122	124·5	112	359	287	498	30·5	29·1	99·0	9·0	3·5	74·7	73·0	102·3	1·7	61·1	79·7	38·9
129	126·5	111	366	305	500	35·7	28·1	97·0	—	—	76·6	73·7	103·9	2·9	62·4	78·7	—
125	—	—	348	288	—	33·0	29·0	98·2	9·8	6·1	72·6	68·3	106·3	4·3	—	87·9	62·2
120·5	116·5	116	353	290	505	36·0	34·0	96·3	10·1	5·3	76·0	67·8	112·1	8·2	57·4	94·4	52·5
129·5?	136·5	112	378	306	518	36·1	25·9	94·0	—	—	74·4	66·8	111·4	7·6	57·4	71·7	—
129	135	104·5	369	294	495	37·0	28·8	94·1	—	—	73·9	75·4	98·1	−1·4	63·5	77·8	—
127	126·5	119	373	304	510	41·0	34·6	97·2	—	—	74·2	74·7	99·3	−0·5	59·4	84·4	—
—	122·5	107·5	—	284	484	37·1	29·8	—	—	—	78·3	73·0	107·3	5·3	58·7	80·3	—
—	122	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
132	119	—	—	308?	—	—	27·1	89·0	—	—	79·1	75·6	104·7	3·5	—	—	—
124·5	130·5	112·5	367	—	517?	—	—	—	—	—	78·7	—	—	—	63·5	—	—
127	127·5	115·5	369	297	514	34·8	31·7	102·1	—	—	74·0	72·0	102·7	1·9	57·5	91·1	—
130·5	113·5	118	362	295	504	36·1	29·1	90·0	—	—	78·2	71·5	109·5	6·8	61·8	80·6	—
124	128·5	110·5	363	300	512	40·2	31·2	104·1	9·3	5·7	72·5	72·8	99·6	−0·3	60·0	77·6	61·3
127·5	134	115·5	378	298	516	33·0	26·5	97·3	—	—	72·6	67·2	108·0	5·4	58·1	80·3	—
127	128	113	368	302?	—	37·4	28·1	92·9	12·2	4·0	78·6	72·7	108·1	5·9	62·1	75·1	32·8
—	131	—	—	299	—	—	—	—	—	—	72·5	—	—	—	—	—	—
117	131	110	358	294	487	34·1	27·1	89·0	—	—	80·1	73·1	109·6	7·0	57·9	79·5	—
128	126	118	372	293	490	32·6	27·1	94·2	—	—	72·4	70·7	102·4	1·7	58·2	83·1	—
122·5	124	105	351	299	498	40·8	34·2	100·3	9·2	4·0	77·6	77·0	100·7	0·6	66·0	83·8	43·5
117	124	104·5	346	290	494	36·2	28·2	98·1	—	—	73·8	70·3	104·9	−3·5	61·4	77·9	—
—	—	—	359	295?	—	34·0	28·0	101·0	—	—	—	68·1	—	—	—	82·4	—
127·5	125	104	357	299	488	34·0	28·8	93·2	—	—	79·8	76·9	103·9	3·0	61·9	84·7	—
128·5	120?	115·5?	364	293	497	34·2	28·0	100·2	8·2	2·3	72·9	71·5	102·0	1·4	59·1	81·9	28·0
113·5	121	111	346	283?	480?	34·7	—	92·0	—	—	75·9	70·3	107·9	5·6	59·9	—	—
122·5	127	108·5	358	296	492	33·7	29·1	91·8	—	—	79·3	70·3	112·9	9·0	60·3	86·4	—
129	123	—	—	283	488	—	—	100·0	—	—	68·7	71·5	96·1	−2·8	—	—	—
130	122	108	360	—	493	—	—	—	8·6	5·7	79·4	—	—	—	62·8	—	66·3
125	120	103	348	298	491	36·0	29·8	98·2	—	—	79·6	73·5	108·3	6·1	61·6	82·8	—
119·5	116	114·5	350	294	495	33·0	28·7	95·0	8·3	2·6	78·6	68·9	114·0	9·7	59·0	87·0	31·3
—	138·5	—	—	—	—	—	—	—	—	—	75·9	—	—	—	—	—	—
121	124·5	112·5	358	296	488	35·0	28·3	94·6	—	—	74·6	78·1	95·5	−3·5	60·5	80·9	—
126	104·5	123·5	355	295	500	35·2	30·3	94·3	—	—	74·6	70·3	106·1	4·3	57·9	86·1	—
134	—	—	365	—	—	37·9	—	95·2	—	—	73·8	69·9	105·6	3·9	—	—	—
117·5	113	102	332	281	—	36·0	29·8	93·5	—	—	80·2	70·9	113·1	9·3	62·6	82·8	—
—	124·5	109	—	292	496	36·3	29·1	—	—	—	74·5	75·1	99·2	−0·6	63·4	80·2	—
123	120	109	352	297	500?	39·0	—	97·0	11·2	4·1	75·3	69·3	108·6	6·0	56·3	—	36·6
126·5	122	109·5	358	308	491	33·0	27·8	88·4	—	—	85·8	76·3	112·4	9·5	59·6	84·2	—
127	124	117	367	289	489	35·6	29·2	90·3	—	—	74·4	69·9	106·5	4·5	58·7	82·0	—
140·5	134	104	378	308	497	38·0	28·0	97·2	—	—	74·7	73·3	101·9	1·4	60·5	73·7	—
117?	125?	—	—	—	—	—	—	—	—	—	77·9	—	—	—	—	—	—
—	117·5	—	—	—	—	37·0	30·0	—	—	—	—	—	108·7	—	—	81·1	—
128	—	—	366	289	499	32·6	26·0	98·2	—	—	71·3	72·2	98·8	−0·8	—	79·8	—
122	114·5	114·5	351	301	497	36·0	28·1	101·1	10·7	4·4	78·0	76·8	101·5	1·2	59·8	78·1	41·1
117	121·5	—	—	303	514	—	30·0	104·0	—	—	73·9	71·4	103·5	2·5	—	—	—
—	125	—	—	300	510	—	—	—	—	—	75·6	—	—	—	—	—	—
126	133	118	377	309	518	—	—	—	—	—	73·4	—	—	—	61·8	—	—
118·5	134·5	107	360	291	—	34·2	30·9	104·9	—	—	69·5	75·4	92·2	−5·9	62·1	90·4	—
—	122·5	114	—	299	—	35·4	32·9	—	—	—	—	—	107·4	—	59·1	92·9	—
132·5	121	113·5	367	—	504?	—	—	—	—	—	74·2	—	—	—	58·5	—	—
112·5	105	115	332	271	465	33·9	29·0	96·0	—	—	74·5	73·6	101·2	0·9	58·5	85·5	—
117	132·5	112·5	363	297	501	35·2	30·6	93·0	—	—	77·9	71·3	109·2	6·6	60·3	86·9	—
118	120	118·5	356	291	—	—	—	—	—	—	72·9	—	—	—	60·2	—	—
120	119	119	358	295	495	36·3	29·0	97·0	—	—	72·9	68·4	106·6	4·5	57·1	79·9	—
127·5	129	110·5	367	295?	503?	37·8	26·0	91·1	—	—	74·1	70·7	104·8	3·4	60·2	68·8	—
130	120	—	—	303	497	—	28·0	—	—	—	73·7	—	—	—	—	—	—
116	119	121	356	287	481	33·1	27·7	87·0	—	—	76·0	73·6	103·2	2·4	56·6	83·7	—
117	120	127·5	364	306	509	36·7	30·1	103·2	—	—	75·5	73·0	103·4	2·5	57·0	82·0	—
127·5	124·5	121	373	307	507	33·9	30·0	92·9	—	—	78·4	73·9	106·2	4·5	57·3	88·5	—
134	133·5	102·5	370	297	489	—	—	—	—	—	72·6	—	—	—	66·5	—	—
—	132·5	105	—	—	—	—	—	—	—	—	—	—	—	—	59·0	—	—
134·5	121·5	111	367	309	503	37·2	29·0	103·0	—	—	74·0	76·8	96·4	−2·8	61·1	78·0	—
121·5	125·5	122	369	291	496	34·8	29·0	96·7	—	—	69·4	70·6	98·4	−1·1	56·8	83·3	—
130	127·5	110	367	309	500	—	—	—	10·3	3·9	79·3	—	—	—	61·5	—	37·
130·5	124	—	—	—	—	—	—	—	—	—	79·9	—	—	—	—	—	—
135	121·5	114·5	371	312	503	—	32·0	—	—	—	79·4	—	—	—	61·4	—	—
128	121·5	110	359	288	488	32·7	26·1	95·1	—	—	73·7	76·3	96·6	−2·6	60·5	79·8	—
132	127·5	—	—	296	495	—	—	96·1	—	—	73·9	74·4	99·2	−0·5	—	—	—
127·5	122·5	108·5	358	—	—	38·9	28·2	100·5	—	—	75·6	70·9	106·6	4·7	59·9	72·5	—

$100\frac{fmb}{fml}$	$100\frac{SS}{SC}$	No.	Remarks
—	—	532	Temporal bones and base missing
—	50·9	533	Metopic; single epipteric bone R; region of L pterion defective
87·7	35·4	534	Metopic; single epipteric bones R and L
79·7	38·9	535	—
78·7	—	536	Coronal obliterated, sagittal and lambdoid nearly obliterated; single pre-condyle L; several small button exostoses on frontal bone
87·9	62·2	537	Fused nasal bones; burnt patch under lambda
94·4	52·5	538	Coronal and lambdoid beginning to close, sagittal closing; abscess cavity at root of M2, R
71·7	—	539	Coronal beginning to close; trace of horizontal suture of interparietal bones R and L
77·8	—	540	—
84·4	—	541	Single epipteric bone R
80·3	—	542	Single epipteric bone R
—	—	543	Temporal bones and base missing
—	—	544	Trace of horizontal suture of interparietal bones L; traumatic depression (?) on frontal
—	—	545	—
91·1	—	546	Coronal closing, sagittal beginning to close
80·6	—	547	Single epipteric bone R
77·6	61·3	548	Metopic; single epipteric bones R and L
80·3	—	549	Occipito-mastoid obliterated R and L
75·1	32·8	550	Occipito-mastoid R nearly obliterated
—	—	551	Coronal closing
79·5	—	552	Sagittal beginning to close
83·1	—	553	Sagittal closing
83·8	43·5	554	—
77·9	—	555	Coronal beginning to close
82·4	—	556	Coronal and lambdoid closed, sagittal obliterated
84·7	—	557	Sagittal closing, lambdoid beginning to close
81·9	28·0	558	Coronal closing
—	—	559	—
86·4	—	560	—
—	—	561	—
—	66·3	562	—
82·8	—	563	—
87·0	31·3	564	Suture between ex- and supra-occipital R open, trace of same suture L; single epipteric bones R and L
—	—	565	—
80·9	—	566	Single (?) epipteric bone R
86·1	—	567	Single epipteric bone R; single pre-condyle L
—	—	568	—
82·8	—	569	—
80·2	—	570	—
—	36·6	571	Sagittal beginning to close
84·2	—	572	Slight trace of metopic above nasion
82·0	—	573	Lateral half of suture between ex- and supra-occipital L visible; single epipteric bone R and L
73·7	—	574	Exostosis frontal R, circular in form, diameter 29, height *c.* 5 mm. This is the largest exostosis found in the series
—	—	575	Base missing; single epipteric bone R
81·1	—	576	Coronal and sagittal beginning to close; occipito-mastoid L partly obliterated
79·8	—	577	Sagittal suture anterior third clearly open, posterior two-thirds completely obliterated; small perforation of basi-occipital between anterior ends of condyles
78·1	41·1	578	—
—	—	579	Single large epipteric bones R and L
—	—	580	Small epipteric bone L; small and apparently partly healed wound perforating parietal
—	—	581	Epipteric bone L
90·4	—	582	Slight traces of sutures of ex- and supra-occipital R and L
92·9	—	583	Sagittal closing
—	—	584	Coronal closed R, clearly open L; single epipteric bone R
85·5	—	585	—
86·9	—	586	—
—	—	587	—
79·9	—	588	Two small epipteric bones R
68·8	—	589	Single epipteric bone L; L half of suture between ex- and supra-occipital, probably op. and continued by crack; markedly asymmetrical
—	—	590	Coronal beginning to close
83·7	—	591	Single epipteric bones R and L; 2 shallow depressions (wounds ?) parietal L
82·0	—	592	Single epipteric bone R
88·5	—	593	Single epipteric bones R and L
—	—	594	—
—	—	595	Suture between ex- and supra-occipital obliterated R, L clearly open; small pre-condyl.
78·0	—	596	—
83·3	—	597	Fronto-temporal articulation R, L pterion normal; wound parietal R
—	37·9	598	Two epipteric bones R and L
—	—	599	—
—	—	600	—
79·8	—	601	Depression on parietal L, due to wound?
—	—	602	Single epipteric bone L
72·5	—	603	—

APPENDIX II. *Individual Measurements of Lachish Skulls.* IX. *Female (cont.), c*

Tomb no.	Sex	No.	*L*	*B*	*B′*	*H′*	S_1'	S_2'	S_3'	S_1	S_2	S_3	*S*	*βQ′*	*U*	*fml*	*fmb*	*LB*	*SC*
120	♀	604	173·5	130	92·3?	129	—	106·0	100·4	—	116·5	121	—	287	491	34·2	32·2	—	—
,,	,,	605	173	129·5	92·2	130	105·0	111·2	92·0	117	125·5	108·5	351	291	489	36·0	—	101·4	—
,,	,,	606	174·5	133·5	86·2	124	107·2	107·0	92·1	125	117·5	111	354	294	499	40·0	—	95·2	—
,,	,,	607	175	139	97·1	135	111·2	—	96·1?	127	—	115	372	315	510	36·2	30·0	93·1	8·7
,,	,,	608	174·5	123	87·0	125·5	101·0	117·2	90·8	116·5	134·5	115·5	367	285	481	34·0	28·4	93·0	—
,,	,,	609	174	138	—	131·5	111·0	111·1	90·8	129·5	124	113·5	367	298	—	32·7	29·1	95·1	12·2
,,	,,	610	176	133·5	90·6	130·5	107·1	118·2	97·1	123	132·5	113	368	295	498	35·0	30·0	95·9	12·0
,,	,,	611	176	135·5	94·1	—	107·5	117·0	84·9	127	130	105	362	296	511	—	29·2	—	—
,,	,,	612	178·5	132	90·0	139	108·2	—	—	122	—	—	367	301	—	36·9	30·0	103·1	—
,,	,,	613	172·5	140	—	—	105·9	109·0	92·0	124	123·5	109	356	303	—	—	29·0	—	—
,,	,,	614	173	134	95·0	127	105·0	114·2	96·0	118·5	130	111·5	360	299	501	39·2	32·0	95·2	—
,,	,,	615	167·5	134·5	91·3	127	97·9	112·0	94·2	111	126	113	350	298	487	33·8	29·0	93·0	—
,,	,,	616	177·5	128·5	—	124	107·2?	111·7	95·3	125·5	124·5	115	365	287	—	35·2	29·1	93·3?	—
,,	♀?	617	186·5	140·5	92·8	—	114·0	—	—	129·5	—	—	377	308	—	—	—	—	—
,,	,,	618	177·5	136	92·8	131	107·1	112·0	93·1	120	128·5	115·5	364	314	506	38·4	30·2	100·2	—
,,	,,	619	179	135	88·0	112·5	103·9	117·1	85·1	118·5	132·5	103	354	284	—	34·9	—	92·6	—
,,	,,	620	179	137	101·1	—	108·3	117·8	95·1	124	130	112	366	318	517	—	28·9	—	—
,,	,,	621	179·5	133·5	86·1	135·5	110·3	108·6	105·1	127·5	118·5	130	375	304	502	35·5	—	98·2	10·0
,,	,,	622	195	140	96·1	129	113·3	119·0	105·1	129·5	131	129	390	306	532	36·0	28·4	103·2	—
,,	,,	623	182	128·5	87·1	—	110·1	118·2	89·1	125	132·5	110·5	369	291	504	—	25·0	—	9·0
,,	,,	624	183·5	139	93·6	126	116·3	112·1	92·9	134	122	117	373	305	516	36·1	27·7	96·9	—
,,	,,	625	174	134	97·9	128·5	103·1	112·0	93·0	114·5	125·5	112	352	303	489	35·4	27·3	98·0	—
,,	,,	626	181	134·5	94·7	135	114·0	113·0	96·0	130·5	123·5	115·5	370	310	508	34·5	28·2	99·0	—
,,	,,	627	185·5	134·5	101·0	128	105·9	106·3?	102·3?	121	116?	128?	365	303	522	37·0	31·6	103·9	11·4
,,	,,	628	179	131	—	—	104·1	—	—	119	—	—	357	—	—	—	31·9	—	—
,,	,,	629	188·5	136·5	99·0	130	109·1	122·0	96·1	129	141	117·5	388	299	529	34·7	27·0	99·7	6·2
,,	,,	630	175·5	132	94·9	126·5	112·2	122·0	88·2	131	138·5	109	379	304	501	34·8	29·1	91·4	13·3
,,	,,	631	177·5	142·5	92·8	127	105·0	117·2?	94·9?	119	134·5?	110	364	307	514	33·6	—	97·2	—
,,	,,	632	184·5	135·5	—	—	115·0	119·0	100·6	131·5	132·5	118	382	302	—	—	—	—	—
,,	,,	633	178	139	85·2	135	107·2	116·1	95·3	120·5	133·5	114·5	369	304?	504?	36·5	26·8	101·7	11·8
,,	,,	634	176·5	135·5	93·0	134·5	112·0	113·1	—	131·5	128·5	—	—	311	511	—	—	98·8	6·5
,,	,,	635	179·5	129	98·0	133·5	111·2	116·8	95·9	130·5	131	113	375	301	505	39·2	32·1	97·3	6·6
,,	,,	636	174	125·5	85·4	127	106·0	114·2	93·1	119·5	129·5	109·5	359	284	486	34·3	26·3	93·0	—
,,	,,	637	177·5	138	98·1	—	109·1	113·7	—	130	127·5	—	—	304	512	—	—	—	—
,,	,,	638	172·5	139	94·1	130	110·9	105·3	93·2	125	117·5	108·5	351	305	502	40·2	—	96·1	—
,,	,,	639	176·5	131	88·9	131·5	—	112·0	—	—	124·5	—	—	303	494	—	—	—	—
,,	,,	640	177·5	140·5	100·5	135	111·1	113·3	96·7	129·5	127·5	112·5	370	319?	519?	38·9	33·9	99·4	—
,,	,,	641	175	139	94·6	143·5	113·0	118·2	96·6	127·5	137	109·5	373	319	511	39·8	30·2	98·8	10·
,,	,,	642	180·5	134·5	94·0	133	108·1	112·7	100·4?	122·5	123	124?	370?	304	508	35·0?	28·9	99·3	12·
,,	,,	643	176	139	87·0	130?	112·0	111·4	91·9	130	124	112·5	367	—	—	34·0	—	95·2?	—
,,	,,	644	186	133·5	100·0	—	102·3	120·0	97·8	119	136·5	117	372	302	526	—	28·5	—	12·
,,	,,	645	173	129·5	93·2	—	102·0	110·7	—	118	120·5	—	—	—	—	—	—	—	—
107	♀	646	182	132·5	90·2	128·5	109·1	106·3	93·5	123·5	116	115	355	290	506	38·0	28·6	99·8	11·
,,	,,	647	172	131	96·4	129	110·1	110·3	89·2	126	125·5	104	355	—	—	34·1	25·8	98·1	12·
,,	,,	648	177	134·5	91·0	124·5	108·5	110·8	90·0	124	120·5	107	351	296	497	35·3	27·0	98·8	—
,,	,,	649	182	137·5	92·1?	133	115·0	111·9	—	130·5	124	—	—	308?	—	—	—	99·2	—
,,	,,	650	177	129·5	92·0	126	107·2	112·1	93·7	123	125·5	117·5	366	296	497	31·6	30·0	94·1	—
,,	,,	651	181	132·5	85·1	125	114·1	106·1	97·2	132	116	127	375	300	504	34·2	27·8	92·1	10·
,,	,,	652	188·5	134	93·2?	132	112·0	120·9	96·1	131	136	113·5	381	303	526	39·3	—	101·9	—
,,	,,	653	171	138	98·3	130·5	105·0	106·1	92·9	120	115	111·5	347?	297?	—	38·0	30·0	96·8	—
,,	♀?	654	—	128	—	—	—	122·0	—	—	134	—	—	—	—	—	—	—	—
116	♀	655	168·5	132·5	89·2	130	104·9	109·2	92·4	122·5	125	109	356	292	482	35·2	—	96·1	—
,,	,,	656	184·5	131·5	93·1	132·5	109·0	124·3	93·0	122·5	138	112·5?	373?	296	504	39·0?	—	98·1	—
,,	,,	657	178	137	87·9	134·5	114·0	114·3	90·9	134·5	127·5	112	374	311	499	35·0	27·0	94·2	—
,,	,,	658	185	138·5	97·4	—	112·3	120·1	—	130	131·5	—	—	305	522	—	—	—	—
,,	,,	659	169	134·5	96·1	128·5	106·8	107·1	95·3	126	120	113	360	304	488	34·6	—	90·3	12
,,	♀?	660	178·5	141	94·3	125	115·3	120·0	—	133·5	134	—	—	314	509	—	—	86·8	—
,,	,,	661	182·5	134·5	95·1	126·5	106·3	110·2	100·6	116·5	119·5	123·5	359	303	509	36·8	—	101·7	—
,,	,,	662	172	136	96·9	—	—	—	100·7?	—	—	117?	—	310?	—	—	30·0	—	—

male (cont.), and Queried Female Calvariae from Tomb 120*; and Female and Queried Female from Tombs* 107 *and* 11

ub	*LB*	*SC*	*SS*	100 $\frac{B}{L}$	100 $\frac{H'}{L}$	100 $\frac{B}{H'}$	100 $\frac{B-H'}{L}$	*Oc.I.*	100 $\frac{fmb}{fml}$	100 $\frac{SS}{SC}$	No.	Remarks
·2	—	—	—	74·9	74·4	100·8	0·6	59·6	94·2	—	604	Single small epipteric bone R; small pre-condyle supra-occipital R and L
—	101·4	—	—	74·9	75·1	99·6	−0·3	61·8	—	—	605	—
—	95·2	—	—	76·5	71·1	107·7	5·4	59·6	—	—	606	Coronal beginning to close
·0	93·1	8·7	4·2	79·4	77·1	103·0	2·3	60·2	82·9	48·3	607	—
·4	93·0	—	—	70·5	71·9	98·0	−1·4	56·1	83·5	—	608	—
·1	95·1	12·2	2·0	79·3	75·6	104·9	3·7	57·1	89·0	16·4	609	Coronal and sagittal beginning to close; L half of open and continued by crack
·0	95·9	12·0	5·3	75·9	74·1	102·3	1·7	63·3	85·7	44·2	610	—
·2	—	—	—	77·0	—	—	—	57·7	—	—	611	Coronal beginning to close; single large epipteric b
·0	103·1	—	—	73·9	77·9	95·0	−3·9	—	81·3	—	612	—
·0	—	—	—	81·2	—	—	—	61·2	—	—	613	—
·0	95·2	—	—	77·5	73·4	105·5	4·0	63·6	81·6	—	614	Single epipteric bone R
·0	93·0	—	—	80·3	75·8	105·9	4·5	60·0	85·8	—	615	Single small epipteric bone L, and 2 of moderate s
·1	93·3?	—	—	72·4	69·9	103·6	2·5	59·5	82·7	—	616	—
—	—	—	—	75·3	—	—	—	—	—	—	617	—
·2	100·2	—	—	76·6	73·8	103·8	·2·8	57·5	78·6	—	618	Coronal beginning to close; trace of horizontal sutu between temporal squama and parietal R
—	92·6	—	—	75·4	62·8	120·0	12·6	59·3	—	—	619	Coronal and sagittal beginning to close, lambdoid o lambda; occipito-mastoid obliterated R and L; parietals R and L
·9	—	—	—	76·5	—	—	—	61·9	—	—	620	Coronal beginning to close; occipito-mastoid nearl
—	98·2	10·0	5·8	74·4	75·5	98·5	−1·1	57·7	—	58·0	621	*Os épactal* divided by vertical suture
·4	103·2	—	—	71·8	66·2	108·5	5·6	58·2	78·9	—	622	Coronal beginning to close; small pre-condyle L
·0	—	9·0	5·3	70·6	—	—	—	57·5	—	58·9	623	Coronal beginning to close; fused nasal bones
·7	96·9	—	—	75·7	68·7	110·3	7·1	56·7	76·7	—	624	Single epipteric bone R
·3	98·0	—	—	77·0	73·9	104·3	3·2	59·7	77·1	—	625	Coronal and sagittal beginning to close; fronto-te
·2	99·0	—	—	74·3	74·6	99·6	−0·3	59·8	81·7	—	626	Small traumatic depression at ophryon
·6	103·9	11·4	5·0	72·5	69·0	105·1	3·5	57·0	85·4	43·9	627	Metopic; undivided *os épactal*
·9	—	—	—	73·2	—	—	—	—	—	—	628	Several wormian bones in sagittal; epipteric bone
·0	99·7	6·2	1·3	72·4	69·0	105·0	3·4	58·5	77·8	21·0	629	Metopic; pterion in *K*, and 2 epipteric bones L, a bones between temporal squama and parietal L
·1	91·4	13·3	3·9	75·2	72·1	104·3	3·1	57·7	83·6	29·3	630	Fronto-temporal articulation R and L
—	97·2	—	—	80·3	71·5	112·2	8·7	63·9	—	—	631	Coronal beginning to close
—	—	—	—	73·4	—	—	—	62·3	—	—	632	Coronal and sagittal beginning to close
·6	101·7	11·8?	5·0?	78·1	75·8	103·0	2·2	59·9	73·4	42·4	633	Two large wormian bones in sagittal above lamb parietal bones R and L; single small epipteric bo
—	98·8	6·5	2·1	76·8	76·2	100·7	0·6	—	—	32·3	634	Sagittal closing
·1	97·3	6·6?	—	71·9	74·4	96·6	−2·5	61·9	81·9	—	635	Coronal closing, sagittal beginning to close
·3	93·0	—	—	72·1	73·0	98·8	−0·9	62·1	76·7	—	636	Single median pre-condyle
—	—	—	—	77·7	—	—	—	—	—	—	637	Metopic; single epipteric bone R and 2 L
—	96·1	—	—	80·6	75·4	106·9	5·2	63·3	—	—	638	—
—	—	—	—	74·2	74·5	99·6	−0·3	—	—	—	639	Single epipteric bone R
·9	99·4	—	—	79·2	76·1	104·1	3·1	63·4	87·1	—	640	Metopic; metopic and coronal beginning to close
·2	98·8	10·0	4·1	79·4	82·0	96·9	−2·6	67·3	75·9	41·0	641	Fused nasal bones
·9	99·3	12·4	5·0	74·5	73·7	101·1	0·8	57·8	82·6	40·3	642	Coronal beginning to close; single epipteric bone
—	95·2?	—	—	79·0	73·9	106·9	5·1	58·4	—	—	643	Coronal and sagittal beginning to close
·5	—	12·5	4·0	71·8	—	—	—	60·3	—	32·0	644	Metopic; M3, R with additional cusp
—	—	—	—	74·9	—	—	—	—	—	—	645	Base and temporals missing
·6	99·8	11·0	4·2	72·8	70·6	103·1	2·2	58·1	75·3	38·2	646	—
·8	98·1	12·1?	5·2	76·2	75·0	101·6	1·2	63·1	75·7	43·0	647	—
·0	98·8	—	—	76·0	70·3	108·0	5·6	60·9	76·5	—	648	—
—	99·2	—	—	75·5	73·1	103·4	2·5	—	—	—	649	All sutures closing; traumatic depression (?) pari
·0	94·1	—	—	73·2	71·2	102·8	2·0	56·9	94·9	—	650	—
·8	92·1	10·3	4·4?	73·2	69·1	106·0	4·1	55·0	81·3	42·7	651	—
—	101·9	—	—	71·1	70·0	101·5	1·1	61·6	—	—	652	—
·0	96·8	—	—	80·7	76·3	105·7	4·4	60·0	78·9	—	653	Metopic; R half of suture between ex- and supra tinued as a crack
—	—	—	—	—	—	—	—	—	—	—	654	R temporal and base missing; small exostosis par
—	96·1	—	—	78·6	77·2	101·9	1·5	61·7	—	—	655	Coronal closing, sagittal and lambdoid beginning
—	98·1	—	—	71·3	71·8	99·0	−0·5	59·3	—	—	656	Sagittal beginning to close
·0	94·2	—	—	77·0	75·6	101·9	1·4	57·9	77·1	—	657	Coronal beginning to close, sagittal and lambdoid
—	—	—	—	74·9	—	—	—	—	—	—	658	—
—	90·3	12·6	7·5	79·6	76·0	104·7	3·6	61·1	—	59·5	659	Incomplete R half of palate, with canine below a
—	86·8	—	—	79·0	70·0	112·8	9·0	—	—	—	660	Occipito-mastoid obliterated R and L
—	101·7	—	—	73·7	69·3	106·3	4·4	58·2	—	—	661	—
·0	—	—	—	79·1	—	—	—	63·5	—	—	662	Coronal closing; large diseased (?) area on frontal

ulls. IX. *Female (cont.), and Queried Female Calvariae from Tomb* 120*; and Female and Queried Female*

U	*fml*	*fmb*	*LB*	*SC*	*SS*	100 $\frac{B}{L}$	100 $\frac{H'}{L}$	100 $\frac{B}{H'}$	100 $\frac{B-H'}{L}$	*Oc.I.*	100 $\frac{fmb}{fml}$	100 $\frac{SS}{SC}$	No.	
491	34·2	32·2	—	—	—	74·9	74·4	100·8	0·6	59·6	94·2	—	604	Single small epipteri supra-occipital R ar
489	36·0	—	101·4	—	—	74·9	75·1	99·6	−0·3	61·8	—	—	605	
499	40·0	—	95·2	—	—	76·5	71·1	107·7	5·4	59·6	—	—	606	Coronal beginning to
510	36·2	30·0	93·1	8·7	4·2	79·4	77·1	103·0	2·3	60·2	82·9	48·3	607	
481	34·0	28·4	93·0	—	—	70·5	71·9	98·0	−1·4	56·1	83·5	—	608	
—	32·7	29·1	95·1	12·2	2·0	79·3	75·6	104·9	3·7	57·1	89·0	16·4	609	Coronal and sagittal open and continued
498	35·0	30·0	95·9	12·0	5·3	75·9	74·1	102·3	1·7	63·3	85·7	44·2	610	
511	—	29·2	—	—	—	77·0	—	—	—	57·7	—	—	611	Coronal beginning to
—	36·9	30·0	103·1	—	—	73·9	77·9	95·0	−3·9	—	81·3	—	612	
—	—	29·0	—	—	—	81·2	—	—	—	61·2	—	—	613	
501	39·2	32·0	95·2	—	—	77·5	73·4	105·5	4·0	63·6	81·6	—	614	Single epipteric bone
487	33·8	29·0	93·0	—	—	80·3	75·8	105·9	4·5	60·0	85·8	—	615	Single small epipteri
—	35·2	29·1	93·3?	—	—	72·4	69·9	103·6	2·5	59·5	82·7	—	616	
—	—	—	—	—	—	75·3	—	—	—	—	—	—	617	
506	38·4	30·2	100·2	—	—	76·6	73·8	103·8	2·8	57·5	78·6	—	618	Coronal beginning to between temporal s
—	34·9	—	92·6	—	—	75·4	62·8	120·0	12·6	59·3	—	—	619	Coronal and sagittal lambda; occipito-m parietals R and L
517	—	28·9	—	—	—	76·5	—	—	—	61·9	—	—	620	Coronal beginning to
502	35·5	—	98·2	10·0	5·8	74·4	75·5	98·5	−1·1	57·7	—	58·0	621	*Os épactal* divided by
532	36·0	28·4	103·2	—	—	71·8	66·2	108·5	5·6	58·2	78·9	—	622	Coronal beginning to
504	—	25·0	—	9·0	5·3	70·6	—	—	—	57·5	—	58·9	623	Coronal beginning to
516	36·1	27·7	96·9	—	—	75·7	68·7	110·3	7·1	56·7	76·7	—	624	Single epipteric bone
489	35·4	27·3	98·0	—	—	77·0	73·9	104·3	3·2	59·7	77·1	—	625	Coronal and sagittal
508	34·5	28·2	99·0	—	—	74·3	74·6	99·6	−0·3	59·8	81·7	—	626	Small traumatic dep
522	37·0	31·6	103·9	11·4	5·0	72·5	69·0	105·1	3·5	57·0	85·4	43·9	627	Metopic; undivided
—	—	31·9	—	—	—	73·2	—	—	—	—	—	—	628	Several wormian bo
529	34·7	27·0	99·7	6·2	1·3	72·4	69·0	105·0	3·4	58·5	77·8	21·0	629	Metopic; pterion in bones between tem
501	34·8	29·1	91·4	13·3	3·9	75·2	72·1	104·3	3·1	57·7	83·6	29·3	630	Fronto-temporal art
514	33·6	—	97·2	—	—	80·3	71·5	112·2	8·7	63·9	—	—	631	Coronal beginning t
—	—	—	—	—	—	73·4	—	—	—	62·3	—	—	632	Coronal and sagittal
? 504?	36·5	26·8	101·7	11·8?	5·0?	78·1	75·8	103·0	2·2	59·9	73·4	42·4	633	Two large wormian parietal bones R ar
511	—	—	98·8	6·5	2·1	76·8	76·2	100·7	0·6	—	—	32·3	634	Sagittal closing
505	39·2	32·1	97·3	6·6?	—	71·9	74·4	96·6	−2·5	61·9	81·9	—	635	Coronal closing, sagi
486	34·3	26·3	93·0	—	—	72·1	73·0	98·8	−0·9	62·1	76·7	—	636	Single median pre-c
512	—	—	—	—	—	77·7	—	—	—	—	—	—	637	Metopic; single epip
502	40·2	—	96·1	—	—	80·6	75·4	106·9	5·2	63·3	—	—	638	
494	—	—	—	—	—	74·2	74·5	99·6	−0·3	—	—	—	639	Single epipteric bon
? 519?	38·9	33·9	99·4	—	—	79·2	76·1	104·1	3·1	63·4	87·1	—	640	Metopic; metopic ar
511	39·8	30·2	98·8	10·0	4·1	79·4	82·0	96·9	−2·6	67·3	75·9	41·0	641	Fused nasal bones
508	35·0?	28·9	99·3	12·4	5·0	74·5	73·7	101·1	0·8	57·8	82·6	40·3	642	Coronal beginning t
—	34·0	—	95·2?	—	—	79·0	73·9	106·9	5·1	58·4	—	—	643	Coronal and sagittal
526	—	28·5	—	12·5	4·0	71·8	—	—	—	60·3	—	32·0	644	Metopic; M3, R wit
—	—	—	—	—	—	74·9	—	—	—	—	—	—	645	Base and temporals
506	38·0	28·6	99·8	11·0	4·2	72·8	70·6	103·1	2·2	58·1	75·3	38·2	646	
—	34·1	25·8	98·1	12·1?	5·2	76·2	75·0	101·6	1·2	63·1	75·7	43·0	647	
497	35·3	27·0	98·8	—	—	76·0	70·3	108·0	5·6	60·9	76·5	—	648	
—	—	—	99·2	—	—	75·5	73·1	103·4	2·5	—	—	—	649	All sutures closing;
497	31·6	30·0	94·1	—	—	73·2	71·2	102·8	2·0	56·9	94·9	—	650	
504	34·2	27·8	92·1	10·3	4·4?	73·2	69·1	106·0	4·1	55·0	81·3	42·7	651	
526	39·3	—	101·9	—	—	71·1	70·0	101·5	1·1	61·6	—	—	652	
? —	38·0	30·0	96·8	—	—	80·7	76·3	105·7	4·4	60·0	78·9	—	653	Metopic; R half of tinued as a crack
—	—	—	—	—	—	—	—	—	—	—	—	—	654	R temporal and bas
482	35·2	—	96·1	—	—	78·6	77·2	101·9	1·5	61·7	—	—	655	Coronal closing, sag
504	39·0?	—	98·1	—	—	71·3	71·8	99·0	−0·5	59·3	—	—	656	Sagittal beginning t
499	35·0	27·0	94·2	—	—	77·0	75·6	101·9	1·4	57·9	77·1	—	657	Coronal beginning t
522	—	—	—	—	—	74·9	—	—	—	—	—	—	658	
488	34·6	—	90·3	12·6	7·5	79·6	76·0	104·7	3·6	61·1	—	59·5	659	Incomplete R half
509	—	—	86·8	—	—	79·0	70·0	112·8	9·0	—	—	—	660	Occipito-mastoid ol
509	36·8	—	101·7	—	—	73·7	69·3	106·3	4·4	58·2	—	—	661	
? —	—	30·0	—	—	—	79·1	—	—	—	63·5	—	—	662	Coronal closing; lar

eried Female from Tombs 107 *and* 116

Remarks
gle small epipteric bone R; small pre-condyle L; trace of suture between ex- and pra-occipital R and L
onal beginning to close
—
—
onal and sagittal beginning to close; L half of suture between ex- and supra-occipital en and continued by crack
—
onal beginning to close; single large epipteric bone R; single median pre-condyle
—
—
gle epipteric bone R
gle small epipteric bone L, and 2 of moderate size R
—
—
onal beginning to close; trace of horizontal suture of interpárietal bones; wormian bone tween temporal squama and parietal R
onal and sagittal beginning to close, lambdoid obliterated, except for parts approaching mbda; occipito-mastoid obliterated R and L; posterior parts of temporals fused to rietals R and L
onal beginning to close; occipito-mastoid nearly obliterated R and L
épactal divided by vertical suture
onal beginning to close; small pre-condyle L
onal beginning to close; fused nasal bones
gle epipteric bone R
onal and sagittal beginning to close; fronto-temporal articulation R, L pterion normal
all traumatic depression at ophryon
topic; undivided *os épactal*
eral wormian bones in sagittal; epipteric bone L
topic; pterion in *K*, and 2 epipteric bones L, single epipteric bone R; 2 small wormian ones between temporal squama and parietal L
onto-temporal articulation R and L
onal beginning to close
onal and sagittal beginning to close
o large wormian bones in sagittal above lambda; trace of horizontal suture of inter-rietal bones R and L; single small epipteric bone L, single large epipteric bone R
gittal closing
onal closing, sagittal beginning to close
gle median pre-condyle
topic; single epipteric bone R and 2 L
—
gle epipteric bone R
topic; metopic and coronal beginning to close
sed nasal bones
onal beginning to close; single epipteric bone R
onal and sagittal beginning to close
topic; M3, R with additional cusp
se and temporals missing
—
—
—
l sutures closing; traumatic depression (?) parietal R
—
—
—
etopic; R half of suture between ex- and supra-occipital R open and apparently continued as a crack
temporal and base missing; small exostosis parietal R
oronal closing, sagittal and lambdoid beginning to close
gittal beginning to close
oronal beginning to close, sagittal and lambdoid closing
—
ncomplete R half of palate, with canine below alveolar margin and grossly deflected
ccipito-mastoid obliterated R and L
—
oronal closing; large diseased (?) area on frontal; possibly deformed artificially

107	686	176·5	129·5	129·5	126	125·5	115	367	35·7	29·0	—	—	—	—	—	73·4	73·4	100·0	0

Group 3. First permanent molars completely erupted; second permanent molars with occlusal surfaces above alveolar margin, but not fully erupted.

120	687	—	—	—	—	—	110	—	37·9	27·2	26·9	40·0	—	—	—	—	—	—	—
,,	688	176·5	130·5	129	130·5	129·5	—	—	—	—	—	—	—	—	—	73·9	73·1	101·2	0·8
107	689	185	127·5	124·5	129	137	113	379	42·8	28·0	22·2	41·0	—	—	—	68·9	67·3	102·4	1·6

Group 4. Second permanent molars completely erupted, but some front teeth not completely erupted.

120	690	172	135	—	129	121	—	—	—	—	—	43·1	37·2	30·9	—	78·5	—	—	—
107	691	182·5	—	130	120	123	123	366	37·2	28·0	21·1?	43·1	37·2	32·0?	113·5	—	71·2	—	—
,,	692	182	133·5	129	124·5	141	112·5	378	38·1	32·2	22·8	44·8	36·2	32·8	—	73·4	70·9	103·5	2·5

Group 5. Second molars and all other permanent teeth anterior to them fully erupted, but third molars not fully erupted. Basal suture not obliterated, unless otherwise stated.

120	693	171·5?	138	—	123	111	—	—	—	28·4	23·1	46·5	39·7	32·0	114·5	80·5	—	—	—
,,	694	180·5	139·5	128·5	—	—	109·5	—	41·2	29·2	—	—	41·8	34·3?	118·5	77·3	71·2	108·6	6·1
,,	695	179	136	125·5	129·5	123·5	118	372	35·2	24·8	—	42·8	37·1	31·3	110	76·0	70·1	108·4	5·9
,,	696	174·5	126·5	125	117	133	—	—	—	30·3	22·1	42·4	37·7	30·2	—	72·5	71·6	101·2	0·9
,,	697	174·5	133	132·5	128	132	104	364	40·0	26·1	—	44·0	38·1	28·0	116	76·2	75·9	100·4	0·3
,,	698	183	131·5	122·5	123·5	112	143·5	379	33·0	30·2	26·3	46·0	37·1	30·0	115·5	71·9	66·9	107·3	4·9
,,	699	176·5	132·5	127·5	121·5	125·5	115	361	32·8	29·5	24·8?	46·1	—	33·5	110·5	75·1	72·2	103·9	2·8
,,	700	174?	138·5	122·5	132?	122	104·5	358?	35·8	26·5	—	—	—	—	—	79·6	70·4	113·1	9·2
,,	701	181·5	140	—	128·5	129	110·5	367	—	28·3	23·1	50·0	38·3	31·0	—	77·1	—	—	—
,,	702	176	131	—	125	122	111?	358?	—	—	23·7	44·1	40·0	32·8	—	74·4	—	—	—
,,	703	179·5	130·5	132	125·5	124·5	108·5	358	37·1	30·0	22·8	49·9	40·2	32·9	—	72·7	73·5	98·9	−0·8
107	704	173?	132	130·5	—	129	—	—	—	32·3	—	—	—	—	—	76·3	75·4	101·1	0·9
,,	705	181	139·5	134·5	126·5	121·5	127	375	39·0	30·8	24·1	51·1	38·5	30·5	—	77·1	74·3	103·7	2·8
,,	706	—	—	—	—	—	—	—	—	—	22·2	46·4	39·1	—	—	—	—	—	—
116	707	172	129	128	124·5	127	—	—	—	—	—	45·0	38·3	31·0	—	75·0	74·4	100·8	0·6
108	708	171·5	131·5	—	—	—	113	—	32·8	25·1	24·0	44·5	—	—	—	76·7	—	—	—

Group 6. Apparently complete separation of basi-occipital from supra-occipital.

120	711	171	134·5?	—	—	121	—	—	—	—	—	—	—	—	—	78·7	—	—	—
,,	712	157	132	—	112?	123·5	100	335?	—	—	—	—	—	—	—	84·1	—	—	—

Group 7. Traces of sutures between basi-occipital and supra-occipital.

120	713	179	139	136·5	137?	129?	112	378?	35·8	32·3	—	—	—	—	—	77·7	76·3	101·8	1·4
,,	714	178·5	135·5	—	128·5	137·5	—	—	—	—	—	—	—	—	—	75·9	—	—	—
,,	715	183	143	123·5	130·5	131	122·5	384	39·9	26·9	—	—	—	—	—	78·1	67·5	115·8	10·7
,,	716	181?	125·5	129?	—	143	105·5	—	45·0	33·2	—	—	—	—	—	69·3	71·3	97·3	−1·9

Group 8. Sutures between basi-occipital and supra-occipital obliterated, but apparently not nearly mature.

120	717	171·5	—	128	—	121	110	—	38·5	27·0	—	—	—	—	—	—	74·6	—	—
,,	718	179·5	140·5	130	126	129·5	—	—	—	—	—	—	—	—	—	78·3	72·4	108·1	5·8
,,	719	172·5	126·5	125·5	—	105	134·5	—	36·1	25·9	—	—	—	—	—	73·3	72·8	100·8	0·6
,,	720	164	124·5	127·0	118·5?	118·5?	105·5	343	31·7	25·8	—	—	—	—	—	75·9	77·4	98·0	−1·5
,,	721	169·5?	132·5	125·5	—	131?	113	—	35·0	31·0	—	—	—	—	—	78·2	74·0	105·6	4·1
,,	722	181?	128	127·5	130·5	128·5	122·5	382	35·3	24·6	—	—	—	—	—	70·7	70·4	100·4	0·3
,,	723	169	—	124	119?	123	112·5	354?	35·1	27·2	—	—	—	—	—	—	73·4	—	—
,,	724	167·5	133·5	125	128?	128	—	—	—	—	—	—	—	—	—	79·7	74·6	106·8	5·1
,,	725	167	129·5	—	125·5	—	—	—	—	—	—	—	—	—	—	77·5	—	—	—
,,	726	177	130·5	129·5	134	134·5	—	—	—	—	—	—	—	—	—	73·7	73·2	100·8	0·6
,,	727	176·5	129·5	127	123?	122	115	360	35·1	26·5	—	—	—	—	—	73·4	72·0	102·0	1·4
,,	728	154	129	117	—	109·5	104	—	32·1	25·0	—	—	—	—	—	83·8	76·0	110·3	7·8
,,	729	155·5	128·5	—	115	108·5	103	327	—	—	—	—	—	—	—	82·6	—	—	—
107	730	171·5	128	120	109·5	120·5	118·5	349	35·8	28·0	—	—	—	—	—	74·6	70·0	106·6	4·7
,,	731	164	125	—	—	115·5	108	—	—	—	—	—	—	—	—	76·2	—	—	—

Group 9. Basal suture open, but apparently nearly mature.

120	732	170·5	136	130·5	125?	125·5?	110	360	33·4	30·7	—	—	—	—	—	79·8	76·5	104·2	3·2
,,	733	181	137·5	140	129·5?	137·5	107	374?	36·5	28·9	—	—	—	—	—	76·0	77·3	98·2	−1·4
,,	734	—	132	139·5	—	—	121	—	41·5	—	—	—	—	—	—	—	—	94·6	—
,,	735	176·5	137	—	123·5?	125	109	357	—	—	—	—	—	—	—	77·6	—	—	—
,,	736	182·5	130	—	129·5	133	—	—	—	—	—	—	—	—	—	71·2	—	—	—

0·0	0	81·2	—	—	686	—
—	—	71·8	67·2	—	687	Trace of suture between ex- and supra-occipital R
1·2	0·8	—	—	—	688	Trace of horizontal suture of interparietal bones R; L side of palate missing
2·4	1·6	65·4	54·1	—	689	—

ted.

—	—	—	—	83·1	690	*Os pentagonale* separate; R side of palate missing
—	—	75·3	49·0	86·0	691	—
3·5	2·5	84·5	50·9	90·6	692	Trace of sutures between ex- and supra-occipital R and L

d

—	—	—	49·7	80·6	693	Fronto-temporal articulation, and single epipteric bone L; single epipteric bone R; basal suture obliterated
8·6	6·1	70·9	—	82·1	694	Metopic; trace of horizontal suture of interparietal bones R
8·4	5·9	70·5	—	84·4	695	—
1·2	0·9	—	52·1	80·1	696	Single pre-condyle L
0·4	0·3	65·2	—	73·5	697	Healed injury above R orbit and L parietal; tubercles of Carabelli
7·3	4·9	91·5	57·2	80·9	698	Trace of horizontal suture of interparietal bones L; healed injury above L orbit
3·9	2·8	89·9	53·8	—	699	Large pre-condyle L; R jugular foramen abnormally large and L abnormally small
3·1	9·2	74·0	—	—	700	—
—	—	—	46·2	80·9	701	—
—	—	—	53·7	82·0	702	—
8·9	−0·8	80·9	45·7	81·8	703	Metopic
1·1	0·9	—	—	—	704	Sutures between ex- and supra-occipitals half open
3·7	2·8	79·0	47·2	79·2	705	Metopic; supernumerary tooth (?) erupting obliquely behind I 1, R
—	—	—	47·8	—	706	—
0·8	0·6	—	—	80·9	707	Additional and separate cusp on M 2, R
—	—	76·5	53·9	—	708	—

Without facial skeletons

—	—	—	—	—	711	—
—	—	—	—	—	712	Metopic
1·8	1·4	90·2	—	—	713	—
—	—	—	—	—	714	—
5·8	10·7	67·4	—	—	715	—
7·3	−1·9	73·8	—	—	716	Metopic; trace of horizontal suture of interparietal bones R and L. The foraminal length (*fml*) of this skull is actually greater than the largest found for an adult male specimen, viz. 44·9 mm.

—	—	70·1	—	—	717	—
8·1	5·8	—	—	—	718	Trace of horizontal suture of interparietal bones R
0·8	0·6	71·7	—	—	719	*Os épactal* divided by horizontal suture half obliterated
8·0	−1·5	81·4	—	—	720	Fronto-temporal articulation R and L
5·6	4·1	88·6	—	—	721	—
0·4	0·3	69·7	—	—	722	—
—	—	77·5	—	—	723	Healed wound on frontal bone
6·8	5·1	—	—	—	724	—
—	—	—	—	—	725	Metopic
0·8	0·6	—	—	—	726	—
2·0	1·4	75·4	—	—	727	Trace of horizontal suture of interparietal bones R and L
0·3	7·8	77·9	—	—	728	Metopic
—	—	—	—	—	729	—
6·6	4·7	78·2	—	—	730	—
—	—	—	—	—	731	—

4·2	3·2	91·9	—	—	732	—
8·2	−1·4	79·2	—	—	733	—
4·6	—	—	—	—	734	—
—	—	—	—	—	735	Metopic
—	—	—	—	—	736	—

LIST OF PLATES

Plate I. Views of the interior of Tomb 120.
A. Entrance of tomb showing skulls round side.
B. Interior of tomb with skulls collected round side.

Plate II. Tomb 120, collections of skulls *in situ*.
A. Skulls collected round wall.
B. A closer view of skulls seen to the left of the beam in Plate IA.

Plate III. Exceptional crania.
A. A female skull (No. 485) showing extensive burnt patch.
B. A male skull (No. 108) showing injuries probably inflicted shortly before death: see p. 104.
C, D. Male skulls (No. 156, Tomb, 107 and No. 179, Tomb 116) of exceptional type; no other specimens of this type were found.

Plate IV. A male cranium (No. 340) with hole in right parietal, probably a trepan, and a sword-cut on the same bone close to the coronal suture.

Plate V. Male crania with trepanned openings.
A. No. 115. There is a septic area round the trepan in the right parietal. A cut probably made by the trepanner can be seen by the lateral outline of the left parietal.
B. No. 114.

Plate VI. Crania showing marked artificial deformation.
A. A male cranium (No. 381).
B. A female cranium (No. 673).

Plate VII. *Norma lateralis* views of five male crania (A–E) believed to be artificially deformed to a slight extent, and of a male cranium (F), probably deformed owing to premature closing of the coronal suture. A, No. 378; B, No. 375; C, No. 377; D, No. 376; E, No. 379; F, No. 380.

Plate VIII. *Norma verticalis* views of three anomalous crania.
A. A male cranium (No. 380) with coronal suture obliterated. A *norma lateralis* view of this deformed specimen is shown in Plate VIIF.
B. A female cranium (No. 454) with healed injury on left side of the frontal bone.
C. A metopic female cranium (No. 670) with wound on left parietal bone and the sagittal suture prematurely obliterated.

Plate IX. *Norma lateralis* views of a typical female (No. 388) and a typical male (No. 28) cranium of the Lachish series. Other views of these two specimens are shown in Plates X–XII. They were selected from among the more complete skulls on account of the fact that their measurements of shape show no marked divergences from the means for the series. In the case of No. 388, none of these measurements differ from the female mean by more than the standard deviation of the distribution. The same is true for No. 28 in comparison with the male means. All the photographs of the typical skulls are approximately 0·6 natural size (linear dimensions).

Plate X. *Norma facialis* views of a typical female (No. 388) and a typical male (No. 28) cranium of the Lachish series.

Plate XI. *Norma verticalis* views of a typical female (No. 388) and a typical male (No. 28) cranium of the Lachish series.

Plate XII. *Norma occipitalis* views of a typical female (No. 388) and a typical male (No. 28) cranium of the Lachish series.

Plate XIII. Crania showing premature obliteration of the sagittal suture.

A. A female cranium (No. 672) showing complete obliteration of the suture with distortion.
B. A male cranium (No. 364) showing complete obliteration of the suture with distortion.
C. A female cranium (No. 667) showing complete obliteration of the suture with distortion.
D. A male cranium (No. 366) showing complete obliteration of the suture without apparent distortion.
E. A female cranium (No. 577) showing almost complete obliteration of the suture and post-coronal constriction.

Plate XIV. Three male crania with sutural anomalies.

A. No. 299. Large wormian bone in right side of coronal suture.
B. No. 60. Two symmetrical interparietal bones of unusual form.
C. No. 330. Left side, showing temporal bone largely fused to parietal. The right side of this specimen is affected in a similar way.

Plate XV. Anomalous regions of two male crania.

A. No. 324. Complete absence of right auricular passage: twice natural size. The left auricular passage is normal.
B. No. 301. Complete absence of left jugular foramen: 1·3 times natural size. The right foramen is normal.

Plate XVI. A male cranium (No. 382) of unusual form, possibly affected by hydrocephaly. A, *Norma lateralis*; B, *Norma facialis*; C, *Norma verticalis*.

Plate XVII. A female cranium (No. 662) with large diseased area on frontal bone (see p. 115). A, *Norma lateralis*; B, *Norma facialis*; C, *Norma verticalis*.

Plate XVIII. Exceptional crania.

A. A male cranium (No. 5) with large healed wound on frontal bone.
B. A male cranium (No. 1) with diseased area on right parietal.
C. A metopic female cranium (No. 419) with injury to the right frontal bone.
D. A female cranium (No. 513) with depression in left parietal.

Plate XIX. A female cranium (No. 518) showing a tooth (upper right second molar) with a filling presumed to be adventitious.

A. The palate (1·2 diameters).
B. The second molar with the filling *in situ* (6·0 diameters).
C. A skiagram (2·7 diameters) showing the depth of the filling.
D. The three molars (2·7 diameters) after removal of the filling in the second molar.

The skiagram in this plate and others in Plates XXII and XXIII were kindly provided by Mr C. Bowdler Henry, M.R.C.S.

Plate XX. A female skull (No. 437) with anomalous jaws.

A. The mandible from above, showing denticles outside the dental arch.
B. The palate showing a diastema between the central incisors, and diastemae between the lateral incisors and canines.
C. The right side of the mandible showing denticles.

Plate XXI. Palates with anomalous dentitions.

A. A female cranium (No. 445) with sockets for three incisors only.
B. A male cranium (No. 132) with diastemae between canines and premolars, and third molars absent.
C. A juvenile cranium (No. 705) with supernumerary tooth behind the right central incisor.
D. A female cranium (No. 401) with grossly deflected left canine.

Plate XXII. Photographs and skiagrams of jaws with anomalous dentitions.

A. A female cranium (No. 506) showing incomplete eruption of the second and third right molars.
B. Skiagram of the same specimen (A) showing that the roots of the partially erupted teeth were fully formed.
C. A female cranium (No. 383) showing a denticle in the palate.
D. Skiagram of the same (C) showing the limits of the denticle and its crypt.
E. A female cranium (No. 496) with impaction of the upper left third molar.
F. Skiagram of the right side of a mandible showing two denticles in the corpus, female (No. 1056).

Plate XXIII. Jaws with anomalous dentitions or cysts.

A. A skiagram of a juvenile jaw (No. 1068), right side, showing canine and first premolar unerupted.
B. A skiagram of the same mandible (A), left side, showing the same teeth on this side unerupted (see p. 120).
C. A male skull (No. 72) showing the third molar on the left side in abnormal position.
D. Occlusal view of the same jaw (C) showing the abnormal position of the third molar, rotation of the second premolar on the left side, a retained milk canine, and absence of the third molar on the right side (see description on p. 119).
E. A female cranium (No. 469) showing a large cyst in the right molar region.
F. A female cranium (No. 467) showing a large cyst in the anterior part of the palate penetrating to the nasal aperture.

REFERENCES

BATRAWI, A. M. EL (1935). "Report on the human remains." *Miss. Archéol. de Nubie*, 1929–34. Cairo.

BONIN, G. VON & MORANT, G. M. (1938). "Indian races in the United States. A survey of previously published cranial measurements." *Biometrika*, **30**, 94–129.

BUXTON, L. H. D. & MORANT, G. M. (1933). "The essential craniological technique. Part I. Definitions of points and planes." *J. Roy. Anthrop. Inst.* **63**, 19–47.

CLEAVER, F. H. (1937). "A contribution to the biometric study of the human mandible." *Biometrika*, **29**, 80–112.

COLLETT, M. (1933). "A study of the 12th and 13th dynasty skulls from Kerma." *Biometrika*, **25**, 254–84.

DAVIN, A. G. & PEARSON, K. (1924). "On the biometric constants of the human skull." *Biometrika*, **16**, 328–63.

DINGWALL, E. J. (1931). *Artificial Cranial Deformation*. London.

DOUBLE, A.-F. LE (1903). *Traité des Variations des Os du Crâne de l'Homme*. Paris.

GENNA, G. E. (1938). *Spedizioni Scientifiche del Comitato Italiano per lo Studio dei Problemi della Popolazione. Seconda Spedizione: i Samaritani. Anthropologia*, **1**. Rome.

HOOKE, B. G. E. (1926). "A third study of the English skull with special reference to the Farringdon Street crania." *Biometrika*, **18**, 1–54.

HRDLIČKA, A. (1932–3). "Seven prehistoric American skulls with complete absence of external auditory meatus." *Amer. J. Phys. Anthrop.* **17**, 355–77.

HUXLEY, H. M. (1906). "Zur Anthropologie der Samaritaner." *Z. Demogr. Statist. Jud.* **2**, 137–9.

KAPPERS, C. U. ARIENS (1934).. *An Introduction to the Anthropology of the Near East in Ancient and Recent Times*. Amsterdam.

MACALISTER, R. A. S. (1912). *Excavations at Gezer*, **1**. London.

MACDONELL, W. R. (1904). "A study of the variation and correlation of the human skull, with special reference to English crania." *Biometrika*, **3**, 191–244.

MARTIN, E. S. (1936). "A study of an Egyptian series of mandibles with special reference to mathematical methods of sexing." *Biometrika*, **28**, 149–78.

MATIEGKA, J. (1926). "On the craniology of the Jews. I. The skulls from the Old Cemetery, Prague, V." *Anthrop., Prague*, **4**, 163–219.

MORANT, G. M. (1925). "A study of Egyptian craniology from prehistoric to Roman times." *Biometrika*, **17**, 1–52.

—— (1928). "A preliminary classification of European races based on cranial measurements." *Biometrika*, **20B**, 301–75.

—— (1935). "A study of predynastic Egyptian skulls from Badari based on measurements taken by Miss B. N. Stoessiger and Prof. D. E. Derry." *Biometrika*, **27**, 293–308.

MORANT, G. M., COLLETT, M. & ADYANTHĀYA, N. K. (1936). "A biometric study of the human mandible." *Biometrika*, **28**, 84–122.

MORANT, G. M. & HOADLEY, M. F. (1931). "A study of the recently excavated Spitalfields crania." *Biometrika*, **23**, 191–248.

PARRY, T. WILSON (1936). "Three skulls from Palestine showing two types of primitive surgical holing." *Man*, **36**, 170–1.

PEARSON, K. & STOESSIGER, B. N. (1927). "On further formulae for the reconstruction of cranial capacity from external measurements of the skull." *Biometrika*, **19**, 211–14.

STARKEY, J. L. (1936). "Discovery of skulls with surgical holing at Tell Duweir, Palestine." *Man*, **36**, 169–70.

STOESSIGER, B. N. (1927). "A study of the Badarian crania recently excavated by the British School of Archaeology in Egypt." *Biometrika*, **19**, 110–50.

STOESSIGER, B. N. & MORANT, G. M. (1932). "A study of the crania in the vaulted ambulatory of St Leonard's Church, Hythe." *Biometrika*, **24**, 135–202.

SZPIDBAUM, H. (1927). "Die Samaritaner." *Mitt. anthrop. Ges. Wien*, **57**, 139–58.

THOMSON, A. & MACIVER, D. RANDALL (1905). *The Ancient Races of the Thebaid*. Oxford.

WEISSENBERG, S. (1909). "Die autochtone Bevölkerung Palastinas in anthropologischer Beziehung." *Z. Demogr. Statist. Jud.* **5**, 129–39.

WOO, T. L. (1930). "A study of seventy-one ninth dynasty Egyptian skulls from Sedment." *Biometrika*, **22**, 65–93.

—— (1937). "A biometric study of the human malar bone." *Biometrika*, **29**, 113–23.

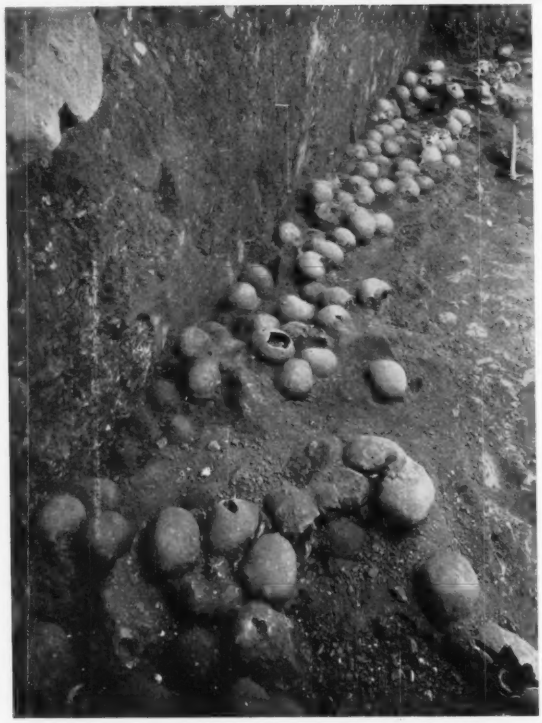
WOO, T. L. & MORANT, G. M. (1932). "A preliminary classification of Asiatic races based on cranial measurements." *Biometrika*, **24**, 108–34.

—— —— (1934). "A biometric study of the 'flatness' of the facial skeleton in man." *Biometrika*, **26**, 196–250.

A

B

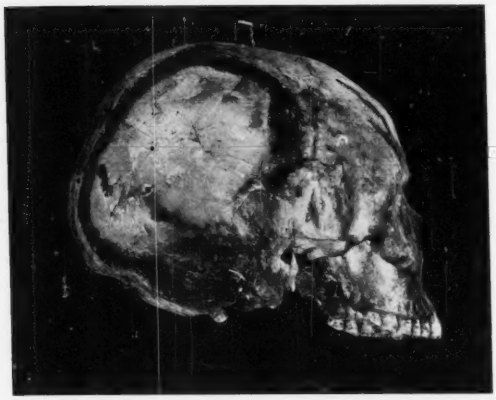
Views of the interior of Tomb 120.



A

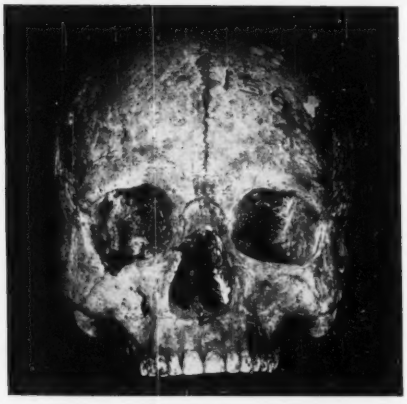
B

Tomb 120, collections of skulls *in situ*.



A. A female skull (No. 485) showing extensive burnt patch. .

B. A male skull (No. 108) showing injuries which probably caused death.

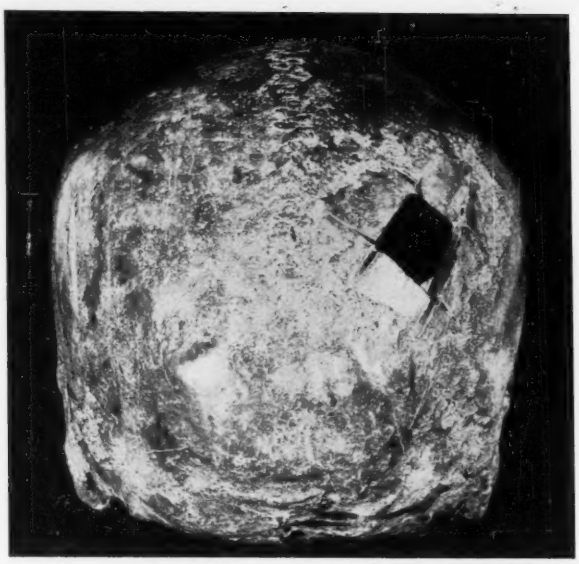


C. A male skull (No. 156) of exceptional type.

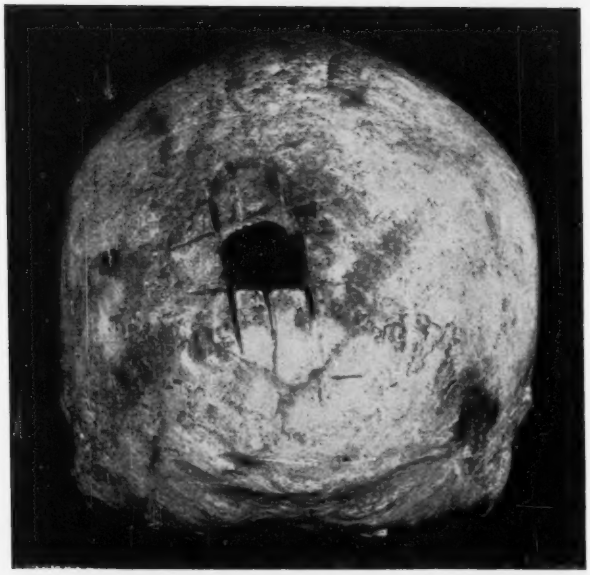
D. A male skull (No. 179) of exceptional type and similar to C.

Exceptional crania.

A male cranium (No. 340) with hole in right parietal, probably a trepan, and sword-cut on same bone close to coronal suture.

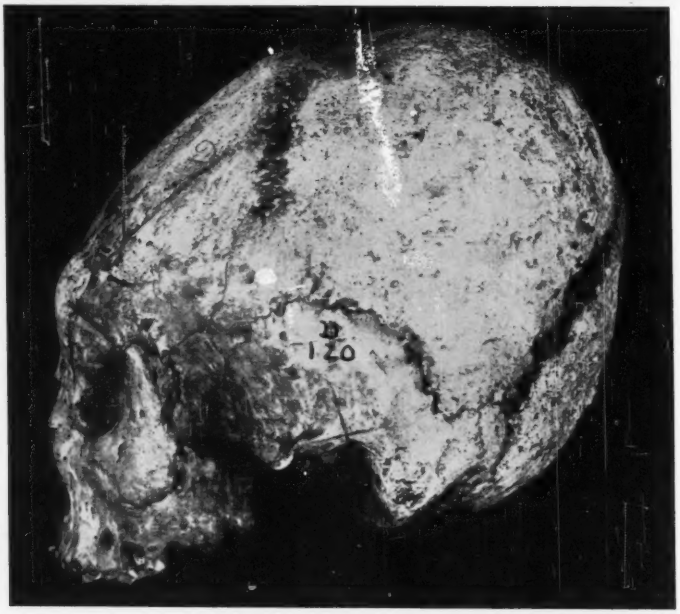


A. No. 115.



B. No. 114.

Male crania with trepanned openings.

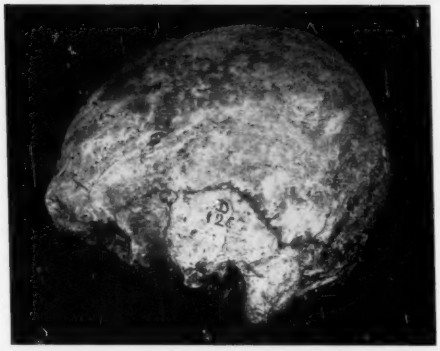


A. No. 381.

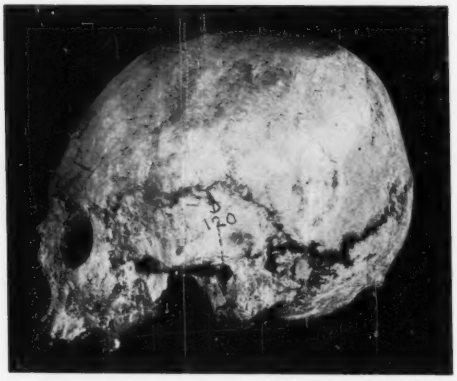
B. No. 673.

Marked artificial deformation of a male (A) and a female (B) cranium.

A. No. 378.



B. No. 375.



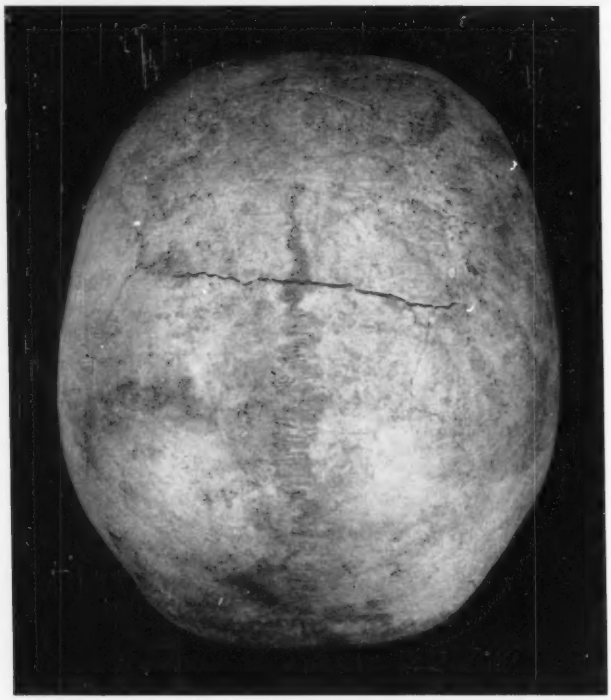
C. No. 377.

D. No. 376.

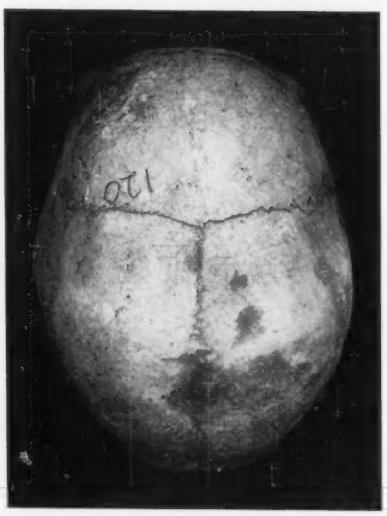
E. No. 379.

F. No. 380.

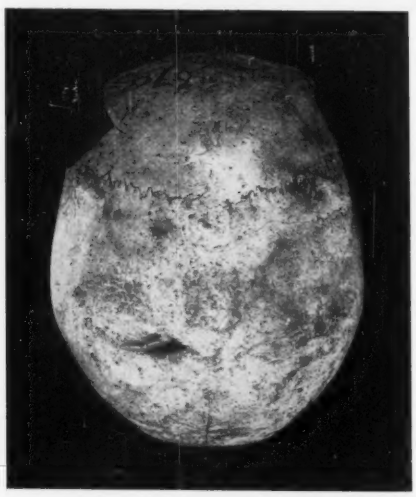
Five male crania (A–E) believed to be artificially deformed to a slight extent and a male cranium (F) probably deformed owing to premature closing of the coronal suture.



A. Coronal suture obliterated (No. 380, male).



B. Wound on left of frontal bone (No. 454, female).

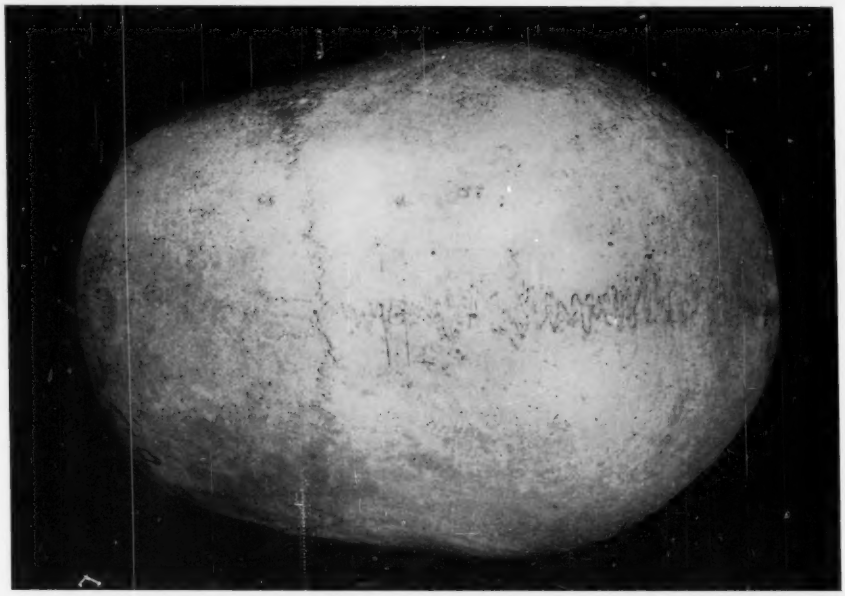


C. Wound on left parietal bone and sagittal suture obliterated (No. 670, female).

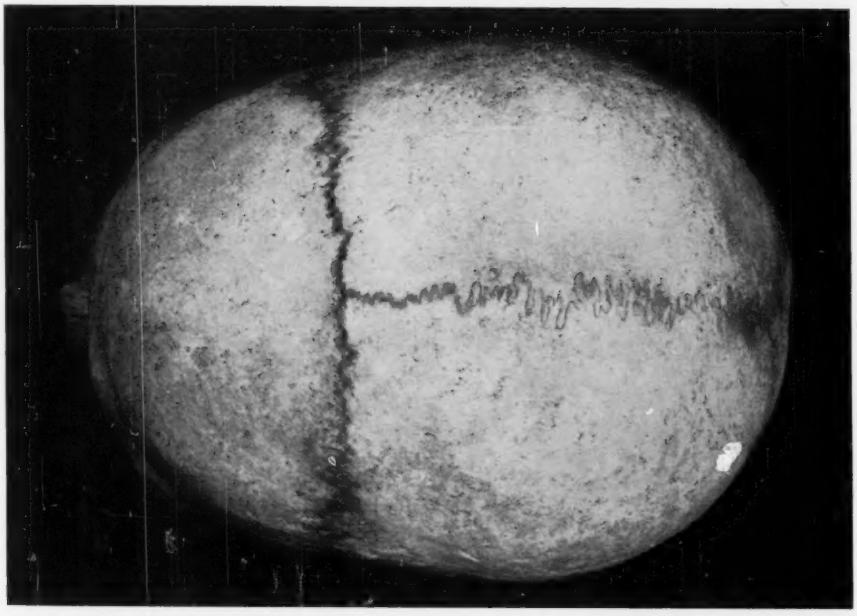
Anomalous crania.

A typical female (No. 388, above) and a typical male cranium (No. 28) of the Lachish series.

A typical female (left) and a typical male cranium of the Lachish series.



A. No 388, female.

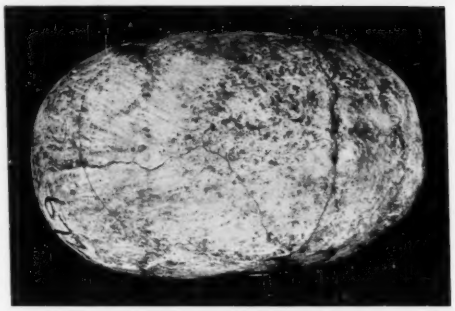


B. No. 28, male.

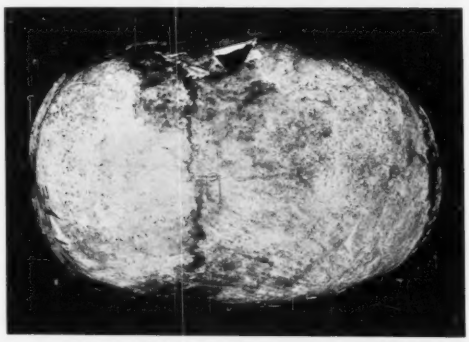
A typical female and a typical male cranium of the Lachish series.

A typical female (left) and a typical male cranium of the Lachish series.

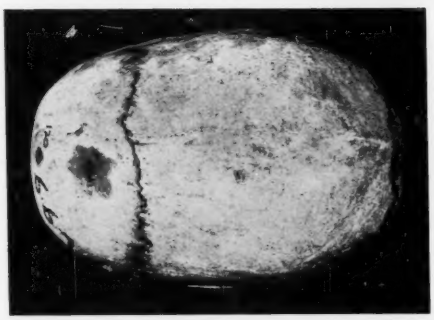
A typical female (No. 388, above) and a typical male cranium (No. 28) of the Lachish series.



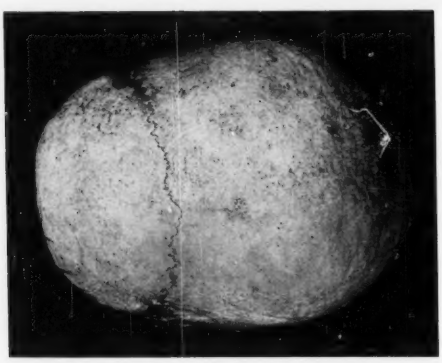
A. No. 672, female.



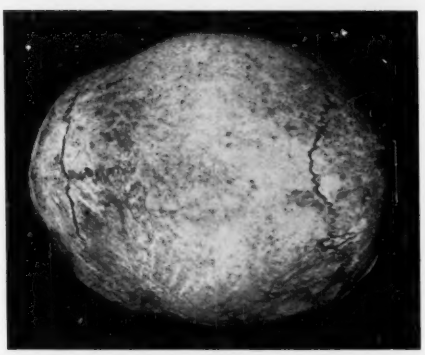
B. No. 364, male.



C. No. 667, female.

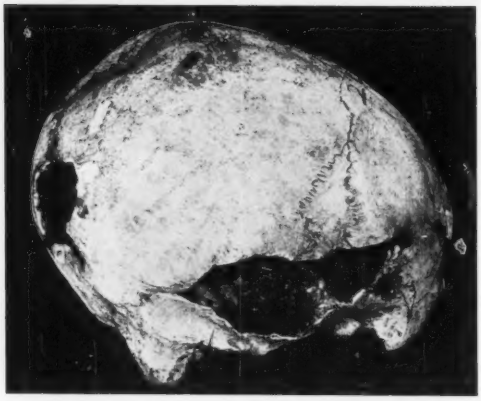


D. No. 366, male.

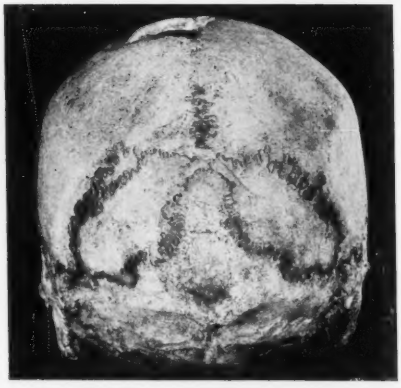


E. No. 577, female.

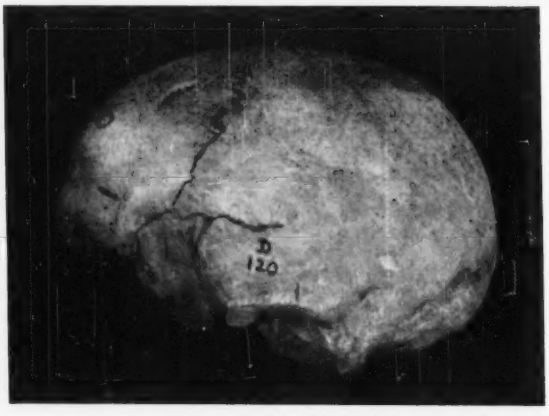
Crania showing premature obliteration of the sagittal suture.



A. No. 299, large wormian bone in right side of coronal suture.

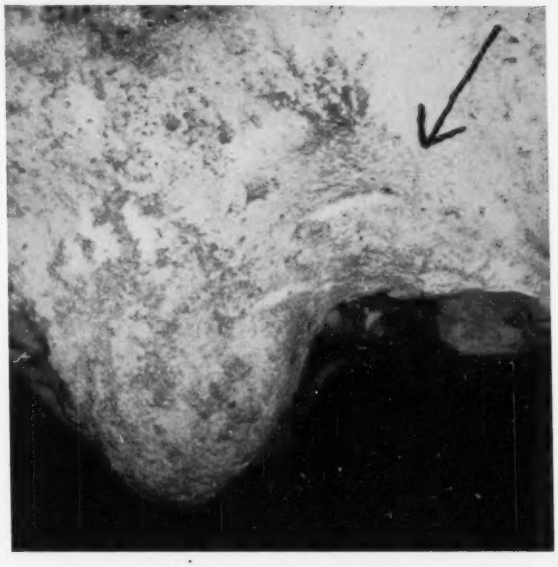


B. No. 60, two symmetrical interparietal bones of unusual form.

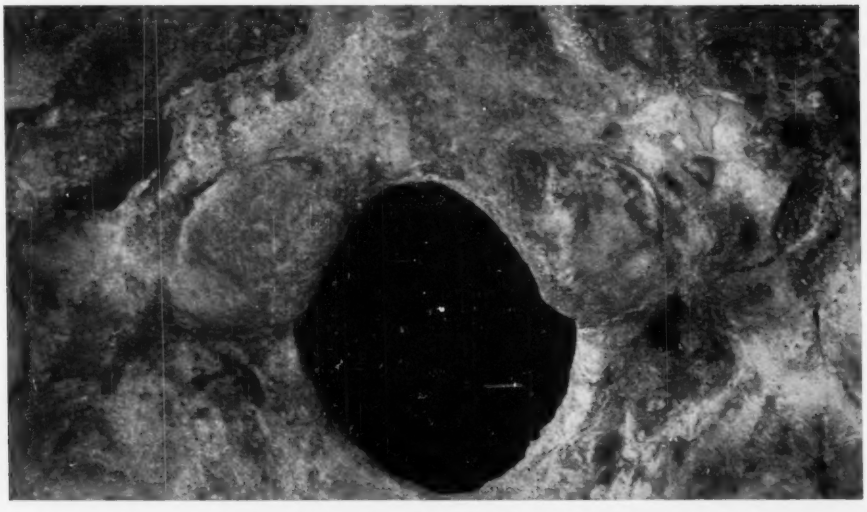


C. No. 330, temporal bone largely fused to parietal.

Male crania with sutural anomalies.

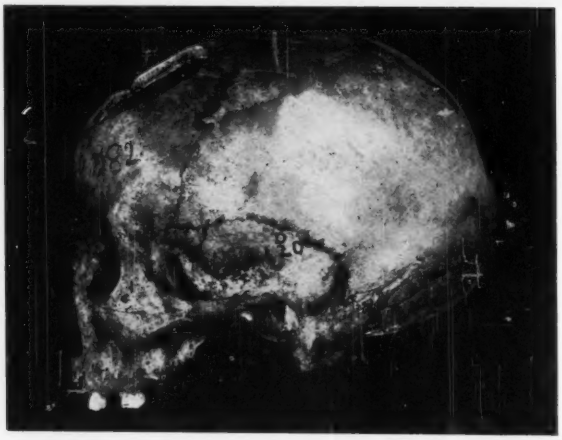


A. No. 324, complete absence of right auricular passage: twice natural size.



B. No. 301, complete absence of left jugular foramen, 1·3 natural size.

Anomalous regions of two male crania.



A

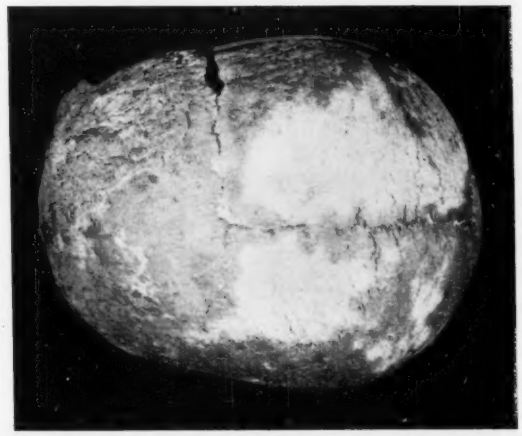
B

C

A male cranium (No. 382) of unusual form, possibly affected by hydrocephaly.

A

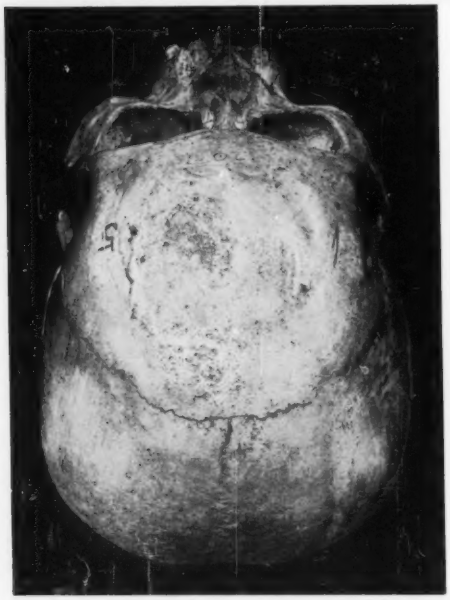
B



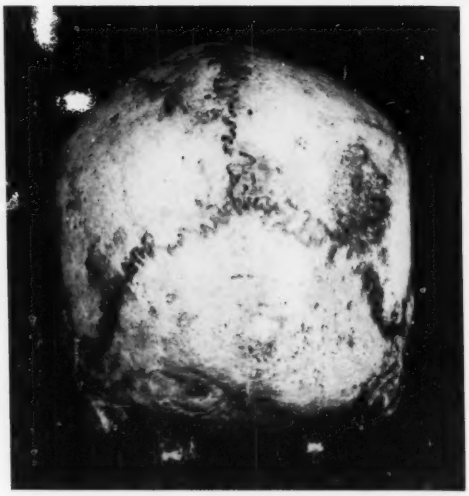
C

A female cranium (No. 662) with a large diseased area on the frontal bone.

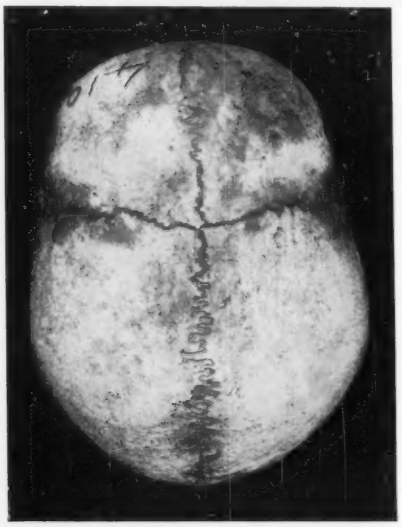
See p. 115 of text.



A. No. 5, male: wound on frontal bone.



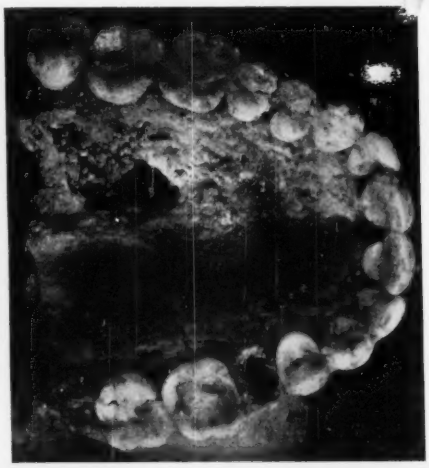
B. No. 1, male: diseased area on right parietal.



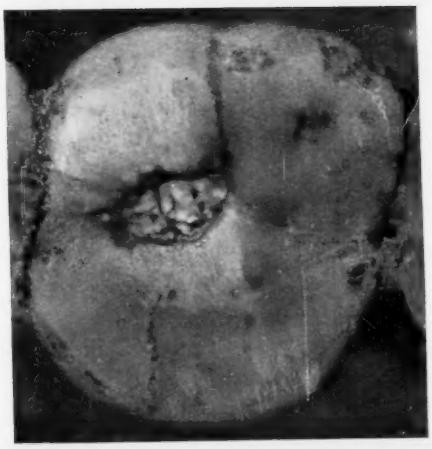
C. No. 419, female: wound on right frontal.

D. No. 513, female: depression in left parietal.

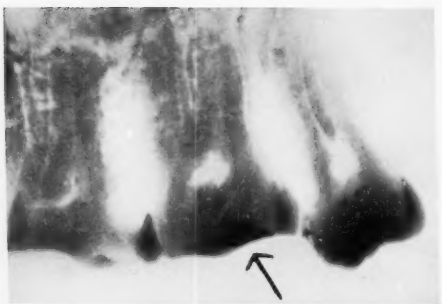
Exceptional crania. See p. 115 of text.



A. The palate: 1·2 natural size.



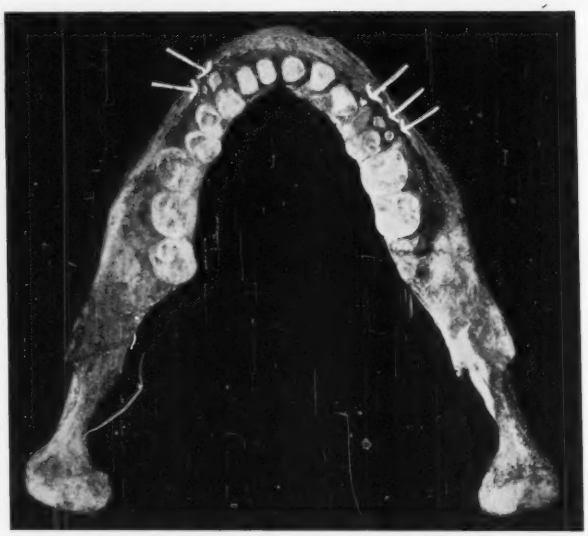
B. The second molar with filling *in situ*: 6·0 natural size.



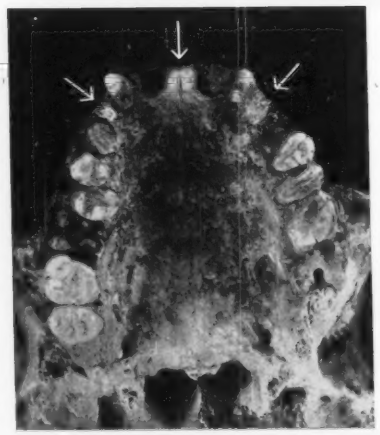
C. A skiagram showing the filling: 2·7 natural size.

D. Showing the second molar after removal of the filling: 2·7 natural size.

A tooth (upper right second molar) with a filling presumed to be adventitious: No. 518, female.



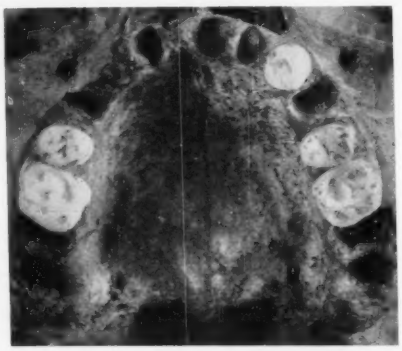
A. Denticles outside the lower dental arch.



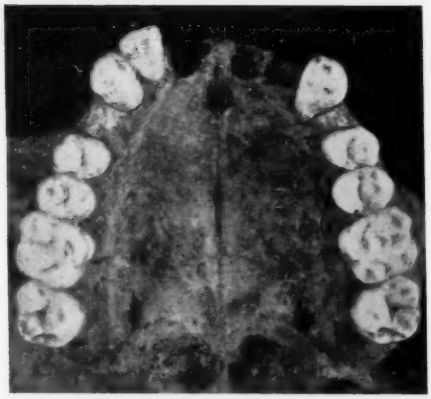
B. Diastemae between central incisors and between lateral incisors and canines.

C. Denticles outside the right side of the lower dental arch.

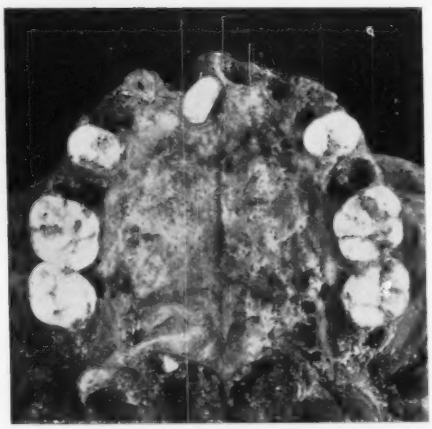
The anomalous jaws of a female skull (No. 437).



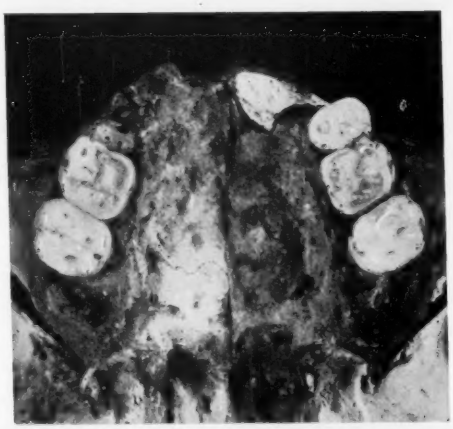
A. Sockets for three incisors only: No. 445, female.



B. Diastemae between canines and premolars and third molars absent: No. 132, male.

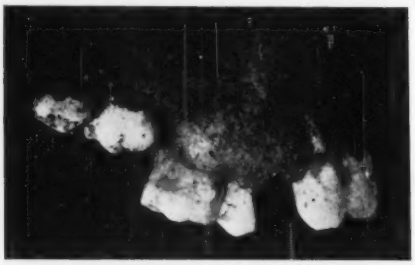


C. Supernumerary tooth behind right central incisor: No. 705, juvenile.

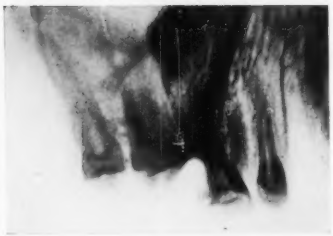


D. Grossly deflected left canine: No. 401, female.

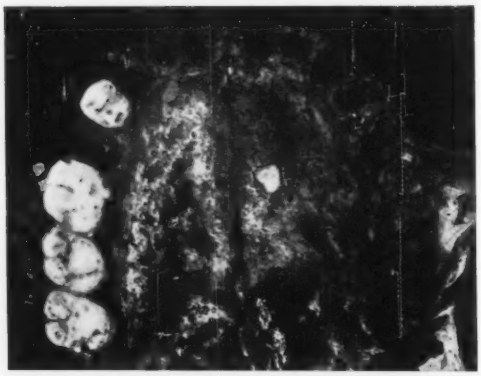
Palates with anomalous dentitions.



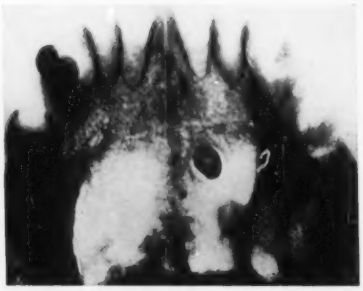
A. Incomplete eruption of second and third right molars: No. 506, female



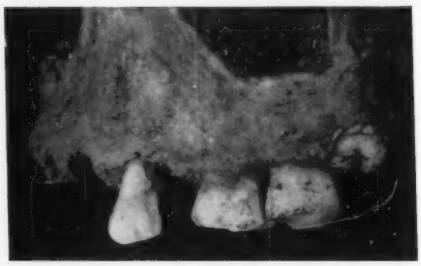
B. Skiagram of the same specimen (A) showing that the roots of the partially erupted teeth were formed.



C. Denticle in palate: No. 383, female.



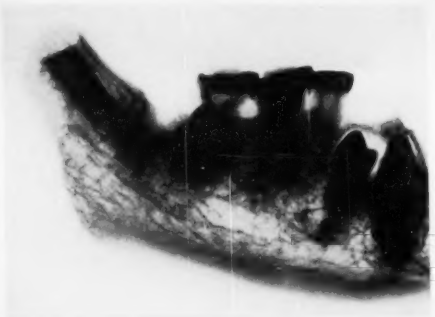
D. Skiagram of the same specimen (C).



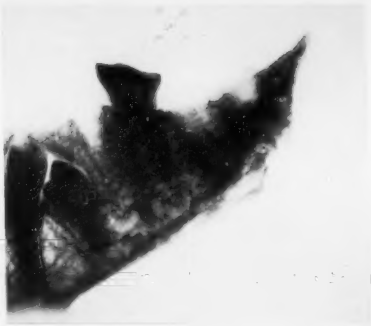
E. Third molar impacted: No. 496, female.

F. Skiagram of the right side of a mandible showing two denticles in the corpus: No. 1056, female.

Jaws with anomalous dentitions.

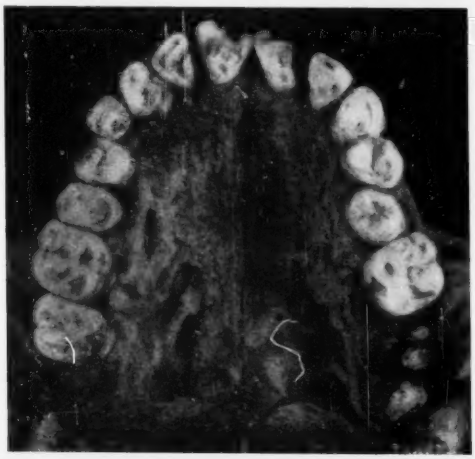


A. Skiagram of right side of mandible showing the canine and first premolar unerupted: No. 1068, immature.

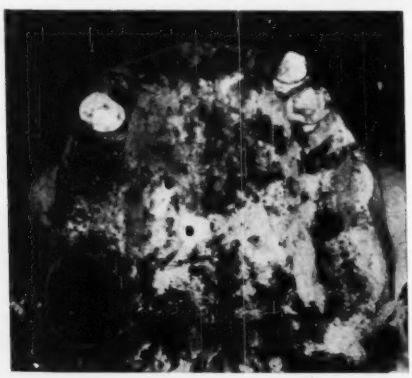


B. Skiagram of the left side of the same mandible (A) showing the same teeth on this side unerupted.

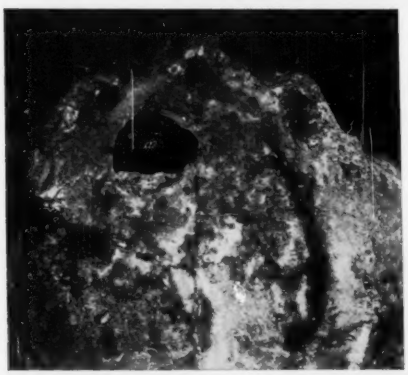
C. Third molar in abnormal position and other dental anomalies: No. 72, male. See description on p. 119.



D. Occlusal view of the same jaw (C). The broken roots of the second left molar can be seen.



E. A cyst in the right molar region: No. 469, female.



F. A cyst in the anterior part of the palate: No. 467, female.

Jaws with anomalous dentitions or cysts.

ESTIMATING BACTERIAL POPULATIONS BY THE DILUTION METHOD

By ROBERT DEAN GORDON
Scripps Institution of Oceanography, University of California, La Jolla, California

CONTENTS

		PAGE
Foreword by C. E. ZoBell		167
I.	Introduction. Nature and critique of the problem	168
II.	Methods of estimating populations from results of successive-dilution technique	170
III.	The estimation of log ρ	172
IV.	The standard deviation of log ρ	174
V.	Transformations. Methods of computing the formulae derived in (III) and (IV)	174
VI.	Remarks	178

Foreword by C. E. ZoBell

Several different methods have been described for estimating the densities of bacterial populations in solutions. The most popular of these are plating procedures, the minimum (successive) dilution method, and various direct microscopic counting methods. The accuracy of the two former methods is predicated upon the ability of the bacteria to multiply in nutrient media. Since no one medium under any one set of conditions can provide for the multiplication of bacteria with highly diverse nutritional and environmental requirements, it is not surprising that direct microscopic counts on materials containing a heterogeneous bacterial flora are usually appreciably higher than plate counts or dilution method counts. However, direct microscopic counts fail to differentiate between dead and living bacteria and are beset with almost insurmountable technical difficulties due to the minuteness of the bacteria. Therefore the choice of an enumeration procedure usually rests between plating procedures and the dilution method.

Briefly, and in its simplest form, the dilution method consists of the following procedure: A sample (e.g. 1 c.c.) of the solution under investigation is taken and inoculated into sterile nutrient medium in a test tube. Part of the original solution is then diluted in a certain ratio (usually 10-fold) and the same size sample of this diluted solution is inoculated into nutrient medium in a second test tube. This process is repeated as many times as seems necessary; so that in the end the experimenter will have prepared a series of inocula representing successive 10-fold (or some other ratio) dilutions of the original solution. The highest dilution

in this series which shows growth is then taken to indicate the magnitude of the bacterial density in the original solution.

Such an experiment, however, obviously tells nothing but an integral power of 10 which more or less accurately determines what physicists call the "order of magnitude" of the density. To obtain greater accuracy, investigators (cf. Halvorson & Ziegler, 1933) have suggested that two to ten or more tubes of nutrient medium be inoculated with each dilution of the material. Upon the basis of results obtained in such multiple tube experiments, several methods of estimating the bacterial densities which gave rise to these results have been suggested, as described by Mr Gordon.

In comparing the minimum dilution method with plating procedures for the enumeration of marine bacteria, in this laboratory, the dilution method probability tables of Halvorson & Ziegler (1933) were used. These tables give estimates of bacterial densities based on the numbers of "positives" (i.e. showing growth) observed in ten inocula of each of three successive 10-fold dilutions. More dilutions than this (e.g. five or six) were of course in most cases prepared, and the estimates were based on the most critical set of three of these dilutions.

The results revealed that the dilution method estimates averaged about 20 % higher than plate counts on the same material, although on some individual samples they were actually lower. Duplicate determinations by the two methods on the same samples of material showed that the degree of reproducibility of the plate counts was much higher than that for the dilution method counts. In view of the fact that the dilution method has applications where the plate count cannot be used, further efforts were made to increase its accuracy.

This can be accomplished in either of two ways; namely, by inoculating more tubes with each dilution or by using more dilutions. The latter alternative was tried in which, instead of diluting each time by 10-fold, the dilutions were made $\sqrt{10}$-fold. Under these conditions it seemed that more reliable and more reproducible counts were obtained by the dilution method even when calculated by a crude arithmetical method.

It was at this stage in the experimental work when Mr Gordon was consulted to aid with the calculations. After examining the methods by which Halvorson & Ziegler obtained their results, he expressed the opinion that these methods appeared questionable. He has developed the procedures presented in the following paper for determining the geometric mean estimates, as he has described. It is to be hoped that his method will yield more consistent and more reproducible results. A brief résumé of the results has already been published (Gordon, 1938).

I. Introduction

The underlying assumption by which estimates are made of a bacterial population density, from the results of the successive-dilution technique, is that the individual bacteria are distributed in the space occupied by the fluid medium

containing them, in the same way that we should conceive molecules to be distributed in a solution, if each molecule were unaffected by the presence of any other molecule. (This is not, for instance, true of molecules possessing magnetic or electrical polarity, or of ions.) In other words, the individual bacteria are assumed to be *randomly* distributed in their medium; so that if, for example, a sample of 1 c.c. is taken from 100 c.c. of solution, each bacterium in the whole solution enjoys an *independent* chance of 1 out of 100 (probability = 0·01) of being caught up in this sample.

Now, in so far as the bacteria exist as individuals they undoubtedly do not satisfy such an assumption "to the letter", but probably exert a certain mutual uniformizing influence on one another. That is, returning to the example just cited, if there are 10,000 bacteria in the 100 c.c. of solution, the probability is likely to be in reality 0, and not $0{\cdot}01^{10,000}$, that all the bacteria should be caught up in the 1 c.c. sample. This, however, simply implies that in a series of such 1 c.c. samples the numbers of bacteria caught up in them are somewhat more closely clustered about the true average density, than we should compute them to be on the basis of the above assumption of complete randomness.

In this sense our assumption of complete randomness amounts simply to "considering a least favourable case" if we are computing the probable "spreads" (standard deviations) of actual counts. On the other hand, the assumption causes us to over-estimate somewhat the probabilities of obtaining no bacteria at all in our 1 c.c. samples, and to underestimate the probabilities of obtaining "one or more" viable bacteria in the samples. It is these probabilities that enter into the estimate of population density by the dilution method. No possibility appears for rationally correcting for such discrepancies; fortunately, however, the two errors committed will to some extent cancel each other in the formulae which follow. But there is suggested in these considerations the possibility of developing rational means of measuring the uniformizing influences associated with various specific cultures in nutrient solution. The values obtained might be correlated with population densities and various specific properties, and might yield very interesting interpretations.

Another, perhaps more serious, objection to the assumption of "randomness", is the known fact that most species of bacteria show tendencies to gather in multicellular groups within the solution, as well as to congregate on glass walls and thus to go out of solution. These processes must affect all methods of counting equally, however, so far as the samples are taken in the same way; hence they should not interfere with the comparability of counts made by different methods.

It might be mentioned here, though, that in comparing plate counts with dilution estimates, account should be taken of the fact that the dilution estimates include obligative anaerobes, while plate counts do not.

II. Methods of estimating populations from results of successive-dilution technique

Assuming random distribution of individuals in a large volume of solution, as described above, let ρ represent the total number of individuals in the whole solution, divided by the total volume occupied. That is, ρ represents the mean density of population in the solution, expressed in a number of individuals per unit volume. Then the probability that in a sample of one unit volume (e.g. 1 c.c.) of solution there will be x individuals (where x is a positive integer) is given by

$$P(x) = \frac{\rho^x}{x!} e^{-\rho}, \tag{1}$$

which is the Poisson distribution.

The probability that there are 0 bacteria is accordingly $P(0) = e^{-\rho}$; and the probability of obtaining one or more bacteria in the sample (i.e. any number except 0) is $1 - P(0) = 1 - e^{-\rho}$, since the distribution (1) is normalized. Hence if we take ten test-tubes containing sterile nutrient solution, and inoculate each with 1 unit volume of our solution, then the probability that n ($\leqq 10$) of these tubes will have received some number of bacteria, and $(10-n)$ will have received none, is

$$G_\rho(n) = \frac{10!}{n!(10-n)!} (1-e^{-\rho})^n (e^{-\rho})^{10-n}, \tag{2}$$

by well-known rules for computing probabilities. If we understand ρ to represent mean density of *viable* bacteria, then $G_\rho(n)$ in (2) represents likewise the probability that n of the ten test-tubes so inoculated will show growth, and simultaneously the other $(10-n)$ of the tubes will fail to show growth.

Let us now inoculate three rows of ten tubes each, the first row with samples of the original solution, the second row with samples of a 10-fold dilution of the original solution, and the third row with samples of a 100-fold dilution of the original solution. Let ρ represent the mean density of individuals in the middle (10-fold) dilution, and denote by n_{10}, n_1 and $n_{0\cdot1}$ the numbers of tubes showing growth in the three respective rows. Then the probability of observing a result represented by the triplicate $(n_{10}, n_1, n_{0\cdot1}) = n$ is

$$G^{(3)}_\rho(n_{10}, n_1, n_{0\cdot1}) = G^{(3)}_\rho(n) = G'_{10\rho}(n_{10})\, G_\rho(n_1)\, G_{0\cdot1\rho}(n_{0\cdot1})$$

$$= \frac{(10!)^3 (e^{-10\rho})^{10-n_{10}} (e^{-\rho})^{10-n_1} (e^{-0\cdot1\rho})^{10-n_{0\cdot1}} (1-e^{-10\rho})^{n_{10}} (1-e^{-\rho})^{n_1} (1-e^{-0\cdot1\rho})^{n_{0\cdot1}}}{n_{10}!\, n_1!\, n_{0\cdot1}!(10-n_{1\cdot0})!\,(10-n_1)!\,(10-n_{0\cdot1})!}. \tag{3}$$

It is upon these three equations (1), (2), and (3) that several methods of estimating population densities from the results $(n_{10}, n_1, n_{0\cdot1}) = n$ of successive dilution experiments are based. (These equations of course refer to successive 10-fold dilutions, and ten tests to each dilution. Analogous equations are easily formed corresponding to other dilution stages and numbers of tests.)

Wells & Wells (1921) have published a method of estimating population density. Their method, applied to the present case, amounts simply to computing three values of ρ, one to fit each of the equations

$$e^{-10\rho} = \frac{10-n_{10}}{10}; \quad e^{-\rho} = \frac{10-n_1}{10}; \quad e^{-0{\cdot}1\rho} = \frac{10-n_{0{\cdot}1}}{10}, \tag{4}$$

where $(n_{10}, n_1, n_{0{\cdot}1})$ are the numbers of "positives" in the three series of tests. The geometric mean $\sqrt[3]{(\rho_{10}\rho_1\rho_{0{\cdot}1})}$ of these three values is then accepted as an estimate of the true value of ρ.

On the surface of it, such a way of arriving at an estimate appears rather naive. Such direct computations are usually unsafe when the data are subject to large uncertainties. Apparently the only reason these authors used a geometric mean of the three values instead of some other mean is that from the equation

$$e^{-r\rho} = \frac{10-n_r}{10} \tag{5}$$

we obtain

$$\log\rho = \log\left(\log\frac{10}{10-n_r}\right) - \log r, \tag{6}$$

which is linear in $\log\rho$, $\log r$, and $\log\left(\log\frac{10}{10-n_r}\right)$, so that the arithmetic means of these terms are related in the same way as the terms themselves. (Note that $\frac{1}{3}[\log\rho_{10}+\log\rho_1+\log\rho_{0{\cdot}1}] = \log(\rho_{10}\rho_1\rho_{0{\cdot}1})^{\frac{1}{3}}$.)

However, bacteria counts are usually expressed "to a certain number (e.g. 3) of significant figures"; that is, interest is centred on the *proportionate* errors, *not* on absolute errors. The difference between no bacteria and one is more important than that between 1000 and 1001. This consideration indicates that the expectation of $\log\rho$, given the data, is more like what we want than, say, the expectation of ρ. But it does not follow that the mean of three separate estimates of $\log\rho$ is the best estimate of $\log\rho$, because the estimates are not equally accurate; they should be weighted by the inverse square of the standard deviation of $\log\rho$ corresponding to an observation n_r, which would have to be computed from an "inverse" distribution derived from (5) by use of Bayes's formula.

Halvorson & Ziegler (1933) published a mimeographed tract in which they presented tables giving the modes of $G_\rho^{(3)}(n)$ (see equation (3)) together with the corresponding maximum values of $G_\rho^{(3)}(n)$, corresponding to various combinations $(n_{10}, n_1, n_{0{\cdot}1}) = n$ (that is, considering $G_\rho^{(3)}(n)$ as a function of ρ). Their notation is not the same as that here used; it is as follows:

$$\bar{\rho} = X = \rho \text{ corresponding to maximum of } G_\rho^{(3)}(n),$$
$$G_{\bar{\rho}}^{(3)}(n) = G_X^{(3)}(n) = P,$$
$$n_{10} = p_1; \quad n_1 = p_2; \quad n_{0{\cdot}1} = p_3.$$

This estimate $\bar{\rho}$ of ρ is obviously that obtained by the "method of maximum likelihood", which is associated with the name of R. A. Fisher. The relation of

this method to the principle of inverse probability has been discussed by Jeffreys (1938*a*, *b*). It is equivalent to taking a uniform prior probability for ρ, and then adopting the mode of the posterior probability as the estimate.

In the present instance the significance of a modal estimate of ρ is reduced by the fact that the *a posteriori* distributions of ρ (computed in particular from uniform *a priori* probability) vary widely in skewness with different values of the n_r's. This is indicated by the comparison of three computed geometric means with corresponding modal estimates in the last section of this paper. The point is verified by a series of laboratory comparisons of modal (Halvorson & Ziegler) estimates with direct plate counts on the same materials, reported elsewhere (Gordon & ZoBell, 1939). Thus the relationship of the mode to the distribution of which it is a representative is a varying quality, and the mode is deprived of any immediate significance.

It has already been mentioned that the particular value of ρ that is best adapted to the purposes of bacteriology corresponds to the expectation of the logarithm, on the ground that we are more interested in the ratio of ρ in two samples than in the absolute difference. Further, since ρ can have values from 0 to ∞, but $\log\rho$ from $-\infty$ to $+\infty$, the probability of $\log\rho$ has an opportunity of being nearly normally distributed that is denied to ρ; similarly for correlation $\tanh^{-1}\rho$, and for the ratio of two standard deviations $\log(s_1/s_2)$, have nearly normal probability distributions, and have been extensively used for that reason on Fisher's recommendation. If we can find the expectation of $\log\rho$ and the second moment of its probability distribution, a normal curve with corresponding mean and standard deviation should give a good representation of the law as a whole.

III. The estimation of log ρ

Using the Bayes-Laplace formula with uniform prior probability for ρ, we obtain from (3) for the posterior probability distribution of ρ,

$$P(\rho) = \frac{G_\rho^{(3)}(n)}{\int_0^\infty G_\rho^{(3)}(n)\,d\rho}. \tag{7}$$

The expectation of $\log\rho$ is

$$\log\bar{\rho} = \int_0^\infty \log\rho\; P(\rho)\,d\rho,$$

which, cancelling factors in (3) and (7), equals

$$\log\bar{\rho} = \frac{\int_0^\infty \log\rho\, e^{-(111-10n_{10}-n_1-0{\cdot}1n_{0{\cdot}1})\rho}\,(1-e^{-10\rho})^{n_{10}}\,(1-e^{-\rho})^{n_1}\,(1-e^{-0{\cdot}1\rho})^{n_{0{\cdot}1}}\,d\rho}{\int_0^\infty e^{-(111-10n_{10}-n_1-0{\cdot}1n_{0{\cdot}1})\rho}\,(1-e^{-10\rho})^{n_{10}}\,(1-e^{-\rho})^{n_1}\,(1-e^{-0{\cdot}1\rho})^{n_{0{\cdot}1}}\,d\rho}. \tag{8}$$

The integrands can obviously be expressed in the form

$$[\log\rho \text{ or } 1]\,(1-E^{100})^{n_{10}}(1-E^{10})^{n_1}(1-E)^{n_{0\cdot1}}e^{-\frac{1}{10}x_0\rho},$$

where E is the Boolean operator $=(1+\Delta)$, Δ meaning difference with regard to

$$x_0 = 1110 - 100n_{10} - 10n_1 - n_{0\cdot1}.$$

Since E is commutative with the sign of integration, we may therefore write

$$\log\bar{\rho} = \frac{(1-E^{100})^{n_{10}}(1-E^{10})^{n_1}(1-E)^{n_{0\cdot1}}\int_0^\infty \log\rho\, e^{-\frac{1}{10}x_0\rho}\,d\rho}{(1-E^{100})^{n_{10}}(1-E^{10})^{n_1}(1-E)^{n_{0\cdot1}}\int_0^\infty e^{-\frac{1}{10}x_0\rho}\,d\rho}. \tag{9}$$

That this is so may be seen by expanding the operator

$$H = (1-E^{100})^{n_{10}}(1-E^{10})^{n_1}(1-E)^{n_{0\cdot1}} \tag{10}$$

as a polynomial in E, then operating term by term on the integrands. The result will be the same as the result of expanding the integrands of (8) in powers of e.

The integrals in (9) may be determined by means of the following identities:

$$\left.\begin{aligned}\int_0^\infty e^{-x\rho}\rho^u\,d\rho &= \frac{\Gamma(u+1)}{x^{u+1}},\\ \int_0^\infty \log\rho\, e^{-x\rho}\rho^u\,d\rho &= \frac{d}{du}\left(\frac{\Gamma(u+1)}{x^{u+1}}\right)\\ &= \frac{\Gamma(u+1)}{x^{u+1}}\,(F(u+1)-\log x),\end{aligned}\right\} \tag{11}$$

where $F(u+1)$ is the "digamma function" defined by $F(u+1) = d\log\Gamma(u+1)/du$. (see *British Association Tables*, vol. **1** (1931) with a different notation).

Place $u=0$ in these identities; there results

$$\left.\begin{aligned}\int_0^\infty e^{-\frac{1}{10}x_0\rho}\,d\rho &= \frac{10}{x_0},\\ \int_0^\infty \log\rho\, e^{-\frac{1}{10}x_0\rho}\,d\rho &= \frac{10}{x_0}\left(F(1)-\log\frac{x_0}{10}\right),\end{aligned}\right\} \tag{12}$$

where $$F(1) = -0{\cdot}57722.$$

In this way we obtain simply from (9), for $\log\bar{\rho}$

$$\log\bar{\rho} = F(1)+\log 10 - \frac{H\left\{\dfrac{\log x_0}{x_0}\right\}}{H\left\{\dfrac{1}{x_0}\right\}},\ldots \tag{13}$$

where H is the operator defined in (10).

IV. The standard deviation of $\log \rho$

The standard deviation of $\log \rho$ is the square root of the second moment of $\log \rho$ about its mean $\log \bar{\rho}$. Using the operator-notation of the previous section, with standard formulae of statistics, we obtain

$$\sigma^2_{\log \rho} = \frac{H\left(\int_0^\infty (\log \rho)^2 e^{-\Upsilon_0 x_0 \rho}\, d\rho\right)}{H\left(\int_0^\infty e^{-\Upsilon_0 x_0 \rho}\, d\rho\right)} - (\log \bar{\rho})^2. \tag{14}$$

Now, as before,

$$\begin{aligned}\int_0^\infty (\log \rho)^2 e^{-\Upsilon_0 x_0 \rho}\, d\rho &= \left[\frac{d^2}{du^2}\int_0^\infty \rho^u e^{-\Upsilon_0 x_0 \rho}\, d\rho\right]_{u=0} \\ &= \frac{10}{x_0}[\{F(1)\}^2 + F(1) + \{\log 10\}^2 + 2F(1)\log 10 \\ &\quad -2\{\log 10 + F(1)\}\log x_0 + \{\log x_0\}^2],\end{aligned} \tag{15}$$

whence we easily find

$$\begin{aligned}\sigma^2_{\log \rho} = &-(\log \bar{\rho})^2 + [F(1) + \log 10]^2 + F(1) \\ &-2[F(1) + \log 10]\frac{H\left\{\frac{\log x_0}{x_0}\right\}}{H\left\{\frac{1}{x_0}\right\}} \\ &+\frac{H\left\{\frac{(\log x_0)^2}{x_0}\right\}}{H\left\{\frac{1}{x_0}\right\}},\end{aligned} \tag{16}$$

where $F(1) = -0{\cdot}577216, \quad F(1) = 1{\cdot}644934.$

V. Transformations

In the fractional terms in (13) and (16), the numerators and denominators each represent finite alternating series; but these series are useless for computing purposes, because the terms are so nearly equal that each would have to be expressed with at least twenty significant figures to yield sufficiently accurate results. Hence we carry out the following transformations:

Referring to (10) we take out $(1-E)^{\Sigma n} = (-\Delta)^{\Sigma n}$ as a factor from the operator H, and obtain

$$H = H_0(-\Delta)^N, \tag{17}$$

where $N = \Sigma n = n_{10} + n_1 + n_{0\cdot1}$, and

$$H_0 = (1 + E + \ldots + E^{99})^{n_{10}}(1 + E + \ldots + E^9)^{n_1}. \tag{18}$$

If this last expression is expanded, evidently all terms must turn out to have positive coefficients; so that if the functions

$$\Delta^N\left(\frac{1}{x}\right), \quad \Delta^N\left(\frac{\log x}{x}\right), \quad \Delta^N\left(\frac{(\log x)^2}{x}\right),$$

can be accurately determined, the required end result is simply a weighted sum of terms of like sign, and there is no cancellation.

In the first place it is very simple to obtain

$$(-\Delta)^N\left(\frac{1}{x}\right) = \frac{N!(x-1)!}{(x+N)!} \tag{19}$$

by mathematical induction.

To obtain $(-\Delta)^N(\log x/x)$ it suffices to expand $\log(x+v)/(x+v)$ as a power series in v and operate with $(-\Delta)^N$ termwise on successive powers of $v=0$, using the formula

$$\Delta^N 0^p = \frac{p!}{(p-N)!}\, B_{p-N}^{(-N)}; \tag{20}$$

where $B_{p-N}^{(-N)}$ are the "generalized Bernoulli numbers" of order $(-N)$ and degree $(p-N)$ (cf. L. M. Milne-Thomson, 1933, p. 134). By this means we finally obtain

$$(-1)^N \Delta^N \frac{\log x}{x} = \frac{N!(x-1)!}{(x+N)!}\log x + \sum_{p=N}^{\infty} \frac{(-1)^{p-N+1}}{x^{p+1}} \frac{p!}{(p-N)!}\left(1+\frac{1}{2}+\ldots+\frac{1}{p}\right)B_{p-N}^{(-N)}. \tag{21}$$

By similarly treating $\log(x+N-v)/(x+N-v)$ it is easy to obtain also

$$(-1)^N \Delta^N \frac{\log x}{x} = \frac{N!(x-1)!}{(x+N)!}\log(x+N) - \sum_{p=N}^{\infty} \frac{1}{(x+N)^{p+1}} \frac{p!}{(p-N)!}\left(1+\frac{1}{2}+\ldots+\frac{1}{p}\right)B_{p-N}^{(-N)}. \tag{21a}$$

These formulae (21) and (21a) can evidently both be used simultaneously to obtain upper and lower bounds to the required result. Both are convergent if $x > N$.

The sums

$$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{p}$$

in (21) and (21a) can be computed directly, or by means of various short formulae of which the following may be suggested (Boole, 1872, p. 92):

$$1+\frac{1}{2}+\ldots+\frac{1}{p} = 0{\cdot}577216+\log p+\frac{1}{2p}-\frac{1}{12p^2}+\frac{1}{120p^4}\ldots. \tag{22}$$

The Bernoulli numbers are easily computed by means of the reversion formula (Milne-Thomson, 1933, p. 129),

$$B_v^{(-N)} = \frac{v B_{v-1}^{(-N)} + B_v^{(-N+1)}}{1 + \frac{v}{N}}, \tag{23}$$

together with the values $B_v^{(0)} = 0$ if $v \neq 0$; $B_0^{(-N)} = 1$. Also polynomial formulae are given in Davis's tables. We have computed them as far as $N = 27$, $v = 7$, of which a tabulation is presented at the end of this paper.

To deal with the function $\Delta^N\left\{\frac{(\log x)^2}{x}\right\}$,

we make use of Boole's operator identity

$$\Delta_x^N = (e^{d/dx} - 1)^N. \tag{24}$$

If this is expanded in powers of d/dx and made to operate on $(\log x)^2/x$, the result is a series in the derivatives of the latter function. These derivatives are found to have the form

$$\frac{d^p}{dx^p}\frac{(\log x)^2}{x} = \frac{(-1)^p p!(\log x)^2}{x^{p+1}} + \frac{p}{x^{p+1}}\{2\log x\, B_{p-1}^{(p+1)} + (p-1)\, B_{p-2}^{(p+1)}\},$$

and the final result is

$$\Delta^N \frac{(\log x)^2}{x} = \sum_{p=N}^{\infty} \frac{p!}{(p-N)!}\frac{B_{p-N}^{(-N)}}{x^{p+1}}\left\{(-1)^p (\log x)^2 + \frac{2\log x\, B_{p-1}^{(p+1)}}{(p-1)!} + \frac{B_{p-2}^{(p+1)}}{(p-2)!}\right\}. \tag{25}$$

In an analogous manner we also obtain

$$\Delta^N \frac{(\log x)^2}{x} = (-1)^N \sum_{p=N}^{\infty} \frac{p!}{(p-N)!}\frac{B_{p-N}^{(-N)}}{(x+N)^{p+1}}\left\{[\log(x+N)]^2 + \frac{(-1)^p 2\log(x+N)\, B_{p-1}^{(p+1)}}{(p-1)!} + \frac{(-1)^p B_{p-2}^{(p+1)}}{(p-2)!}\right\}. \tag{25a}$$

As (21) and (21*a*), so also (25) and (25*a*), can always serve together to give upper and lower bounds to the values sought, so long as $x_0 > N$.

For use in the last equations, the following formulae may be found useful:

$$\frac{B_{p-1}^{(p+1)}}{(p-1)!} = (-1)^{p-1}\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{p}\right), \tag{26}$$

$$\frac{B_{p-2}^{(p+1)}}{(p-2)!} = (-1)^p \sum_{r=1}^{p-1} \frac{1}{r} \sum_{s=1}^{p-r} \frac{(-1)^{s-1}}{s}. \tag{27}$$

(i) *Expansion of the operator H_0*

The operator H_0 (equation (18)) taken as a polynomial in E, has the form of

$$\begin{aligned}\mathfrak{H}_0(z) &= (1 + z + \ldots + z^{99})^{n_{10}} (1 + z + \ldots + z^9)^{n_1} \\ &= \frac{(1 - z^{100})^{n_{10}} (1 - z^{10})^{n_1}}{(1 - z)^{n_{10}+n_1}} \\ &= P_0 + P_1 z + P_2 z^2 + \ldots + P_v z^v + \ldots + P_{99n_{10}+9n_1} z^{99n_{10}+9n_1},\end{aligned}$$

where $$P_v = P_{99n_{10}+9n_1-v} = \sum_{r=0}^{\cdot 01v} \sum_{s=0}^{\cdot 1v-10r} (-1)^{r+s} C_r^{n_{10}} C_s^{n_1} C_{v-100r-10s+n_{10}+n_1-1}^{v-100r-10s}, \quad (28)$$

with $$C_a^b = \frac{b!}{a!(b-a)!}.$$

Thus, with these values of the P_v's, we have

$$H_0 = P_0 + P_1 E + P_2 E^2 + \ldots + P_v E^v + \ldots. \quad (29)$$

(ii) *Computation of P_v for large v*

When v is fairly large, that is, considerably different from both 0 and $99n_{10}+9n_1+1$, the formula (28) becomes unwieldy and inaccurate. By means of a Laplace transformation, that is, by making use of the so-called "characteristic function" for P_v (see Uspensky, 1937, chapter XII), we are able to obtain the following Fourier series representing the continuous function which is obtained by plotting the points (v, P_v) and then connecting them with straight lines:

$$P_v = P_{99n_{10}+9n_1-v} = \sum_{\mu=0}^{\infty} A_\mu \cos\frac{2\pi\mu}{K}(v-b), \quad (30)$$

where $$K = 99n_{10}+9n_1+2; \quad b = \tfrac{1}{2}(99n_{10}+9n_1);$$

and $$A_0 = \frac{10^{2n_{10}+n_1}}{99n_{10}+9n_1+2}; \quad A_\mu = \frac{K}{8\pi^2\mu^2} \frac{\left(\sin\frac{100\pi\mu}{K}\right)^{n_{10}} \left(\sin\frac{10\pi\mu}{K}\right)^{n_1}}{\left(\sin\frac{\pi\mu}{K}\right)^{n_{10}+n_1-2}}$$

for $\mu \neq 0$.

If μ/K is an integer, $\neq 0$, then

$$A_\mu = \frac{K}{8\pi^2\mu^2} \frac{0^{n_{10}+n_1}}{0^{n_{10}+n_1-2}} = 0.$$

Otherwise obviously

$$|A_\mu| < \frac{10^{2n_{10}+n_1}K}{8\pi^2\mu^2}\left(\sin\frac{\pi\mu}{K}\right)^2$$

$$< \frac{10^{2n_{10}+n_1}K}{8\pi^2\mu^2}\left(\sin\frac{\pi}{200}\right)^2 \text{ if } |\mu/K - n| < \frac{1}{200}, \; n = \text{integer};$$

$$|A_\mu| < \frac{10^{n_1}K}{8\pi^2\mu^2}\left(\sin\frac{\pi}{200}\right)^{-n_{10}+2} \text{ if } \frac{1}{200} \leqq |\mu/K - n| < \frac{1}{20};$$

$$|A_\mu| < \frac{K}{8\pi^2\mu^2}\left(\sin\frac{\pi}{20}\right)^{-n_{10}-n_1+2}, \text{ if } |\mu/K - n| \geqq \frac{1}{20}.$$

The first limit is the greatest. It follows that

$$\sum_{\mu=p+1}^{\infty} A_\mu \cos\frac{2\pi\mu}{K}(\xi-b) < \sum_{\mu=p+1}^{\infty} |A_\mu| < \frac{0{\cdot}000247 \times 10^{2n_{10}+n_1}K}{8\pi^2} \sum_{\mu=p+1}^{\infty} \frac{1}{\mu^2}$$

$$< \frac{0{\cdot}000247 \times 10^{2n_{10}+n_1}K}{8\pi^2} \int_p^{\infty} \frac{dx}{x^2} < \frac{0{\cdot}000247 \times 10^{2n_{10}+n_1}K}{8\pi^2 p} \quad (31)$$

which can serve as an error function for (30).

VI. Remarks

Sufficient mathematical tools are presented in equations (19), (21), (21*a*), (25), (25*a*), (28), and (30), for actually computing the geometric mean estimates $\bar{\rho}$ of densities of bacteria from the results of successive dilution experiments, together with the standard deviations of their natural logarithms, from formulae (13) and (16). The standard deviation of the logarithm of course will be a measure of the proportionate accuracy of the estimate, in the same sense that the ordinary standard deviation is a measure of the arithmetical accuracy. (It may be remarked, incidentally, that *all logarithms occurring in the above formulae are understood to be natural logarithms.*)

If we assume in general that the function $P(\rho)$ defined in (7) can be very nearly approximated by a normal distribution function with respect to $\log \rho$—which should be satisfactory for bacteriological purposes—then corresponding to $\sigma_{\log \rho}$ there will be a "probable error" $0{\cdot}675\sigma_{\log \rho} = s$. To this will correspond a "probable error ratio"

$$\epsilon = e^s - 1 = 10^{0{\cdot}293\sigma} - 1,$$

that is

$$\log_{10}(1+\epsilon) = 0{\cdot}293\sigma_{\log \rho}.$$

The meaning of ϵ is that the probability is approximately $\frac{1}{2}$, that

$$-\frac{\epsilon}{1+\epsilon}\bar{\rho} < \rho - \bar{\rho} < \epsilon\bar{\rho}.$$

To save labour, it remains yet to determine limits to the extents of errors committed by adding only, say, every twenty-fifth term corresponding to the operator

$$H_0 = \sum_{(v)} P_v E^v,$$

in making the computations; or else to determine short formulae for the required sums. For the purposes of rigour, upper bounds will also have to be determined for the Bernoulli numbers used in the formulae.

Mrs Naomi Lancaster has made rough computations of several values of $\bar{\rho}$ as shown below:

Argument			$\bar{\rho}$ computed by us	$\bar{\rho}$ from Halvorson & Ziegler	% deviation from Halvorson & Ziegler
n_{10}	n_1	$n_{0\cdot1}$			
10	7	3	1·43	1·53	−7·0%
8	5	1	0·291	0·267	+9·0%
4	2	1	0·086	0·080	+7·5%

If we were to plot these percentage deviations against n_{10}, we should be led to expect a maximum positive deviation of perhaps 12 % or more in the region of $n_{10} = 6$ or 7.

Dr ZoBell has compared plate counts with corresponding estimates from Halvorson & Ziegler's tables, and their ratios show exactly similar trends to those indicated by the above three comparisons, but relatively even more pronounced. These will probably be reported elsewhere in detail.

In order to make possible the practical use of these results by bacteriologists and others, as well as to test their validity experimentally, it will be necessary to prepare tables of $\bar{\rho}$ and ϵ as formulated in the preceding pages, corresponding to the tables previously prepared by Halvorson & Ziegler. This will require financial aid from some source.

The writer wishes to express his appreciation to Dr C. E. ZoBell for acquainting him with this very interesting problem, and to Dr George F. McEwen for his encouragement and occasional aid in carrying out the work and making available the necessary literature. Dr Harry Bateman of the Mathematics Department, California Institute of Technology, and Dr Risselman of the University of California at Los Angeles, were also helpful in bringing the analysis through one or two difficult points.

APPENDIX

Table of Bernoulli numbers $B_v^{(-N)}$

$v=$	1	2	3	4	5	6	7
$N=1$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
2	1	$\frac{7}{6}$	$\frac{3}{2}$	$\frac{31}{15}$	3	$\frac{127}{28}$	$\frac{85}{12}$
3	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{9}{2}$	$\frac{43}{5}$	$\frac{69}{4}$	$\frac{3025}{84}$	$\frac{311}{4}$
4	2	$\frac{13}{3}$	10	$\frac{243}{10}$	$\frac{185}{3}$	$\frac{6821}{42}$	$\frac{1325}{3}$
5	$\frac{5}{2}$	$\frac{20}{3}$	$\frac{75}{4}$	$\frac{331}{6}$	$\frac{675}{4}$	$\frac{11215}{21}$	$\frac{5225}{3}$
6	3	$\frac{19}{2}$	$\frac{63}{2}$	$\frac{1087}{10}$	$\frac{777}{2}$	$\frac{30083}{21}$	5432
7	$\frac{7}{2}$	$\frac{77}{6}$	49	$\frac{1939}{10}$	$\frac{4753}{6}$	$\frac{9992}{3}$	$\frac{43120}{3}$
8	4	$\frac{50}{3}$	72	$\frac{4819}{15}$	1476	$\frac{146240}{21}$	33664
9	$\frac{9}{2}$	21	$\frac{405}{4}$	$\frac{2514}{5}$	2565	$\frac{93886}{7}$	$\frac{573975}{8}$
10	5	$\frac{155}{6}$	$\frac{275}{2}$	752	$\frac{12650}{3}$	$\frac{677465}{28}$	$\frac{567325}{4}$
11	$\frac{11}{2}$	$\frac{187}{6}$	$\frac{363}{2}$	$\frac{16258}{15}$	$\frac{26499}{4}$	$\frac{1158509}{28}$	$\frac{3164029}{12}$

Table of Bernoulli numbers $B_v^{(-N)}$ (cont.)

$v=$	1	2	3	4	5	6	7
$N=12$	6	37	234	$\frac{15149}{10}$	10023	$\frac{2842373}{42}$	465725
13	$\frac{13}{2}$	$\frac{130}{3}$	$\frac{1183}{4}$	$\frac{2063}{10}$	$\frac{176267}{12}$	$\frac{2238730}{21}$	$\frac{9453353}{12}$
14	7	$\frac{301}{6}$	$\frac{735}{2}$	$\frac{82439}{30}$	$\frac{41895}{2}$	$\frac{975623}{6}$	$\frac{7704025}{6}$
15	$\frac{15}{2}$	$\frac{115}{2}$	450	$\frac{7181}{2}$	29175	$\frac{10129615}{42}$	2026550
16	8	$\frac{196}{3}$	544	$\frac{23066}{5}$	$\frac{119408}{3}$	$\frac{7330868}{21}$	$\frac{9329056}{3}$
17	$\frac{17}{2}$	$\frac{221}{3}$	$\frac{2601}{4}$	$\frac{87601}{15}$	$\frac{106641}{2}$	$\frac{10384229}{21}$	$\frac{37236205}{8}$
18	9	$\frac{165}{2}$	$\frac{1539}{2}$	$\frac{36483}{5}$	70281	$\frac{19239635}{28}$	$\frac{27257571}{4}$
19	$\frac{19}{2}$	$\frac{551}{6}$	$\frac{1805}{2}$	$\frac{45049}{5}$	$\frac{1095635}{12}$	$\frac{26279679}{28}$	$\frac{39123375}{4}$
20	10	$\frac{305}{3}$	1050	$\frac{66049}{6}$	117075	$\frac{17672415}{14}$	$\frac{41371225}{3}$
21	$\frac{21}{2}$	112	$\frac{4851}{4}$	$\frac{133217}{10}$	$\frac{7714707}{52}$	$\frac{5022439}{3}$	$\frac{38264149}{2}$
22	11	$\frac{737}{6}$	$\frac{2783}{2}$	$\frac{159819}{10}$	$\frac{1115983}{6}$	$\frac{46037134}{21}$	$\frac{78466685}{3}$
23	$\frac{23}{2}$	$\frac{805}{6}$	1587	$\frac{570653}{30}$	$\frac{461817}{2}$	$\frac{59587135}{21}$	$\frac{105841262}{3}$
24	12	146	1800	$\frac{112379}{5}$	284100	$\frac{76306988}{21}$	47006000
25	$\frac{25}{2}$	$\frac{475}{3}$	$\frac{8125}{4}$	26380	$\frac{1040000}{3}$	$\frac{96763700}{21}$	$\frac{1486135625}{24}$
26	13	$\frac{1027}{6}$	$\frac{4563}{2}$	$\frac{461578}{15}$	419796	$\frac{486388487}{84}$	$\frac{322887513}{4}$
27	$\frac{27}{2}$	$\frac{369}{2}$	$\frac{5103}{2}$	$\frac{178452}{5}$	$\frac{2019087}{4}$	$\frac{202034577}{28}$	$\frac{416849895}{4}$

REFERENCES

Boole, G. (1872). *Finite Differences.* London: Macmillan.

Gordon, R. D. (1938). *Proc. Nat. Acad. Sci., Wash.*, **24**, 212–15.

Gordon, R. D. & Zobell, C. E. (1939). *Zbl. Bakt.* **2** (in the Press).

Halvorson, H. O. & Ziegler, N. R. (1933). *Quantitative Bacteriology.* Minneapolis: Burgess.

Jeffreys, H. (1938*a*). *Ann. Eugen., Lond.*, **8**, 146–51.

—— (1938*b*). *Proc. Roy. Soc.* A, **167**, 464–83.

Milne-Thomson, L. M. (1933). *Calculus of Finite Differences.* London: Macmillan.

Uspensky, J. V. (1937). *Introduction to Mathematical Probability.* New York: McGraw-Hill.

Wells, P. V. & Wells, W. F. (1921). *J. Wash. Acad. Sci.* **11**, 265–73.

NOTE ON THE INVERSE AND DIRECT METHODS OF ESTIMATION IN R. D. GORDON'S PROBLEM

By E. S. PEARSON

THE practical solution of the problem of estimating the mean density of a bacterial population by the dilution method requires not only the determination of a single-valued estimate, but also some measure of the reliability of this estimate. By the introduction of an ingenious mathematical procedure Mr Gordon has taken, in the preceding paper, a first step in the solution of this problem on lines involving the application of Bayes's theorem.* Thus if ρ is the mean density per unit volume at a given dilution, he obtains an *a posteriori* distribution for ρ and, since it seems likely that the derived distribution of $\log \rho$, rather than that of ρ, will be approximately normal, he shows how the expectation and standard deviation of $\log \rho$ may be calculated. With the help of these a probability statement regarding the unknown ρ of the form given on p. 178 may be made. To put the results into working form extensive computation will, however, be necessary.

Contrasted with this inverse approach is what may be termed the direct solution involving the determination of a fiducial or confidence interval. This solution requires (*a*) the choice of an appropriate sample estimate of ρ, say R, and (*b*) the determination of its sampling distribution, say $p(R \mid \rho)$. Mr Gordon refers to R. A. Fisher's maximum likelihood estimate, say R_L (Fisher, 1922, pp. 363–5), whose calculation in the case where ten tubes are examined at each of three dilution levels, the dilution factor being 10 : 1, has been made easy by the tables of Halvorson & Ziegler (1933*a*), but convinced as he clearly is that the inverse approach is the only legitimate one, he no doubt did not feel it necessary to discuss the possibility of a fiducial solution as alternative to his own. Since, however, a paper by Matuszewski *et al.* (1935) did present a preliminary working solution of this kind for just the same experimental arrangement—I will call it the 10, 3, 10 arrangement—discussed by Halvorson & Ziegler and by Gordon, it may be useful to make some reference to this result, and also to add a few comments on the difference between the two lines of approach.

In their first paper (1933*a*, p. 121) Halvorson & Ziegler gave a table showing the value of R_L corresponding to each (or rather, most) of the possible combinations of n_{10}, n_1, and $n_{0\cdot1}$, the number of tubes (in Gordon's notation) out of the ten tested at each dilution which show growth. In their third paper (1933*c*) they carried out some investigations into the sampling variation of R_L for a fixed ρ.

* It is interesting to note that the pioneer paper on this subject by Greenwood & Yule (1917) also followed the Bayes's theorem approach.

Taking the 10, 3, 10 arrangement and the four cases $\rho = 0{\cdot}15$, $0{\cdot}25$, $0{\cdot}50$ and $1{\cdot}50$, they calculated from the term of an appropriate multinomial expansion the probability of observing different combinations of n_{10}, n_1, and $n_{0{\cdot}1}$; hence, using their tables, they obtained the probability associated with different values of R_L. Their results, presented in the form of tables and a diagram, show that:

(1) The distributions of R_L are asymmetrical.

(2) The distributions of $\log R_L$ are more nearly symmetrical, but since R_L can only assume a finite number of discrete values, the distributions cannot in either form be represented adequately by smooth curves.

(3) Calculations from the partially grouped values give the results in Table I.

TABLE I

ρ	0·150	0·250	0·500	1·500
Mean (R_L)	0·164	0·284	0·558	1·648
$\sigma(R_L)$	0·066	0·117	0·226	0·689
$\log\rho$	−0·824	−0·602	−0·301	+0·176
Mean ($\log R_L$)	−0·816	−0·578	−0·285	+0·184
$\sigma(\log R_L)$	0·163	0·164	0·163	0·168
$\sigma(\log R_L)$ (limiting formula)	0·1535	0·1532	0·1768	0·1550

N.B. Logarithms are to base 10.

(4) The values of $\sigma(\log R_L)$ calculated from Halvorson & Ziegler's probability distribution remain nearly constant in the range of ρ considered. Having regard to the respective standard deviations, the bias of $\log R_L$ is of less importance than the bias of R_L. However, if a method of obtaining accurate fiducial limits were available, the bias in the single-valued estimate would be of no importance.

(5) The figures in the last row have been calculated from Fisher's formula for the large sample value of the variance of a maximum likelihood estimate, which reduces in this case to (Fisher, 1922, p. 364):

$$\{\sigma^2(\log_e R_L)\}^{-1} = 10\left\{\frac{e^{-10\rho}}{1-e^{-10\rho}}\,100\rho^2 + \frac{e^{-\rho}}{1-e^{-\rho}}\,\rho^2 + \frac{e^{-\frac{1}{10}\rho}}{1-e^{-\frac{1}{10}\rho}}\,\frac{\rho^2}{100}\right\}.$$

This formula gives minima for $\sigma(\log R_L)$ in the neighbourhood of $\rho = 0{\cdot}16$, $1{\cdot}6$ and 16. A series of calculated values including those tabled above are:

TABLE II

ρ	0·10	0·15	0·25	0·40	0·50	1·00	1·50	2·00	2·50	3·00	4·00
$\sigma(\log_{10} R_L)$	0·166	0·153	0·153	0·169	0·177	0·166	0·155	0·153	0·156	0·161	0·174

The probability values tabled by Halvorson & Ziegler are not sufficiently detailed to determine how far the differences between the values of $\sigma(\log R_L)$ compared in the last two rows of Table I are due to inadequacy in the large-sample variance formula. It is, however, clear that the standard error of the logarithm of the maximum likelihood estimate changes very little with ρ. Halvorson & Ziegler reached a similar result by noting that $\sigma(R_L)/\rho$ was very stable. The same result was noticeable in a series of calculations concerned with estimating the density of organisms in milk, made by Barkworth & Irwin (1938). These authors analysed the results of seven separate experiments in which 255 tubes were tested at each of four dilutions, namely 1 : 10, 1 : 50, 1 : 250, and 1 : 1250. They obtained the standard error of the maximum likelihood estimate of ρ from a large sample formula analogous to that given above; for seven values of ρ lying between 19 and 67, the ratio $\sigma(R_L)/\rho$ lay between 0·058 and 0·062.

It is clear that if more detailed computations of the probability distribution of R_L for a wider range of values of ρ were made on these lines, charts or tables giving fiducial or confidence limits for ρ could be readily supplied. In their paper of 1935 referred to above Matuszewski *et al.* have provided one such chart formed, as will be described below, on a basis which is partly empirical. The chart may be used as follows:

(*a*) Having observed experimentally the three numbers n_{10}, n_1, and $n_{0{\cdot}1}$, obtain from Halvorson & Ziegler's tables the maximum likelihood estimate, R_L, of ρ.

(*b*) Taking R_L, read off from the chart lower and upper confidence limits, say $\rho_1(R_L)$ and $\rho_2(R_L)$.*

(*c*) Then, using Neyman's terminology, the statement

$$\rho_1(R_L) < \rho < \rho_2(R_L)$$

may be associated with a confidence coefficient of 0·95. In other words, if this procedure is applied in general bacteriological practice to the three frequencies n_{10}, n_1, and $n_{0{\cdot}1}$ obtained from the dilution method, then the odds will be at

* The authors take λ' as the maximum likelihood estimate of mean density λ in the most concentrated of the three solutions, so that $\lambda' = 10R_L$.

least* 19 : 1 that the interval $\rho_1(R_L)$, $\rho_2(R_L)$ will cover the unknown mean density ρ.

It should be emphasized that the chart provided by Matuszewski *et al.* was not based on an exact mathematical solution, but found by graduating a series of experimental sampling results; the following quotation from their paper (p. 76) explains the method employed:

> The method followed by Miss J. Supińska consisted in a complex sampling experiment, using Tippett's random sampling numbers. The experiment produced a series of values of the variates x_0, x_1 and x_2 [i.e. n_{10}, n_1, and $n_{0\cdot1}$ in Gordon's notation] following the sampling distribution which they would follow in our hypothetical conditions of the experiment. For each series of x_0, x_1 and x_2 it was possible to read up from the table of Halvorson & Ziegler an estimate, say λ', of the concentration λ. The estimates λ' have been then tabulated and an empirical frequency distribution of λ' corresponding to several fixed values of λ has been determined. Following the method described by J. Neyman, these empirical frequency distributions were then used to construct confidence intervals as if they were the accurate ones. As the random variation could not fail to affect the limits of the intervals it was felt necessary to correct them by fitting two parabolae, one marking the lower and the other the upper limits of the confidence intervals.

While therefore the writers did not claim exactness for their results, their chart, combined with Halvorson & Ziegler's table, does provide a provisional working solution not yet available to those who prefer the inverse approach.

It is interesting to obtain from the chart the limits $\rho_1(R_L)$ and $\rho_2(R_L)$ in the three hypothetical cases discussed by Gordon on p. 178 above. In Table III these limits and also their logarithms are given.

TABLE III

Observed frequencies			Gordon's estimate, $\bar{\rho}$	Maximum likelihood estimate, R_L	$\log R_L$	95 % confidence limits				
n_{10}	n_1	$n_{0\cdot1}$				ρ_1	ρ_2	$\log\rho_1$	$\log\rho_2$	$\frac{1}{2}\log\rho_1\rho_2$
10	7	3	1·43	1·53	0·18	0·80	3·00	−0·097	0·477	0·19
8	5	1	0·291	0·267	−0·57	0·125	0·525	−0·903	−0·280	−0·59
4	2	1	0·086	0·080	−1·10	0·029	0·165	−1·54	−0·78	−1·16

It will be noted that:

(1) The 95 % confidence interval for ρ is in all cases relatively broad and, having regard to this, the differences between the single valued estimates $\bar{\rho}$ and R_L are of little importance.

(2) While neither $\bar{\rho}$ nor R_L is central with regard to the interval, log R_L

* 0·95 is a lower limit to the probability of a correct statement, owing to the discontinuous distribution of the frequencies.

differs only slightly from $\frac{1}{2}(\log \rho_1 + \log \rho_2)$, as will be seen by comparing the 6th and last columns. The length of the interval $\log \rho_1 - \log \rho_2$ changes slowly, increasing as R_L decreases. A study of the chart shows that for R_L roughly in the range 0·6–2·4, the breadth of the interval $\log \rho_2 - \log \rho_1$ remains nearly constant at a value of about 0·55, increasing slowly as R_L drops below 0·6 or rises above 2·4. Without further details of Supinska's sampling experiment, it is not possible to make a closer comparison between this confidence chart based on random sampling and the four distributions of R_L tabled by Halvorson & Ziegler. Both results suggest, however, that for a considerable range of values of ρ, $\log_{10} R_L$ is distributed about a mean value of approximately $\log_{10} \rho$ with a standard error of about 0·16.

If finally it be asked whether the direct or inverse solution is to be preferred, the answer must, I think, be that this can only be a matter of personal opinion. As mentioned in the footnote on p. 181 Greenwood & Yule (1917) preferred the latter method. The fundamental difference of the two methods of approach has recently been emphasized by Harold Jeffreys, with whom it is to be supposed that Gordon is in substantial agreement. Writing with regard to the *a priori* distribution of an unknown parameter, such as ρ, Jeffreys says (1938, p. 466):

> I can find nothing in the works of the pioneers of the principle of inverse probability to suggest that they identified the prior probability with a known frequency, and believe that if such an idea had occurred to them they would have repudiated it as definitely as I do. The function of a prior probability used to express ignorance is simply to express formally the transition from an inference about different possible data, given the hypothesis, to one about different hypotheses given the same data, and this transition must be made somehow on any theory.

It follows that if the prior probability distribution cannot be identified with frequency, neither can the posterior distribution. A probability, in Jeffreys' sense, obtained from the integral of the posterior distribution between limits ρ_1 and ρ_2, can be regarded as no more than a rational measure of the degree of belief that the experimenter may place in the truth of the statement $\rho_1 < \rho < \rho_2$. It can have no precise link with long-run relative frequency. This, indeed, it could only have if there were reason to suppose that in repeated dilution experiments the population value of ρ would be distributed uniformly between 0 and ∞.* But the legitimacy of any attempt to make this connexion Jeffreys has denied emphatically. For him, if I understand rightly, the posterior distribution derived from the formal prior distribution provides the one type of numerical scaling which he regards as useful in forming an opinion on the value of an unknown constant. To quarrel with this conviction would be out of place.

On the other hand, those who agree with Jeffreys must recognize that the direct approach with its fiducial argument has resulted from the development

* Gordon has assumed this to be the most appropriate form of prior distribution for ρ; possibly Jeffreys would prefer to make the distribution vary as $1/\rho$.

of a line of thought which finds probability statements useful in a form in which they can be directly related to long-run frequency of occurrence. To know that if a certain experimental technique is carried out and an arithmetical calculation made, then there are strong grounds for believing* that about 19 times out of 20 the limits $\rho_1(R_L)$ and $\rho_2(R_L)$ will include the unknown ρ is a form of information which appeals to a large number of statisticians. In this and other problems it seems likely to appeal particularly to persons who are frequently repeating the same form of operation, and can therefore the more readily appreciate the consequences of "being wrong" once in 10, once in 20, or once in 100 times, according to the risk they choose to allow. It is true that in certain instances there may be other special information which will enable the experimenter to guess at narrower or modified limits, rather than those obtained from the standard fiducial procedure. But whether this information is ever of a kind which can be put into numerical form is doubtful. Certainly this would not be achieved by using the prior probability distribution proportional to $d\rho$ or $d\rho/\rho$. The fact that we can sometimes narrow the range of uncertainty and so get nearer the mark, does not detract from the long-run "safeness" of the fiducial argument. Of course, whatever the approach, it is essential that the sampling should have been random.

From the point of view of the bacteriologist this difference of opinion between experts may be discouraging, but it has been shown more than once that the two lines of approach lead to results which, from the practical point of view, are almost precisely the same. Thus Jeffreys (1937), starting with a prior probability law for σ of $d\sigma/\sigma$, has reached "Student's" distribution for the posterior probability law for the mean; it follows that in this instance the fiducial limits of the direct approach associated with a confidence coefficient of, say, 0·95, will correspond exactly with those obtained from Jeffreys' posterior distribution and associated with this probability measure of 0·95. It is to be hoped, therefore, that if Mr Gordon obtains financial support for the lengthy computation required to produce tables of $\bar{\rho}$ and ϵ, he will at the same time make some research into the correspondence of his limits for ρ with those following from the direct approach. By this means he would undoubtedly widen the range of persons who could use his tables with confidence.

REFERENCES

BARKWORTH, H. & IRWIN, J. O. (1938). *J. Hyg., Camb.*, **38**, 446.
FISHER, R. A. (1922). *Philos. Trans.* A, **222**, 309.
GREENWOOD, M. & YULE, G. U. (1917). *J. Hyg., Camb.*, **16**, 36.
HALVORSON, H. O. & ZIEGLER, N. R. (1933*a*). *J. Bact.* **25**, 101.
—— —— (1933*b*). *J. Bact.* **26**, 331.
—— —— (1933*c*). *J. Bact.* **26**, 559.
JEFFREYS, H. (1937). *Proc. Roy. Soc.* A, **160**, 325.
—— (1938). *Proc. Roy. Soc.* A, **167**, 464.
MATUSZEWSKI, T., NEYMAN, J. & SUPIŃSKA, J. (1935). *J. R. Statist. Soc. Suppl.* **2**, 63.

* The confidence felt cannot be expressed in terms of a numerical probability but is based upon experience of the stability of statistical ratios.

ON THE DISTRIBUTION OF MAXIMUM LIKELIHOOD ESTIMATES

By B. L. WELCH

THE properties of maximum likelihood estimates have been discussed by F. Y. Edgeworth (1908), R. A. Fisher (1922) and others. Most important is the property that in large samples a maximum likelihood estimate tends to be normally distributed with a variance which is given by a very simple formula and that no other estimate can have a smaller variance. If we have a random sample of n from a population whose probability law, $p(x \mid \theta)$, depends on only one parameter θ, and if T is the maximum likelihood estimate of θ, then under certain conditions it may be shown that T is in large samples normally distributed about θ with variance $1/nA_\theta$, where

$$A_\theta = \int \left(\frac{\partial \log p}{\partial \theta}\right)^2 p\,dx = -\int \frac{\partial^2 \log p}{\partial \theta^2} p\,dx. \qquad \text{.....(1)}$$

Similar formulae are available for the variances and covariances of estimates when there are several parameters, but only the single parameter case will be discussed here.

When dealing with small samples the maximum likelihood estimate T is frequently adopted together with the method of approximating its distribution by referring it to a normal curve with mean θ and variance $1/nA_\theta$. The question arises whether this is an adequate procedure. This question splits into two parts. First, we may ask how far we are likely to go wrong by assuming that T is actually distributed in the manner known to be correct in large samples: and secondly, we may ask how far the advantage which T holds over any other estimate in the matter of sensitivity in large samples is retained in small samples. The first of these problems has an interesting historical aspect. Karl Pearson (1936) has told us that because of his doubt of the adequacy of the approximation in finite samples he made no subsequent use of the above large sample variance formula originally given by him and L. N. G. Filon (1898). However, the real reason for his view that the approximation is not good perhaps lay partly in the fact that he had not noted that the formula referred only to maximum likelihood estimates—a point not made clear until Edgeworth returned to the problem in 1908. Indeed this question of how good the approximation is cannot be answered very definitely, for there is, as a rule, no general agreement as to how close an approximation should be before it can be termed good.

The second problem has been discussed extensively by R. A. Fisher (e.g. 1925), who concludes that the maximum likelihood estimate still retains in

small samples desirable properties which recommend its use in preference to other estimates.

The present note is concerned only with the first problem and, in connexion with this, it is not intended to imply that the approximation by the large sample formula is not good but rather to give a method which may supply a closer approximation if this be thought desirable.

Let us write $p(x \mid \theta)$ equal to $\exp[\phi(x, \theta)]$ so that the maximum likelihood estimate T is given by

$$\Sigma \phi'(x, T) = 0. \qquad \ldots\ldots(2)^*$$

In general, the solution of this equation is not algebraically simple, although the numerical solution by iteration may not be difficult. It is therefore not possible to find the distribution of T, and indeed it is generally impracticable to find even the moments of T. However, usually the actual distribution of T is not required, but simply a method of calculating the probability that T shall exceed any specified value T_0 (say). It is often possible to find the moments of another quantity which will facilitate such a calculation.

If the sample be represented by a point in n-dimensional space, then all samples yielding the same value T_0 of T will lie on the hypersurface $\Sigma \phi'(x, T_0) = 0$. Conversely, with most probability laws likely to be encountered, and certainly for that of the example given below, it will not be possible for this hypersurface to contain points yielding a maximum likelihood estimate different from T_0. On one side of the hypersurface we may therefore expect $T > T_0$ and on the other side $T < T_0$. Now since by Taylor's theorem

$$\Sigma \phi'(x, T_0) = \Sigma \phi'(x, T) + (T_0 - T)\, \Sigma \phi''(x, T) + \text{etc.}, \qquad \ldots\ldots(3)$$

and since by definition of T we have $\Sigma \phi'(x, T) = 0$ and $\Sigma \phi''(x, T)$ negative, we shall expect $T > T_0$ on that side of the hypersurface for which $\Sigma \phi'(x, T_0) > 0$. Hence if the maximum likelihood equation establishes a many-one relationship between sample points and values of T, we see that the probability that $T > T_0$ is equal to the probability that $\Sigma \phi'(x, T_0) > 0$. But the quantity $\Sigma \phi'(x, T_0)$ is the simple sum of a number of independent components, and its cumulants therefore follow simply from those of a single $\phi'(x, T_0)$. Any of the usual methods of approximating to probabilities from cumulants may then be employed to evaluate $P\{\Sigma \phi'(x, T_0) > 0\}$ and hence $P(T > T_0)$.

As an example consider the probability law

$$p(x \mid \theta) = \frac{2}{\Gamma(\frac{1}{4})} e^{-(x-\theta)^4}, \qquad \ldots\ldots(4)$$

which has been discussed at some length by Edgeworth (1908). We have $\phi'(x, \theta) = 4(x - \theta)^3$ and therefore

$$\Sigma \phi'(x, T_0) = 4\Sigma (x - T_0)^3. \qquad \ldots\ldots(5)$$

* The dash denotes differentiation with respect to θ, and Σ denotes summation over the whole sample.

Now by a straightforward method it may be shown that the first three cumulants of $4(x-T_0)^3$, when the true value of the parameter is θ, are given by

$$\left.\begin{aligned}\kappa_1 &= \mu_1' = 12c(\theta-T_0)+4(\theta-T_0)^3,\\ \kappa_2 &= \mu_2 = 12c+(60-144c^2)(\theta-T_0)^2+144c(\theta-T_0)^4,\\ \kappa_3 &= \mu_3 = 36(\theta-T_0)[(5-12c^2)+(48c+96c^3)(\theta-T_0)^2\\ &\qquad +(36-144c^2)(\theta-T_0)^4],\end{aligned}\right\} \quad \text{.....(6)}$$

where

$$c = \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} = 0{\cdot}33799.$$

The cumulants of $\Sigma\phi'(x, T_0)/n$ are then κ_1, κ_2/n and κ_3/n^2. By fitting a Pearson Type III curve with these three moments we can then approximate to $P\{4\Sigma(x-T_0)^3/n > 0\}$. If closer approximations are necessary further moments may be calculated and some other kind of curve fitted.

More frequently, perhaps, the converse problem is posed of finding T_0 such that $P(T > T_0)$ has a specified probability. The above method then involves a certain amount of trial and error. For instance, suppose $n = 25$ and T_0 is wanted so that $P(T > T_0)$ equals 0·05. A first approximation is obtained by taking T to be normally distributed about θ with variance $1/25A_\theta$, where

$$A_\theta = \int \{4(x-\theta)^3\}^2 p\,dx = 12\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} = 4{\cdot}056. \qquad \text{.....(7)}$$

The approximate standard deviation of T is then 0·0993 and T_0 is $\theta+0{\cdot}163$. Substituting this value of T_0 in (6) we obtain $\kappa_1 = -0{\cdot}678$, $\kappa_2/25 = 0{\cdot}210$, and $\kappa_3/625 = -0{\cdot}0392$. The standard deviation of $4\Sigma(x-\theta-0{\cdot}163)^3/25$ is therefore exactly 0·458, and the skewness $\sqrt{\beta_1} = -0{\cdot}407$. Using these moments and the very convenient tables of L. R. Salvosa (1930) we find that approximation by a Pearson Type III curve gives $P\{4\Sigma(x-\theta-0{\cdot}163)^3/25 > 0\} = 0{\cdot}056$. From this it is not difficult to see that, by adjusting T_0 to the value $\theta+0{\cdot}168$, the Type III approximation to $\Sigma\phi'(x, T_0)/n$ will give $P\{4\Sigma(x-\theta-0{\cdot}168)^3/25 > 0\} = 0{\cdot}05$, i.e. $P(T > \theta+0{\cdot}168) = 0{\cdot}05$.

There is in the present example little difference between the results given by the large sample approximation to the distribution of T and the Type III approximation to the distribution of $\Sigma\phi'(x, T_0)/n$. This is partly due to the nature of $p(x \mid \theta)$, owing to whose symmetry the maximum likelihood estimate, T, has a distribution which is *exactly* symmetrical with true mean at θ. The main approximation in using the large sample method therefore lies in the value adopted for the variance. Often the distribution of T will not be symmetrical nor have true mean exactly at θ. It is in such cases that a method which uses exact values of the lower order moments of $\Sigma\phi'(x, T_0)/n$ may be expected to yield the greatest improvement in accuracy.

REFERENCES

EDGEWORTH, F. Y. (1908, 1909). "On the probable errors of frequency constants." *J. R. Statist. Soc.* **71**, 381–97, 499–512, 651–78; **72**, 81–90.

FISHER, R. A. (1922). "On the mathematical foundations of theoretical statistics." *Philos. Trans.* A, **222**, 309–68.

FISHER, R. A. (1925). "Theory of statistical estimation." *Proc. Camb. Phil. Soc.* **22**, 700–25.

PEARSON, KARL (1936). "Method of moments and method of maximum likelihood." *Biometrika*, **28**, 34–59.

PEARSON, KARL & FILON, L. N. G. (1898). *Philos. Trans.* A, **191**, 229 *et seq.*

SALVOSA, L. R. (1930). *Tables of Pearson's Type III Function.* Reprinted from *Ann. Math. Stat.* May 1930. Publ. Edwards Brothers, Inc., Ann Arbor, Michigan.

ON NEYMAN'S "SMOOTH" TEST FOR GOODNESS OF FIT

I. DISTRIBUTION OF THE CRITERION ψ^2 WHEN THE HYPOTHESIS TESTED IS TRUE

By F. N. DAVID

1. Introductory

The χ^2 test has passed into common use since its introduction by Karl Pearson in 1900. It has, however, long been recognized that in certain cases it is inadequate as a test for goodness of fit, and in particular in the case when the deviations of the observations from the hypothesis tested are consecutively positive (or negative); for the χ^2 test, by taking into account the square of the difference between the observed and expected values, renders it impossible to pay attention to the sign of this difference. It is because of this inadequacy that Neyman (1937) introduced the "smooth" test for goodness of fit. This "smooth" test has been discussed by E. S. Pearson (1938) in some detail, which absolves us from further discussion here, but the procedure of the test will be set out in order that the substance of this present paper may be understood.

Following the procedure of previous papers (1933, 1936) Neyman (1937, p. 156) insists that a choice of a suitable test for goodness of fit may only be made after a statement of the probability functions specified both by the hypothesis tested and by the admissible hypotheses alternative to that tested. In order to supplement the χ^2 test he confines the alternative frequency laws to a class of functions which he terms "smooth", and whose form it is supposed can be represented by transformed Legendre polynomials. The keystone of his test lies in a probability integral transformation previously given by R. A. Fisher (1932) and K. Pearson (1933), by means of which the distribution of n independent random variables may be made rectangular.*

The criterion ψ^2, which may be used instead of χ^2 if the admissible probability laws are smooth, is designed to test the departure of the distribution of the transformed variables from rectangularity. Before it may be applied, however, it is necessary to choose what may be termed the order of the test, bearing in mind which departures from the hypothesis tested it is most important for the investigator not to overlook. The question of the appropriate order of test to choose has

* For example if

$$\left.\begin{aligned} p(y) &= \text{constant for } a < y < b \\ &= 0 \text{ otherwise} \end{aligned}\right\}$$

we should say that y was distributed rectangularly in the interval $[a; b]$. An alternative method of expression would be to say that in the interval $[a; b]$ all values of y are equally likely.

been discussed to some extent by Neyman (1937) and E. S. Pearson (1938). Further investigation into the matter has been carried out and the results will be published in a second part of this paper. Once the order of the test has been decided, then the calculation of the criterion ψ^2 is a comparatively simple matter and may be set out in the following way.

Assume that there are n independent random variables x_j $(j=1, 2, \ldots, n)$, following a known continuous probability law which is specified by H_0, the hypothesis to be tested. By means of the relation

$$y = \int_{\infty}^{x} p(x/H_0)\,dx, \qquad \ldots\ldots(1)$$

the n independent variables x are transformed into n independent variables y which are rectangularly distributed, *whatever the probability law of the x's*. If it is decided that a test of the kth order is to be applied, the next step involves the calculation of k quantities u_i $(i=1, 2, \ldots, k)$ by means of the substitution of the n variables y which were obtained from (1) into the appropriate Legendre polynomials, π_i, where

$$u_i = n^{-\frac{1}{2}} \sum_{j=1}^{n} \pi_i(y_j) \quad i = 1, 2, \ldots, k. \qquad \ldots\ldots(2)$$

The first four Legendre polynomials* are

$$\left.\begin{array}{ll} \pi_1(y) = \sqrt{12}\,(y-\frac{1}{2}) & \pi_3(y) = \sqrt{7}\,(20(y-\frac{1}{2})^3 - 3(y-\frac{1}{2})) \\ \pi_2(y) = \sqrt{5}\,(6(y-\frac{1}{2})^2 - \frac{1}{2}) & \pi_4(y) = 210(y-\frac{1}{2})^4 - 45(y-\frac{1}{2})^2 + \frac{9}{8} \end{array}\right\} \qquad \ldots\ldots(3)$$

The criterion ψ^2 consists of the sum of the squares of these u's, thus

$$\psi^2 = \sum_{i=1}^{k} u_i^2 = n^{-1} \sum_{i=1}^{k} \left(\sum_{j=1}^{n} \pi_i(y_j) \right)^2. \qquad \ldots\ldots(4)$$

Any such criterion is, however, of little use if its distribution is not known. An approximation to the distribution was given by Neyman and it is this approximation which is reviewed in the next section.

2. Distribution of ψ^2

It was shown by Neyman that when n, the number of observations in the sample, is large, then each of the quantities u_i^2 $(i=1, 2, \ldots, k)$, is distributed independently as χ^2 with one degree of freedom. It will follow therefore that for large n the criterion ψ^2 of a kth order test will be distributed as χ^2 with k degrees of freedom. As with many statistical tests based on the assumption that n is large, the decision as to the numerical value of n is left to the user of the test and, with the introduction of the personal element, we get a variation in ideas both as to

* It is of interest to note that a recurrence formula for these polynomials follows directly from a general formula for Legendre polynomials (see for example Whittaker & Watson, *Modern Analysis*, p. 302, II). If $z=y-\frac{1}{2}$ then

$$\frac{n+1}{\sqrt{(2n+3)}}\,\pi_{n+1}(z) - 2z\sqrt{(2n+1)}\,\pi_n(z) + \frac{n}{\sqrt{(2n-1)}}\,\pi_{n-1}(z) = 0.$$

what is meant by a large sample and the point at which a large sample becomes a small sample. It is therefore useful to obtain some idea of the degree of approximation involved in the supposition that ψ^2 is distributed as χ^2. Although this may only be investigated completely by the discovery of the true distribution of ψ^2 it is not difficult to find Pearson curves which have moments agreeing with the true moments of ψ^2, and these may be used to throw light on the approximation to the χ^2 distribution.

The derivation of the true moments of ψ^2 is a matter of simple but tedious algebra. Using the notation $\mathscr{E}$ to denote mathematical expectation, it is seen that we must calculate

$$\mu_t = \mathscr{E}(\psi^2 - \mathscr{E}(\psi^2))^t \quad \text{for} \quad t = 1, 2, 3, 4, \qquad \text{.....(5)}$$

where ψ^2 is as defined in (4) and the y's are independently distributed within the interval [0; 1] in accordance with the rectangular law. I have calculated only the moments of ψ^2 for first and second order tests, partly because numerical work* leads me to believe that these order tests are more powerful to detect *any* departures from the basic hypothesis H_0 than any other higher order test, and partly because the necessary algebra became so very heavy.

From the properties of the transformed Legendre polynomials we know (see Neyman, 1937) that

$$\mathscr{E}(\pi_i(y_j)) = \int_0^1 \pi_i(y_j)\, dy_j = 0 \quad \text{for any } i \text{ and } j. \qquad \text{.....(6)}$$

Also since the observations are independent of one another

$$\mathscr{E}(\pi_i(y_j) \,.\, \pi_i(y_p)) = 0, \qquad \text{.....(7)}$$

and

$$\mathscr{E}(\pi_i^2(y_j)) = \int_0^1 \pi_i^2(y_j)\, dy_j = 1. \qquad \text{.....(8)}$$

In general it will be noticed that we require to calculate

$$\mathscr{E}(u_i^2)^t = n^{-1}\mathscr{E}\left(\sum_{j=1}^{n} \pi_i(y_j)\right)^{2t} \quad \text{for } i = 1, 2 \text{ and } t = 1, 2, 3, 4 \qquad \text{.....(9)}$$

and

$$\mathscr{E}(u_i^{2p}\, u_m^{2q}) = n^{-1}\mathscr{E}\left(\left(\sum_{j=1}^{n} \pi_i(y_j)\right)^{2p}\left(\sum_{j=1}^{n} \pi_m(y_j)\right)^{2q}\right) \quad \text{for } i, m = 1, 2 \text{ and } p, q = 1, 2, 3. \qquad \text{.....(10)}$$

The calculations required are straightforward and follow at once from the application of the elementary theorems concerning the sum and product of expectations. The final results are given below in Table I.

It will be noticed that for n small the moments of ψ_1^2 and ψ_2^2 differ considerably from those of χ^2. It is, however, only fair to point out here that Neyman supposes, as in the case of the χ^2 test, that a "smooth" test for goodness of fit would not be

* To be discussed in Part II of this paper.

TABLE I

True moments of ψ^2	$\psi_1^2 = u_1^2, \quad (k=1)$	$\psi_2^2 = u_1^2 + u_2^2, \quad (k=2)$
μ_1'	1	2
μ_2	$2 - \frac{6}{5n}$	$4 - \frac{32}{35n}$
μ_3	$8 - \frac{72}{5n} + \frac{48}{7n^2}$	$16 + \frac{704}{49n} - \frac{722208}{35035n^2}$
μ_4	$60 - \frac{936}{5n} + \frac{7524}{35n^2} - \frac{432}{5n^3}$	$144 + \frac{15216}{49n} - \frac{2203468}{35035n^2} + \frac{17946980}{119119n^3}$

carried out except on a large number of observations. It seems to be possible by an approximate method to obtain some idea of how large n should be.

A consideration of the β-coefficients calculated from the moments of Table I suggests that the most appropriate curves to use would be a Pearson Type I for ψ_1^2 and Type VI for ψ_2^2. Since, however, it was hoped that both curves would approximate quickly to the χ^2 Type III distribution, as a first step Type III curves with the start of the curve at the origin were fitted to the moments of both ψ_1^2 and ψ_2^2. As there are only two constants to evaluate for the Type III curve with zero start, the procedure was equivalent to finding a χ^2 distribution with the mean and standard deviation equal to the corresponding true values for ψ^2. For purposes of comparison, the 5 % and 1 % levels of χ^2 with $f = 1$ and $f = 2$ were taken, and the tail areas of the fitted Type III curve corresponding to these abscissae were calculated from *Tables of the Incomplete Γ-Function* (K. Pearson, 1922). The results are summarized in Table II.

TABLE II

	$f=1$, $\chi^2_{0\cdot05}=3\cdot841$, $\chi^2_{0\cdot01}=6\cdot635$					
n	5	10	20	30	50	100
μ_2	1·76	1·88	1·94	1·96	1·976	1·988
$P\{\psi_1^2 > 3\cdot841\}$	0·0523	0·0511	0·0505$^+$	0·0504	0·0502$^+$	0·0502$^-$
$P\{\psi_1^2 > 6\cdot635\}$	0·0098	0·0099	0·0099^5	0·0100	0·0100	0·0100

	$f=2$, $\chi^2_{0\cdot05}=5\cdot991$, $\chi^2_{0\cdot01}=9\cdot210$					
n	5	10	20	30	50	100
μ_2	3·8171	3·9086	3·9543	3·9695$^+$	3·9817	3·9909
$P\{\psi_2^2 > 5\cdot991\}$	0·0511	0·0505$^+$	0·0503	0·0502	0·0501	0·0500
$P\{\psi_2^2 > 9\cdot210\}$	0·0100	0·0100	0·0100	0·0100	0·0100	0·0100

The probability levels of ψ^2 as given by the empirical Type III curve approximate quite quickly to those of χ^2, and at first sight it would appear that for any size of sample little risk would be involved in testing the significance of ψ^2 by means of the χ^2 tables. However, it must not be forgotten that the empirical curve is itself an approximation to the unknown true ψ^2 distribution. To obtain some idea therefore of the approximation involved it is necessary to calculate the β_1 and β_2 of the fitted Type III curves and compare them with the β's of the true distribution of ψ^2 and of the χ^2 distribution. These values of the β's are plotted in the diagram and their numerical values are given in Table IV. It will be noted that the β's of the fitted Type III curves lie nearer the χ^2 points (8, 15) and (4, 9) than do those of the true ψ^2. Hence, while the results of Table II suggest that the approximation of the ψ^2 distribution by a Type III distribution is a fairly good one, it would appear that further investigation is advisable before any definite conclusion may be drawn.

It was therefore decided to utilize the true third moments of ψ^2. Type I curves with zero start were fitted using the first three moments of ψ_1^2, and similarly Type VI curves using those of ψ_2^2. It will be recognized that the procedure for the fitting of a Type VI curve and the evaluation of its integral is very similar to that for the fitting of Type I, and the calculations required were almost the same for both curves. Accordingly it is necessary to set out the steps followed for the case of Type I only.

If the equation of the Type I curve is written as

$$Z = \frac{1}{a^{m_1+1}} \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1)\,\Gamma(m_2+1)}\, Y^{m_1}\left(1-\frac{Y}{a}\right)^{m_2} \qquad \text{.....(11)}$$

easy algebra will give

$$M_1' = a\frac{(m_1+1)}{(m_1+m_2+2)}, \quad M_2' = aM_1'\frac{(m_1+2)}{(m_1+m_2+3)}, \quad M_3' = aM_2'\frac{(m_1+3)}{(m_1+m_2+4)}, \qquad \text{.....(12)}$$

where M_1', M_2' and M_3' are the true moments of ψ_1^2 about the origin. It was not found possible to write down the solutions of a, m_1 and m_2 in any neat form, and the procedure followed was to substitute numerical values for M_1', M_2' and M_3' at an early stage. The partial integral of (11) is recognized as an Incomplete Beta-Function. Hence to calculate the tail areas corresponding to $\chi^2_{0\cdot05}$ and $\chi^2_{0\cdot01}$ it will only be necessary to refer to *Tables of the Incomplete Beta-Function* (K. Pearson, 1934). Table III gives the results of interpolation into these tables.

It will be noted that for strict accuracy tripie interpolation would be necessary. This, however, called for much calculation, so an approximation was made by taking $m_1 = -0\cdot50$ and using a double interpolation formula only. The effect of the approximation is to make the interpolated values slightly *greater** numerically than those values which would have been obtained by the use of triple inter-

* A rough form of quadrature for the case $f=1$, $n=5$ gives $P\{\psi_1^2 > 3\cdot841\} = 0\cdot0481$.

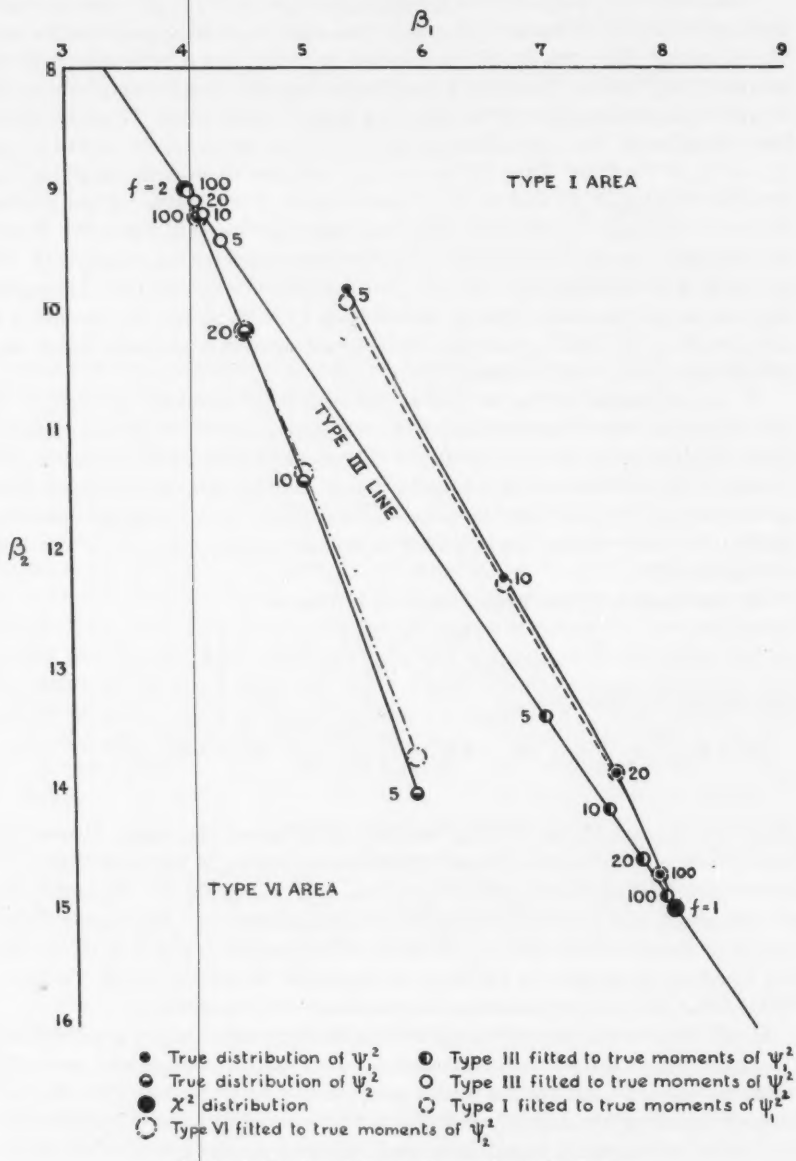


Fig. 1. β_1, β_2 points for true and approximate distributions.

TABLE III

$f=1$, $\chi^2_{0\cdot05}=3\cdot841$, $\chi^2_{0\cdot01}=6\cdot635$				$f=2$, $\chi^2_{0\cdot05}=5\cdot991$, $\chi^2_{0\cdot01}=9\cdot210$			
n	5	10	20	n	5	10	20
μ_3	5·3944	6·6286	7·2971	μ_3	18·0489	17·2312	16·6668
a	20·5920	45·3643	95·2355$^+$	a	16·2813	30·8654	60·3289
m_1	−0·50797	−0·50185$^+$	−0·50044	m_1	0·29947	0·154503	0·07825$^-$
m_2	8·63985$^+$	21·10009	46·07631	m_2	10·57853	17·81708	33·52469
$P\{\psi_1^2>3\cdot841\}$	0·049$^-$	0·049$^+$	0·050$^-$	$P\{\psi_2^2>5\cdot991\}$	0·047$^+$	0·049$^+$	0·050$^-$
$P\{\psi_1^2>6\cdot635\}$	0·007$^+$	0·009$^-$	0·010$^-$	$P\{\psi_2^2>9\cdot210\}$	0·010$^-$	0·010$^-$	0·010$^-$

TABLE IV

	Type of curve		$n=5$	$n=10$	$n=20$	$n=100$
ψ_1^2	True values	β_1	5·34	6·61	7·52	7·86
		β_2	9·84	12·25$^+$	13·86^5	14·71
	Fitted Type III	β_1	6·95	7·48	7·74	7·95$^-$
		β_2	13·42	14·22	14·61	14·92
	Fitted Type I	β_1	5·34	6·61	7·52	7·86
		β_2	9·94	12·30	13·89	14·71
ψ_2^2	True values	β_1	5·86	4·97	4·49	4·10
		β_2	14·06	11·43	10·19	9·24
	Fitted Type III	β_1	4·30	4·14	4·08	4·01
		β_2	9·45	9·21	9·12	9·01
	Fitted Type VI	β_1	5·86	4·97	4·49	4·10
		β_2	13·74	11·37	10·17	9·23

polation formulae. As n increases m_1 gradually approaches the value $-0\cdot5$. For values of n greater than 20 the required Beta-Function ratio fell outside the range of the existing tables. These values therefore could not be evaluated except by quadrature but the results for $n = 5$, 10 and 20 give sufficient indication for our purposes of the approximate error involved. A system of triple interpolation was carried out for the evaluation of the integral of the Type VI curve but the calculations became so very heavy that it was only found practicable to obtain the result correct to three decimal places. The results of interpolation were confirmed approximately by a rough quadrature of the curves. The tail areas of the Type I

and Type VI curves beyond $\chi^2_{0\cdot05}$ are seen to be slightly less than 0·05 but nearer to that value than the corresponding areas from the Type III approximation shown in Table II. On the other hand the Type I and Type VI areas beyond $\chi^2_{0\cdot01}$ are a little further from 0·01 than for the Type III approximation. Except perhaps in the case of $n = 5$ of the Type I curve these differences can hardly be regarded as of importance. To judge how closely the Type I and Type VI curves are likely to approximate to the unknown true distributions we may compare their β_2 values with the true values. This has been done in Table IV and in the diagram. The proximity in position between the solid and dotted circles in the latter, suggests that the Type I and Type VI curves must represent the true ψ^2_1 and ψ^2_2 distributions very closely.

Tnus the probability levels of the Type I and Type VI curves given in Table III may be taken as lying not very far from the true values, and this shows that no great error will be made if we assume for samples of 20 and over that Neyman's ψ^2 criterion of the first and second orders are distributed as χ^2 with one and two degrees of freedom respectively.

3. Sphere of usefulness of the "smooth" test

It was pointed out in § 1 that where the deviations of the observations from the hypothesis tested are consecutively positive (or negative) then the ψ^2 "smooth" test for goodness of fit may be applied. It does not seem possible to give any definite rule as to when it might be used in preference to the χ^2 test. Neyman remarks that it would be interesting to enquire into the relative workings of the χ^2 and ψ^2 tests, possibly with this point in mind, but nothing has been written on the subject and it is difficult to see how a comparison of the powers of the two tests could be obtained.

While recognizing the ingenuity of Neyman's "smooth" test, and the attempt it makes usefully to supplement the χ^2 test, the present writer feels that as it stands it is not wholly satisfactory, and that much work remains to be done before it can pass into common use. It has been pointed out in § 1 that Neyman (1937) and E. S. Pearson (1938) have both discussed the appropriate order of test to choose, without however reaching any definite conclusion. It has been possible to remedy this in part by confirming E. S. Pearson's suggestions through numerical calculations, but an extension of the theory appears necessary before the test is generally applicable.

In § 2 it has been shown that serious error will not be made if the use of the "smooth" test is restricted to samples of twenty and over, and Neyman himself says that he would use the test only if the available sample is large. At present, however, the application of the test presents rather a formidable task for any computer. For example, if the sample is of size 100, and it is desired to apply a test of the second order, the numerical work would entail 100 entries into tables of the appropriate probability integral with possible interpolations and 100

substitutions into each of the first and second polynomials. Hence for a sample greater in size than 100 the labour of computation involved will incline the computer towards the simpler and more familiar χ^2 test. The need for such a "smooth" test cannot be denied, but it would have greater utility if it could be applied after some grouping of the observations has been made. Whether this is a possible development or not I do not know.

At present it is only possible to apply the "smooth" test when the parameters of the hypothesis tested are known, whereas the χ^2 test permits one to calculate them from the data, as for example in the fitting of Pearson curves. What the effect of calculating parameters from the data would be on the ψ^2 criterion I cannot at present suggest, although it is hoped to throw light on this point by means of a sampling experiment. Certainly if parameters are calculated from the data and used in the basic hypothesis, H_0, it would mean that the variables y would be no longer independent or rectangularly distributed. Whether it is possible to determine what form the probability law of y would take, given restrictions on the original sample, remains to be investigated.

I have to thank Prof. E. S. Pearson for helpful criticism and Miss J. Townend for drawing the figure.

REFERENCES

FISHER, R. A. (1932). *Statistical Methods for Research Workers*, Fourth Edition, § 21·1.
NEYMAN, J. (1937). *Skand. Aktuar. Tidskr.* **20**, 149–99.
NEYMAN, J. & PEARSON, E. S. (1933). *Philos. Trans.* A, **231**, 289–337.
—— —— (1936). *Statist. Res. Mem.* **1**, 1–37.
PEARSON, E. S. (1938). *Biometrika*, **30**, 134–48.
PEARSON, K. (1933). *Biometrika*, **25**, 379–410.
—— (1922). *Tables of the Incomplete* Γ*-Function*. Biometrika Office.
—— (1934). *Tables of the Incomplete* B*-Function*. Biometrika Office.

TESTS OF HYPOTHESES CONCERNING LOCATION AND SCALE PARAMETERS

By E. J. G. PITMAN
University of Tasmania

1. The comparison of location parameters

Suppose that k observed numbers

$$x_1, x_2, \ldots, x_k,$$

are values of k chance variables, and that the elementary probability function of the simultaneous distribution of the chance variables is

$$F(x_1 - a_1, \ldots, x_k - a_k),$$

the function F being of known form but the values of the location parameters $a_1, a_2, \ldots, a_k$ being unknown. Suppose, further, that we wish to test the hypothesis that

$$a_1 = a_2 = \ldots = a_k,$$

which we shall call the hypothesis H_0. Any test must be based upon some statistic J which is a function of the observed values $x_1, x_2, \ldots, x_k$, and H_0 will be accepted for certain values of J and rejected for all other values. If the observations are our only source of knowledge of the values of the location parameters, a satisfactory test must give the same answer when the observed values are

$$x_1 + \lambda, x_2 + \lambda, \ldots, x_k + \lambda,$$

as when they are

$$x_1, x_2, \ldots, x_k.$$

Hence the statistic J must have the property

$$J(x_1 + \lambda, \ldots, x_k + \lambda) \equiv J(x_1, \ldots, x_k). \qquad \ldots\ldots(1)$$

Without loss of generality, we may assume that J is always positive, and that a small value of J is regarded as significant, i.e. as indicating that the hypothesis H_0 is untrue. If this is not so, we simply replace J by some suitable function of it. In order to use the observed value of J as a test of H_0, we must know the distribution of J when H_0 is true. Since J has the property (1), its value will be unaffected by the same change of origin of all the x. Hence, when the a are all equal, we may, without loss of generality, assume that their common value is 0. We require then the distribution of J when

$$a_1 = a_2 = \ldots = a_k = 0.$$

Let p be any given number (such as 0·95) between 0 and 1. When this distribution of J is known, we can determine θ such that, when H_0 is true, the probability that $J \geqq \theta$ is p. Having chosen p, we reject the hypothesis H_0 if the observed value of

J is less than θ, and accept it otherwise. The probability of rejecting H_0 when it is true will be $1-p$. If J is so chosen that for a fixed value of θ the probability that $J \geqq \theta$ is greatest when H_0 is true, the hypothesis H_0 will be more likely to be accepted when it is true than when any alternative hypothesis is true. A test which has this property is said to be "unbiased" (Neyman & Pearson, 1936, p. 8). Let us attempt to determine J so that the resulting test is unbiased.

A set of values of the x may be specified by a point (the sample point) whose rectangular co-ordinates in a k dimensional space are $(x_1, x_2, \ldots, x_k)$. For the co-ordinates of a variable point in this space we shall use $(\xi_1, \xi_2, \ldots, \xi_k)$. Since J has the property (1), it will be constant along any line parallel to the line

$$\xi_1 = \xi_2 = \ldots = \xi_k, \qquad \ldots\ldots(2)$$

and therefore the region A, consisting of all points for which $J \geqq \theta$, will be bounded by a cylindrical hypersurface with its generators parallel to the line (2). When H_0 is true, the probability that $J \geqq \theta$ is

$$p = \int_A F(\xi_1, \ldots, \xi_k)\, d\xi_1 \ldots d\xi_k,$$

and when H_0 is not true, the probability is

$$p' = \int_A F(\xi_1 - a_1, \ldots, \xi_k - a_k)\, d\xi_1 \ldots d\xi_k$$

$$= \int_B F(\xi_1, \ldots, \xi_k)\, d\xi_1 \ldots d\xi_k,$$

where B is a region formed from A by translation without rotation. Thus, the necessary and sufficient condition for an unbiased test is that the integral of $F(\xi_1, \ldots, \xi_k)$ over the region A be greater than its integral over any equal and parallel region B.

If L is any line parallel to the line (2), we shall write

$$P(L) = \int_L F(\xi_1, \ldots, \xi_k)\, d\eta,$$

where

$$\eta = \Sigma\xi/\sqrt{k},$$

the distance of the point $(\xi_1, \xi_2, \ldots, \xi_k)$ from the plane $\Sigma\xi = 0$, and the integral is taken along L. Now if A is defined as the locus of all lines L for which $P(L)$ is greater than or equal to some constant h, $P(L)$ will be less than h on any line L in B which is not also in A. From this it easily follows that

$$\int_A F(\xi_1, \ldots, \xi_k)\, d\xi_1 \ldots d\xi_k > \int_B F(\xi_1, \ldots, \xi_k)\, d\xi_1 \ldots d\xi_k,$$

and so the resulting test is unbiased. Hence we shall have an unbiased test if we define J at any point as equal to $P(L)$ for the line L through that point.

The co-ordinates of a variable point on the line L through the point

$$(x_1, x_2, \ldots, x_k)$$

may be expressed in the form

$$\xi_r = x_r - t \quad (r = 1, 2, \ldots, k)$$

where

$$t = \frac{\Sigma x}{k} - \frac{\eta}{\sqrt{k}}.$$

The expression for $P(L)$ becomes

$$P(L) = \sqrt{k}\int_{-\infty}^{\infty} F(x_1 - t, \ldots, x_k - t)\, dt.$$

Instead of $P(L)$ we shall take $P(L)/\sqrt{k}$ for J, and, replacing the symbol t by the symbol a, we have

$$J = \int_{-\infty}^{\infty} F(x_1 - a, \ldots, x_k - a)\, da.$$

The test based on this J will be unbiased.

As a simple illustration, take the case where the x are independent normal variables, each with unit standard deviation, and the mean value of x_r is a_r.

$$F(x_1 - a_1, \ldots, x_k - a_k) = \frac{1}{(2\pi)^{\frac{1}{2}k}} \exp\{-\tfrac{1}{2}\Sigma(x_r - a_r)^2\},$$

therefore

$$J = \frac{1}{(2\pi)^{\frac{1}{2}k}} \int_{-\infty}^{\infty} \exp\{-\tfrac{1}{2}\Sigma(x-a)^2\}\, da$$

$$= \frac{e^{-\frac{1}{2}S}}{k^{\frac{1}{2}}(2\pi)^{\frac{1}{2}(k-1)}},$$

where

$$S = \Sigma(x_r - \bar{x})^2, \quad \bar{x} = \Sigma x_r/k.$$

In practice we would take S, which is a monotonic function of J, as our criterion, large values of S being significant. When H_0 is true, S is distributed like χ^2 with $k-1$ degrees of freedom.

Suppose now that the hypothesis to be tested asserts not merely that the location parameters of the chance variables x are all equal, but that these location parameters all have the same particular given value. By a change of origin of all the x this particular value may be taken as 0. Thus the hypothesis to be tested is that

$$a_1 = a_2 = \ldots = a_k = 0,$$

which we shall call the hypothesis H_0'. It is obvious that if we take

$$J' = F(x_1, \ldots, x_k)$$

and regard small values of J' as significant, the test will be unbiased. Moreover, it is very easy to show that, if the x are independent, so that F is the product of the separate elementary probability functions, and if each elementary probability function is unimodal, then, when H_0' is not true, the probability that $J' \geqq \theta$ increases as any a moves towards the value 0.

2. THE COMPARISON OF SCALE PARAMETERS

Here the elementary probability function of the chance variables x is

$$\Pi\{1/c_r\}\, F\{x_1/c_1, \ldots, x_k/c_k\},$$

where the c are all positive and we wish to test the hypothesis,

$$c_1 = c_2 = \ldots = c_k,$$

which we shall call the hypothesis H_1. Let us assume for the moment that the x are all positive chance variables. Putting

$$y_r = \log x_r, \quad a_r = \log c_r,$$

we have for the elementary probability function of the chance variables y

$$\exp(\Sigma y_r - \Sigma a_r)\,.\, F\{\exp(y_1 - a_1), \ldots, \exp(y_k - a_k)\}.$$

The hypothesis H_1 applied to the parameters c is equivalent to H_0 applied to the parameters a. We shall therefore have an unbiased test if we take as our criterion

$$J = \int_{-\infty}^{\infty} \exp(\Sigma y_r - ka)\,.\, F\{\exp(y_1 - a), \ldots, \exp(y_k - a)\}\, da$$

$$= \Pi\{x_r\} \int_0^{\infty} \frac{F\{x_1/c, \ldots, x_k/c\}}{c^{k+1}}\, dc.$$

The expression for J in the general case where the x are not necessarily positive chance variables is

$$J = \Pi\{|x_r|\} \int_0^{\infty} \frac{F\{x_1/c, \ldots, x_k/c\}}{c^{k+1}}\, dc.$$

Small values of this are significant. The axial planes divide the k dimensional space into 2^k regions, and the probability of the sample point falling in any one of these regions is independent of the particular values of the parameters c. By considering these possibilities separately and using the result for the case of all positive chance variables, it is easy to see that the test is still unbiased. J is a homogeneous function of degree 0 in the x; hence, when H_1 is true, its distribution is independent of the particular common value of the scale parameters c. Therefore in determining this distribution we may take the common value to be 1.

From the corresponding result of § 1, it follows that an unbiased test of the hypothesis H_1', that

$$c_1 = c_2 = \ldots = c_k = 1,$$

will be obtained by using

$$J' = \Pi\{|x_r|\}\,.\, F\{x_1, \ldots, x_k\},$$

small values of which are significant. Again, from the corresponding result of § 1 it follows that if the x are independent positive chance variables with elementary probability functions $c_r^{-1} f_r(x_r/c_r)$ such that $x f_r(x)$ is a unimodal function of x, then,

when H_1' is not true, the probability that $J' \geqq \theta$ increases as any c moves towards the value 1.

Tests of other hypotheses will be discussed in a later paper. The remainder of this paper will be devoted to a discussion of the most important application of the results of this section.

The problem of deciding whether k samples, each known to have been drawn from a normal population, have been drawn from populations with the same standard deviation has been discussed by Neyman & Pearson (1931), who have proposed a criterion λ_{H_1} for this purpose. Applying the method of this paper, we obtain a test which is the same as that proposed by Bartlett (1937, p. 273). Bartlett's test is therefore unbiased. It can be shown that the Neyman-Pearson test is unbiased only when the number in each sample is the same, in which case it is exactly equivalent to Bartlett's test.

A continuous chance variable which takes values from 0 to ∞ with elementary probability function

$$\frac{1}{\Gamma(m)} e^{-x} x^{m-1}$$

will be called a $\Gamma(m)$ variable. If its elementary probability function has the more general form

$$\frac{1}{c\Gamma(m)} e^{-x/c} \left(\frac{x}{c}\right)^{m-1},$$

we shall call it an *unscaled* $\Gamma(m)$ variable with scale parameter c. If

$$y_1, y_2, \cdots, y_n$$

are n numbers with mean $\bar{y}$, we shall call the expression

$$S = \sum_1^n (y_r - \bar{y})^2$$

their *squariance*.† We know that if the y are a sample of n values from a normal population of standard deviation σ, S/σ^2 is distributed like χ^2 with $n-1$ degrees of freedom, or, what is more convenient here, $\frac{1}{2}S/\sigma^2$ has a $\Gamma\{\frac{1}{2}(n-1)\}$ distribution, so that S is an unscaled $\Gamma\{\frac{1}{2}(n-1)\}$ variable with scale parameter $2\sigma^2$. Moreover, in this case, if the mean of the population is unknown, the whole of the information about σ supplied by the sample is contained in the value of S. If the mean of the population has the known value a,

$$\sum_1^n (y_r - a)^2$$

is a sufficient statistic for the estimation of σ; it is an unscaled $\Gamma(\frac{1}{2}n)$ variable with scale parameter $2\sigma^2$. Hence questions about variances of normal populations can

† If this word is objected to, I hope that someone will coin a better, for it is very convenient to have a name for this expression. The usual "sum of squares" is not precise in meaning and is awkward in use. As I have pointed out previously (Pitman, 1937, p. 215), it is the squariance of a sample of n that appears naturally in the mathematical theory, and not the variance S/n, or the "estimated variance" $S/(n-1)$. The last, however, becomes prominent in the applications.

be reduced to questions about scale parameters of unscaled Γ variables. In particular, if we have k samples with squariances

$$S_1, S_2, \ldots, S_k,$$

and if each sample has been drawn from a normal population of unknown mean and variance, the question "Are the variances of the normal populations all equal?" is equivalent to "Are the scale parameters of the unscaled Γ variables

$$S_1, S_2, \ldots, S_k$$

all equal?" The problem of answering this latter question we now proceed to consider.

3. Application to gamma variables

Suppose that $x_1, x_2, \ldots, x_k$

are k observed numbers, that x_r is a value of an unscaled $\Gamma(m_r)$ variable with scale parameter c_r, and that the k chance variables are independent. We wish to test the hypothesis H_1, that

$$c_1 = c_2 = \ldots = c_k.$$

Put $M = \Sigma m.$

Except where explicitly stated otherwise, all summations Σ and all products Π are to be taken over the k values of the operand, which will be written without an index. Thus

$$\Sigma m = \sum_{r=1}^{k} m_r, \quad \Pi\{x^m\} = \prod_{r=1}^{k} \{x_r^{m_r}\}.$$

The elementary probability function of the distribution of the x is

$$\frac{e^{-\Sigma(x/c)}\,\Pi\{(x/c)^{m-1}\}}{\Pi\{c\Gamma(m)\}}.$$

Hence

$$J = \frac{\Pi\{x^m\}}{\Pi\{\Gamma(m)\}} \int_0^\infty e^{-(\Sigma x)/c} \frac{dc}{c^{M+1}}$$

$$= \frac{\Gamma(M)}{\Pi\{\Gamma(m)\}} \frac{\Pi\{x^m\}}{(\Sigma x)^M}.$$

It is sometimes more convenient to deal with

$$K = \frac{\Pi\{x^m\}}{(\Sigma x)^M},$$

which differs from J only by a constant factor. The maximum value of K is

$$\frac{\Pi\{m^m\}}{M^M},$$

and we shall denote the logarithm of the ratio of this maximum to the value of K by L, so that

$$L = M \log (\Sigma x/M) - \Sigma\{m \log (x/m)\}.$$

It is necessarily positive or zero, and large values are significant.

For testing the hypothesis H_1', that

$$c_1 = c_2 = \ldots = c_k = 1,$$

the appropriate criterion is $$J' = \frac{\Pi\{x^m\}\, e^{-\Sigma x}}{\Pi\{\Gamma(m)\}}.$$

We shall write $$K' = e^{-\Sigma x}\, \Pi\{x^m\},$$

$$L' = \log (K'_{\max}) - \log K' = \Sigma x - M - \Sigma\{m \log (x/m)\}.$$

4. The distribution of K when H_1 is true

As explained above, when H_1 is true we may take the common value of the c to be 1. The probability that $K \geqq \kappa$ is

$$p = \frac{1}{\Pi\{\Gamma(m)\}} \int e^{-\Sigma x}\, \Pi\{x^{m-1}\}\, dx_1 \ldots dx_k$$

over the region where $$\frac{\Pi\{x^m\}}{(\Sigma x)^M} \geqq \kappa.$$

Make the change of variables

$$\Sigma x = u,$$

$$x_r = u y_r, \quad (r = 1, 2, \ldots, k-1)$$

and write for convenience $$y_k = 1 - \sum_1^{k-1} y_r,$$

so that $$\Sigma y = 1 \text{ and } x_k = u y_k.$$

Then integrating with respect to u, which is not involved in the boundary condition, we obtain for the value of p

$$\frac{\Gamma(M)}{\Pi\{\Gamma(m)\}} \int \Pi\{y^{m-1}\}\, dy_1 \ldots dy_{k-1} \text{ over } \Pi\{y^m\} \geqq \kappa.$$

As the maximum value of K is $\Pi\{(m/M)^m\}$, we are concerned only with values of κ less than this. It should also be noted that in the applications to normal variables only integral and half-integral values of $m_1, m_2, \ldots$ occur.

When $\kappa = 0$, $p = 1$, and the region of integration is

$$0 \leqq y_r \leqq 1, \quad (r = 1, 2, \ldots, k-1)$$

$$0 \leqq \sum_1^{k-1} y_r \leqq 1.$$

Therefore $$\int \Pi\{y^{m-1}\}\, dy_1 \ldots dy_{k-1}$$

over this region is equal to $$\frac{\Pi\{\Gamma(m)\}}{\Gamma(M)},$$

which may be denoted by $B(m_1, m_2, \ldots, m_k)$. It is the extension of the complete beta function,

$$B(m_1, m_2) = \frac{\Gamma(m_1)\, \Gamma(m_2)}{\Gamma(m_1 + m_2)} = \int_0^1 x^{m_1 - 1} (1-x)^{m_2 - 1}\, dx.$$

When $k = 2$, the expression for p is

$$\frac{1}{B(m_1, m_2)} \int_{x_1}^{x_2} x^{m_1-1}(1-x)^{m_2-1}\,dx,$$

where $$x_1^{m_1}(1-x_1)^{m_2} = x_2^{m_1}(1-x_2)^{m_2} = \kappa.$$

In the tabulations in § 5 for $k = 2$, the value of x_1 is given.

When $k = 3$,

$$p = \frac{1}{B(m_1, m_2, m_3)} \int y_1^{m_1-1} y_2^{m_2-1}(1-y_1-y_2)^{m_3-1}\,dy_1 dy_2$$

over $$y_1^{m_1} y_2^{m_2}(1-y_1-y_2)^{m_3} \geqq \kappa.$$

By the substitution $$y_1 = uv \qquad (0 \leqq u \leqq 1),$$

$$y_2 = u(1-v) \quad (0 \leqq v \leqq 1),$$

this becomes

$$p = \frac{1}{B(m_1, m_2, m_3)} \int u^{m_1+m_2-1}(1-u)^{m_3-1} v^{m_1-1}(1-v)^{m_2-1}\,du\,dv \quad \ldots\ldots(3)$$

over $$u^{m_1+m_2}(1-u)^{m_3} v^{m_1}(1-v)^{m_2} \geqq \kappa.$$

If $m_1 = m_2 = m_3 = m$, and m is integral or half-integral, this is expressible in terms of complete elliptic integrals; but even when $m = 1$ the reduction to standard form is tedious and I have not carried it out. However, Nair (1939) has shown how to obtain the exact distribution in the general case

$$m_1 = m_2 = \ldots = m_k.$$

It is of some interest to determine the exact distribution in other cases in order to check the accuracy of the approximate distribution discussed in § 5. A case which happens to be fairly easy to handle is

$$m_1 = m_2 = m, \quad m_3 = 2m.$$

From (3)

$$p = \frac{1}{B(m, m, 2m)} \int u^{2m-1}(1-u)^{2m-1} v^{m-1}(1-v)^{m-1}\,du\,dv$$

over $$u^2(1-u)^2 v(1-v) \geqq \kappa^{1/m}.$$

Putting $$u = \tfrac{1}{2}(1+x) \quad (-1 \leqq x \leqq 1),$$

$$v = \tfrac{1}{2}(1+y) \quad (-1 \leqq y \leqq 1),$$

we obtain

$$p = \frac{1}{2^{6m-2} B(m, m, 2m)} \int (1-x^2)^{2m-1}(1-y^2)^{m-1}\,dx\,dy$$

over $$(1-x^2)^2(1-y^2) \geqq 64\kappa^{1/m},$$

which is fairly easily expressible in terms of complete elliptic integrals when m is small and integral or half-integral. When $m = 1$, so that

$$m_1 = m_2 = 1, \quad m_3 = 2,$$

$$p = \tfrac{3}{8}\int (1-x^2)\,dx\,dy \text{ over } (1-x^2)^2(1-y^2) \geqq 64\kappa$$

$$= \tfrac{3}{4}\int \sqrt{\{(1-x^2)^2 - 64\kappa\}}\,dx \text{ over } (1-x^2)^2 \geqq 64\kappa$$

$$= \tfrac{3}{2}\int_0^a \sqrt{\{(a^2-x^2)(b^2-x^2)\}}\,dx,$$

where
$$a^2 = 1-8\sqrt{\kappa}, \quad b^2 = 1+8\sqrt{\kappa}.$$

This reduces to

$$p = \tfrac{1}{2}(a^2+b^2)\int_0^a \sqrt{\frac{b^2-x^2}{a^2-x^2}}\,dx - \tfrac{1}{2}b^2(b^2-a^2)\int_0^a \frac{dx}{\sqrt{\{(a^2-x^2)(b^2-x^2)\}}}$$

$$= b\int_0^{\frac{1}{2}\pi} \sqrt{(1-t^2\sin^2\phi)}\,d\phi - 8b\sqrt{\kappa}\int_0^{\frac{1}{2}\pi} \frac{d\phi}{\sqrt{(1-t^2\sin^2\phi)}},$$

where $t = a/b$. In the table in § 5 the value of $\theta = \arcsin t$ is given in order that the results may more readily be checked from tables of the complete elliptic integrals.

5. The semi-invariants of L and the approximate distribution of L when H_1 is true

We shall denote the sth semi-invariant of a chance variable Z by $\lambda_s(Z)$. If x is a $\Gamma(m)$ variable,

$$\lambda_s(\log x) = G_s(m),$$

where $G_s(m)$ is the sth derivative with respect to m of

$$G(m) = \log \Gamma(m).$$

From this we can easily show that the semi-invariants of

$$L = M\log(\Sigma x/M) - \Sigma\{m\log(x/m)\}$$

are
$$\lambda_1(L) = M\{G_1(M) - \log M\} - \Sigma\{m[G_1(m) - \log m]\},$$

$s > 1$,
$$\lambda_s(L) = (-)^s\{\Sigma m^s G_s(m) - M^s G_s(M)\}.$$

Using the asymptotic expansion of $G_s(m)$ and neglecting terms of order $1/m_r^2$, we obtain

$$\lambda_s(L^*) = \frac{k-1}{2}\,\Gamma(s),$$

where
$$L^* = \frac{L}{1+\alpha},$$

$$\alpha = \frac{1}{6(k-1)}\{\Sigma(1/m) - 1/M\}.$$

Hence L^* is approximately a $\Gamma\{\frac{1}{2}(k-1)\}$ variable. $2L^*$ is distributed approximately like χ^2 with $k-1$ degrees of freedom, which is Bartlett's result (1937, p. 274), with a different notation.

The corresponding result used for testing H_1' is that, when H_1' is true,

$$2L^* = \frac{2L'}{1+\alpha}$$

is distributed approximately like χ^2 with k degrees of freedom, where

$$\alpha = \frac{1}{6k}\Sigma(1/m).$$

In the tables below, P is the true probability of obtaining, when H_1 is true, a value of $2L^*$ as great as or greater than that shown. P' is the approximate probability calculated from the approximate distribution of $2L^*$. Although in practice we are concerned only with the upper tail of the distribution of $2L^*$, it is of some interest to see how the approximate distribution as a whole compares with the true distribution.† It will be noted that the approximation improves as α decreases. The meanings of the symbols x_1 and θ are explained in § 4.

TABLE I

Comparing true and approximate probability integrals

$m_1=m_2=\frac{1}{2}$, $\alpha=\frac{1}{2}$				$m_1=m_2=1$, $\alpha=\frac{1}{4}$			
x_1	$2L^*$	P	P'	x_1	$2L^*$	P	P'
0·42	0·01729	0·8977	0·8954	0·475	0·00401	0·95	0·9495
0·35	0·06287	0·8060	0·8020	0·45	0·01608	0·90	0·8991
0·27	0·15850	0·6957	0·6905	0·35	0·15090	0·70	0·6977
0·15	0·44890	0·5064	0·5029	0·25	0·46029	0·50	0·4975
0·05	1·10715	0·2871	0·2927	0·15	1·07735	0·30	0·2993
0·01	2·15262	0·1275	0·1423	0·05	2·65717	0·10	0·1031
0·001	3·68164	0·0403	0·0550	0·025	3·72464	0·05	0·0536
0·0001	5·21670	0·0127	0·0224	0·005	6·26726	0·01	0·0123
0·000019	6·32320	0·0056	0·0119	0·0025	7·37228	0·005	0·0066

$m_1=1$, $m_2=2$, $\alpha=\frac{7}{36}$				$m_1=1$, $m_2=\infty$, $\alpha=\frac{1}{3}$			
x_1	$2L^*$	P	P'	x_1	$2L^*$	P	P'
0·31	0·00631	0·9371	0·9367	0·91	0·00739	0·9318	0·9315
0·30	0·01303	0·9097	0·9091	0·87	0·01588	0·9002	0·8997
0·22	0·16997	0·6817	0·6801	0·70	0·09717	0·7561	0·7553
0·16	0·45502	0·5016	0·5000	0·40	0·54221	0·4625	0·4616
0·095	1·07829	0·2997	0·2991	0·20	1·38761	0·2385	0·2388
0·035	2·53527	0·1096	0·1113	0·10	2·40444	0·1195	0·1210
0·016	3·78064	0·0496	0·0518	0·04	3·87237	0·0472	0·0491
0·004	6·06129	0·0122	0·0138	0·01	6·19744	0·0115	0·0128
0·002	7·21519	0·0061	0·0072	0·004	7·75793	0·0045	0·0053

† See Bishop & Nair (1939), where the results of a very thorough investigation of the upper tail of the distribution are given. The computations of the present paper were completed before the publication of Bishop & Nair's paper.

TABLE I (*cont.*)

$m_1=m_2=5$, $\alpha=\frac{1}{20}$				$m_1=m_2=1$, $m_3=2$, $\alpha=\frac{3}{10}$			
x_1	$2L^*$	P	P'	θ	$2L^*$	P	P'
0·49	0·003809	0·95081	0·95079	7°	0·10005	0·9517	0·9512
0·48	0·015252	0·90177	0·90171	10°	0·20321	0·9043	0·9034
0·43	0·188518	0·66432	0·66415	19°	0·71677	0·7004	0·6988
0·39	0·472482	0·49205	0·49185	27°	1·40868	0·4952	0·4944
0·31	1·485259	0·22306	0·22295	36°	2·42738	0·2958	0·2971
0·25	2·739828	0·09785	0·09788	50°	4·53262	0·1001	0·1037
0·21	3·905482	0·04806	0·04813	57°	5·88759	0·0493	0·0527
0·14	6·958488	0·00828	0·00834	69°	9·02451	0·0093	0·0110
0·11	8·928707	0·00277	0·00281	75°	11·32588	0·0027	0·0035

6. Applications

Suppose that we have k samples, each drawn from a normal population of unknown mean and variance, and that we wish to test the hypothesis that the variances of the normal populations are all the same. Denote the number of members of the rth sample by n_r+1, and the squariance of this sample by S_r; then S_r is an unscaled $\Gamma(\frac{1}{2}n_r)$ variable, and the hypothesis to be tested is equivalent to the hypothesis H_1 applied to the variables S_r. Thus the appropriate criterion is†

$$L = \tfrac{1}{2}N \log (\Sigma S/\tfrac{1}{2}N) - \Sigma\{\tfrac{1}{2}n \log (S/\tfrac{1}{2}n)\},$$

where

$$N = \Sigma n.$$

A large value of L is significant.

$$2L = N \log (\Sigma S/N) - \Sigma\{n \log (S/n)\},$$

$$2L^* = \frac{2L}{1+\alpha},$$

where

$$\alpha = \frac{1}{6(k-1)}\left\{\Sigma \frac{1}{\frac{1}{2}n} - \frac{1}{\frac{1}{2}N}\right\} = \frac{\Sigma(1/n) - 1/N}{3(k-1)}.$$

When H_1 is true, $2L^*$ is distributed approximately like χ^2 with $k-1$ degrees of freedom.‡ From the tabulations in § 5 it is evident that even with very small samples the approximation is sufficiently good for us to use it to determine

† The difference between this expression, L, and the function L_1 as defined by Neyman & Pearson should be noted. The relation is referred to on p. 212 below as well as being discussed by Bishop & Nair (1939).

‡ Cf. Bartlett (1937, p. 274).

whether an observed value of L is significant or not. It may be noted that if we put

$$v_r = S_r/n_r,$$

the "estimated variance" derived from sample r, the expression for $2L$ is

$$2L = N \log \bar{v} - \Sigma(n \log v),$$

where

$$\bar{v} = \Sigma nv/N.$$

Hence L has its minimum value 0 when

$$v_1 = v_2 = \ldots = v_k,$$

and $\partial L/\partial v_r$ has the same sign as $v_r - \bar{v}$, and therefore L increases as the v diverge more and more from a common value. Thus the L test may be regarded as answering the question "Are the estimated variances $v_1, v_2, \ldots, v_k$ significantly different?"

If we wish to test the hypothesis that the variance of each population is unity, we obtain the appropriate criterion from the fact that this hypothesis is equivalent to the hypothesis H' applied to the variables $\frac{1}{2}S_r$. The criterion is therefore

$$L' = \Sigma \tfrac{1}{2}S - \tfrac{1}{2}N - \Sigma\{\tfrac{1}{2}n \log (\tfrac{1}{2}S/\tfrac{1}{2}n)\},$$

and we have

$$2L' = \Sigma S - N - \Sigma\{n \log (S/n)\},$$

$$2L^* = \frac{2L'}{1+\alpha}, \quad \alpha = \frac{\Sigma(1/n)}{3k}.$$

When H' is true, $2L^*$ is distributed approximately like χ^2 with k degrees of freedom.

It should be noted that the field of application of the L test is quite different from that of Fisher's z test, which is used in the analysis of variance. When, in the analysis of variance, we test for significance of treatment differences or significance of an interaction, the question we put is not "Are these several estimated variances significantly different?" but "Is this particular estimate, or are any of this particular group, significantly greater than that particular estimated variance (residual variance)?" The truth is that, when engaged in the analysis of variance, we are interested not in variances of normal populations but in *means* of populations. A straightforward treatment of an analysis of variance problem by the method of likelihood leads to a criterion function which, when the null hypothesis is true (treatments without real differences of effect, or no interaction, as the case may be), is certainly the ratio of two estimated variances; but the question raised is not "Is this ratio significantly different from 1?" but "Is this ratio significantly *greater* than 1?"† We are led to Fisher's z test or an equivalent, and not to the L test. It is true that in the analysis of variance we have certain squariances S_1, S_2, ... which, when the null hypothesis is true, are unscaled gamma variables with equal scale parameters, and so the corresponding L will have the distribution discussed above; but this would not justify the use of the value of L as a test for the null hypothesis. In devising a satisfactory test we must consider the state of

† That being so, it may be remarked in passing, to speak of a "negative interaction" in a way which implies that it has some significance is a breach of statistical good manners, it is questioning the referee's decision after having agreed to accept it as final.

affairs not only when the hypothesis to be tested is true but also when it is false. In this case, when the null hypothesis is false, some at least of S_1, S_2, ... are not unscaled gamma variables but have a different distribution, and so the proof that the test is unbiased no longer holds. When used outside the field in which it naturally arises the L test loses its theoretical satisfactoriness. If, for example, the totals for the various treatments were identical, the L test applied to variance due to treatments and residual variance would judge this significant, i.e. as indicating that the treatments produce real differences in effect, while the z test would rightly judge the result as non-significant. Improbable as such a result is, it is more probable when the null hypothesis is true than when it is false. It would seem that the L test must lose power by judging as significant results in which the variance due to treatments is small compared with the residual variance, and that the z test must be more powerful in this field than the L test, i.e. more likely to reject the null hypothesis when it is false. Wishart (1938) proposes to use the L test as a supplementary test in experiments of factorial design when the z test has given a non-significant result for treatments as a whole, but it seems unlikely that this combination of the tests will be as powerful as the z test used alone.

7. The Neyman-Pearson test

The Neyman-Pearson criterion function for testing the hypothesis that the k normal samples have been drawn from populations with the same variance is (in our notation)

$$\lambda_{H_1} = \frac{\Pi\{[S/(n+1)]^{\frac{1}{2}(n+1)}\}}{\{\Sigma S/(N+k)\}^{\frac{1}{2}(N+k)}},\dagger$$

small values of this being significant. Remembering that S_r is an unscaled $\Gamma(\frac{1}{2}n_r)$ variable, we see that this is equivalent to using the criterion function

$$K' = \frac{\Pi\{x^{m+\frac{1}{2}}\}}{(\Sigma x)^{M+\frac{1}{2}k}}$$

instead of

$$K = \frac{\Pi\{x^m\}}{(\Sigma x)^M}$$

for testing the hypothesis H_1 applied to the variables $x_1, x_2, \ldots, x_k$ of § 2.

Denoting by p' the probability that $K' \geqq \kappa'$, we can easily show that when

$$c_1 = c_2 = \ldots = c_k = c,$$

$$\frac{\partial p'}{\partial c_1} = \frac{1}{c\Pi\{\Gamma(m)\}} \int (z_1 - m_1)\, e^{-\Sigma z}\, \Pi\{z^{m-1}\}\, dz_1 \ldots dz_k$$

over the region

$$\frac{\Pi\{z^{m+\frac{1}{2}}\}}{(\Sigma z)^{M+k/2}} \geqq \kappa' \qquad \ldots\ldots(4)$$

† Neyman & Pearson have denoted $\lambda_{H_1}^{2/(N+k)}$ by L_1.

From consideration of the derivative with respect to z_1 of the function on the left-hand side of (4) it is obvious that a line parallel to the z_1 axis, which cuts the boundary of the conical region (4), cuts it in two points and all points between these lie within the region (4). Let ζ, ζ' $(\zeta < \zeta')$ be the z_1 co-ordinates of these points—they will be functions of $z_2, z_3, \ldots, z_k$. Consider

$$\int_\zeta^{\zeta'} z_1 e^{-\Sigma z}\, \Pi\{z^{m-1}\}\, dz_1.$$

Integrating by parts, we obtain

$$\left[-e^{-\Sigma z} z_1 \Pi\{z^{m-1}\}\right]_\zeta^{\zeta'} + m_1 \int_\zeta^{\zeta'} e^{-\Sigma z}\, \Pi\{z^{m-1}\}\, dz_1$$
$$= \left[\frac{-e^{-\Sigma z} z_1 (\Sigma z)^{M+k/2} \kappa'}{\Pi\{z^{\frac{1}{2}}\}}\right]_\zeta^{\zeta'} + m_1 \int_\zeta^{\zeta'} e^{-\Sigma z}\, \Pi\{z^{m-1}\}\, dz_1,$$

since when $z_1 = \zeta$ or ζ' the equality in (4) is satisfied. Hence

$$\int_\zeta^{\zeta'} (z_1 - m_1)\, e^{-\Sigma z}\, \Pi\{z^{m-1}\}\, dz_1 = \left[\frac{-e^{-\Sigma z} z_1 (\Sigma z)^{M+k/2} \kappa'}{\Pi\{z^{\frac{1}{2}}\}}\right]_\zeta^{\zeta'}$$
$$= \kappa' \int_\zeta^{\zeta'} \frac{e^{-\Sigma z} z_1 (\Sigma z)^{M+k/2}}{\Pi\{z^{\frac{1}{2}}\}} \left\{1 - \frac{M + \frac{1}{2}k}{\Sigma z} + \frac{1}{2z_1}\right\} dz_1.$$

Thus, when $c_1 = c_2 = \ldots = c_k = c$, we have

$$\frac{\partial p'}{\partial c'} = \frac{\kappa'}{c\Pi\{\Gamma(m)\}} \int \frac{e^{-\Sigma z} z_1 (\Sigma z)^{M+k/2}}{\Pi\{z^{\frac{1}{2}}\}} \left\{1 - \frac{M + \frac{1}{2}k}{\Sigma z} + \frac{1}{2z_1}\right\} dz_1 \ldots dz_k$$

over the region (4). By the method of § 4 we obtain

$$\frac{\partial p'}{\partial c_1} = \frac{\kappa' \Gamma(M)}{2c\Pi\{\Gamma(m)\}} \int_R \frac{1 - k y_1}{\Pi\{y^{\frac{1}{2}}\}} dy_1 \ldots dy_{k-1}$$

over the region R determined by

$$\Pi\{y^{m+\frac{1}{2}}\} \geqq \kappa',$$

where $\Sigma y = 1$ as before. A necessary condition for the K' test to be unbiased is

$$\frac{\partial p'}{\partial c_r} = 0, \quad (r = 1, 2, \ldots, k)$$

when $$c_1 = c_2 = \ldots = c_k,$$

and therefore for an unbiased test we must have

$$k \int_R \frac{y_r}{\Pi\{y^{\frac{1}{2}}\}} dy_1 \ldots dy_{k-1} = \int_R \frac{1}{\Pi\{y^{\frac{1}{2}}\}} dy_1 \ldots dy_{k-1} \quad (r = 1, 2, \ldots, k). \ldots\ldots (5)$$

It is obvious from symmetry and from the fact that $\Sigma y = 1$ that (5) is true when

$$m_1 = m_2 = \ldots = m_k = m.$$

In fact the K' test is then equivalent to the K test since

$$K' = K^{(m+1)/m},$$

and the K' test is therefore in that case unbiased. If the m are not all equal, suppose $m_1 > m_2$, then $a^{m_1+1} b^{m_2+1} \gtreqless b^{m_1+1} a^{m_2+1}$

according as $a \gtreqless b$.

Hence if P is any point in R with its y_1 co-ordinate less than its y_2 co-ordinate, the point P' obtained by interchanging the y_1 and y_2 co-ordinates of P will also lie in R, but if P is on the boundary of R (or sufficiently near it and inside R), and has its y_1 co-ordinate greater than its y_2 co-ordinate, the point P' will lie outside R.

Therefore $$\int_R \frac{y_1 - y_2}{\Pi\{y^{\frac{1}{2}}\}} dy_1 \dots dy_{k-1} > 0,$$

and so (5) cannot be true. Thus the K' test is biased when the m are not all equal. Considering the application to samples from normal populations, we see that the Neyman-Pearson test is biased unless the number in each sample is the same, in which case it is equivalent to the L test.

8. The distribution of K when H_1 is false

The probability that K is greater than or equal to κ is

$$p = \frac{1}{\Pi\{c^m \Gamma(m)\}} \int e^{-\Sigma(x/c)} \Pi\{x^{m-1}\} dx_1 \dots dx_k$$

over the region where $$\frac{\Pi\{x^m\}}{(\Sigma x)^M} \geqq \kappa,$$

which is equal to $$\frac{1}{\Pi\{\Gamma(m)\}} \int e^{-\Sigma z} \Pi\{z^{m-1}\} dz_1 \dots dz_k$$

over $$\frac{\Pi\{c^m z^m\}}{(\Sigma cz)^M} \geqq \kappa.$$

By the method of § 4 we obtain

$$p = \frac{\Gamma(M)}{\Pi\{\Gamma(m)\}} \int \Pi\{y^{m-1}\} dy_1 \dots dy_{k-1}$$

over $$\frac{\Pi\{c^m y^m\}}{(\Sigma cy)^M} \geqq \kappa,$$

where $\Sigma y = 1$. When $k = 2$, this can be evaluated by means of the tables of the incomplete beta function. It can be evaluated also in some simple cases of $k = 3$. An approximation to the distribution of L in the general case is being investigated.

SUMMARY

Tests of certain hypotheses concerning location and scale parameters are developed. These tests are unbiased and are applicable to chance variables with any continuous distributions. An application to the comparison of variances of normal variables yields Bartlett's test, which is thus shown to be unbiased. The Neyman-Pearson variances test is shown to be biased except when the samples are all of the same size.

REFERENCES

BARTLETT, M. S. (1937). *Proc. Roy. Soc.* A, **160**, 268–82.
BISHOP, D. J. & NAIR, U. S. (1939). *J. R. Statist. Soc. Suppl.* **6**, 89–99.
NAIR, U. S. (1939). *Biometrika*, **30**, 274–94.
NEYMAN, J. & PEARSON, E. S. (1931). *Bull. int. Acad. Cracovie*, A, 460–81.
—— —— (1936). *Statist. Res. Mem.* **1**, 1–37.
PITMAN, E. J. G. (1937). *Proc. Camb. Phil. Soc.* **33**, 212–22.
WISHART, J. (1938). *J. Agric. Sci.* **28**, 299–306.

NOTE

[As one of the authors of the original L_1 test and of the conception of "bias" in connexion with tests of statistical hypotheses, it is perhaps permissible for me to say that Prof. Neyman and I had for some time realized that the L_1 test was slightly biased when applied to samples of unequal size. The difference between this test and that put forward by M. S. Bartlett, and discussed more fully by Prof. Pitman above, may be expressed by saying that the Bartlett test weights the sums of squares with degrees of freedom where the L_1 test had used sample sizes as weights. Intuitionally the former weighting seems the more appropriate to take but, as in certain problems of estimation with which the reader will probably be familiar, the application of the principle of maximum likelihood, a form of which Neyman and I had used, introduces sample sizes rather than degrees of freedom. In 1936 B. L. Welch (*Statistical Research Memoirs*, **1**, 53) had suggested the use of a test function in the form of the ratio of a weighted geometric mean to weighted arithmetic mean of sums of squares, the weights being adjustable at will; but it was Bartlett who definitely advocated the weighting with degrees of freedom.

In a Ph.D. Thesis for the University of London, completed in 1937, only a part of which has been published (U. S. Nair, 1939), Dr Nair showed that in a particular case with $k=3$ and sample sizes of 5, 10 and 15 the L_1 test was certainly biased. During the past winter he also sent me a mathematical proof showing that in the simplest case, with $k=2$, the Bartlett test was unbiased. Of two or three people investigating the problem, Prof. Pitman has however been the first to reach a complete solution. His proof is of great generality; not only does it cover the case of any number of samples or groups of data, but it is derived from a very interesting general approach not limited to the particular problem of testing for homogeneity of standard deviations among normally distributed variables. E. S. P.]

MISCELLANEA

(i) On the Calculation of the Cumulants of the χ-distribution

By N. L. JOHNSON and B. L. WELCH

In a recent investigation it has been found necessary to calculate the cumulants of the χ-distribution with considerable accuracy. This may be done, of course, by calculating the moments of χ about zero and then correcting in the usual way. It was found, however, that a substantial saving of labour may be effected by another method which will be indicated below. This method is essentially the same as that employed by K. Pearson (1915) to calculate the constants of the standard deviation distribution, being extended to give the fifth and sixth cumulants.

The distribution of χ with f degrees of freedom is given by

$$p(\chi)\,d\chi = \frac{1}{2^{\frac{1}{2}f-1}\Gamma(\frac{1}{2}f)}\,e^{-\frac{1}{2}\chi^2}\,\chi^{f-1}\,d\chi.$$

The kth moment of χ about zero is

$$\mu'_k = 2^{\frac{1}{2}k}\frac{\Gamma\{\frac{1}{2}(f+k)\}}{\Gamma(\frac{1}{2}f)},$$

whence it is seen that the even moments are integers and the odd moments integral multiples of μ'_1. The kth moment μ_k about the mean may therefore be expressed as a polynomial of the kth degree in μ'_1 with integral coefficients. Hence we obtain the following formulae for the first six cumulants:

$$\kappa_1 = \mu'_1,$$

$$\kappa_2 = f - \kappa_1^2,$$

$$\kappa_3 = (-2f+1)\,\kappa_1 + 2\kappa_1^3,$$

$$\kappa_4 = (-2f^2+2f) + (8f-4)\,\kappa_1^2 - 6\kappa_1^4,$$

$$\kappa_5 = (16f^2-16f+3)\,\kappa_1 + (20-40f)\,\kappa_1^3 + 24\kappa_1^5,$$

$$\kappa_6 = (16f^3-24f^2+8f) + (-136f^2+136f-28)\,\kappa_1^2 + (240f-120)\,\kappa_1^4 - 120\kappa_1^6.$$

By substituting $(f-\kappa_2)$ for κ_1^2 we may now obtain formulae for κ_3, κ_4, κ_5 and κ_6 involving only the first power of κ_1 and powers of κ_2 up to the third. Thus

$$\kappa_3 = \kappa_1(1-2\kappa_2),$$

$$\kappa_4 = -2f(1-2\kappa_2) + 4\kappa_2 - 6\kappa_2^2,$$

$$\kappa_5 = \kappa_1[4f(1-2\kappa_2) + 3 - 20\kappa_2 + 24\kappa_2^2],$$

$$\kappa_6 = -8f^2(1-2\kappa_2) - f(20-104\kappa_2+120\kappa_2^2) + (28\kappa_2 - 120\kappa_2^2 + 120\kappa_2^3).$$

The facts that the multiplier of the highest power of f is always a multiple of $(1-2\kappa_2)$ and that κ_3 tends to $\frac{1}{2}$ as f tends to infinity, suggest that some simplification may result by expressing

these cumulants in terms of powers of $(1-2\kappa_2)$ instead of powers of κ_2. Putting $(1-2\kappa_2)=\alpha$ we find

$$\kappa_3 = \kappa_1\alpha,$$

$$\kappa_4 = \tfrac{1}{2}-(2f-1)\alpha-\tfrac{3}{2}\alpha^2,$$

$$\kappa_5 = \kappa_1[-1+(4f-2)\alpha+6\alpha^2]$$

$$= \kappa_1[-2\kappa_4+3\alpha^2],$$

$$\kappa_6 = (2f-1)-(8f^2-8f-1)\alpha-15(2f-1)\alpha^2-15\alpha^3$$

$$= 2(2f-1)\kappa_4+3\alpha-12(2f-1)\alpha^2-15\alpha^3.$$

The calculation of the higher cumulants by means of these formulae is rendered very easy once κ_1 is known. For f small κ_1 may be calculated from the following formulae:

$$f \text{ even} \quad \kappa_1 = \sqrt{\frac{\pi}{2}}\frac{(f-1)(f-3)\ldots 3\cdot 1}{(f-2)(f-4)\ldots 2},$$

$$f \text{ odd} \quad \kappa_1 = \sqrt{\frac{2}{\pi}}\frac{(f-1)(f-3)\ldots 4\cdot 2}{(f-2)(f-4)\ldots 3\cdot 1}.$$

(To 15 decimal places $\sqrt{\frac{2}{\pi}} = 0{\cdot}79788\,45608\,02865,$

and $\sqrt{\frac{\pi}{2}} = 1{\cdot}25331\,41373\,15500.$)

For f large Stirling's expansion for the factorial may be used, giving

$$\log\kappa_1 = \tfrac{1}{2}\log f - \frac{1}{4f}+\frac{1}{24f^3}-\frac{1}{20f^5}+\frac{17}{112f^7}+\ldots,$$

$$\kappa_1 = \sqrt{f}\left\{1-\frac{1}{4f}+\frac{1}{32f^2}+\frac{5}{128f^3}-\frac{21}{2048f^4}-\frac{399}{8192f^5}+\frac{869}{65536f^6}+\ldots\right\}.$$

The above formulae allow the calculation of the cumulants very quickly with as much accuracy as is ever likely to be required, but the following short tables, giving only a few figures, will be useful for many practical purposes. Note that in the second table the cumulants are multiplied by powers of f in order to increase the accuracy of harmonic interpolation.

f	κ_1	κ_2	κ_3	κ_4	κ_5	κ_6
1	0·797885	0·363380	0·218014	0·114771	−0·004438	−0·152659
2	1·253314	0·429204	0·177460	0·045149	−0·037791	−0·068652
3	1·595769	0·453521	0·148340	0·022247	−0·029635	−0·029175
4	1·879971	0·465708	0·128935	0·012860	−0·021825	−0·014156
5	2·127692	0·472926	0·115210	0·008271	−0·016482	−0·007712
6	2·349964	0·477669	0·104953	0·005730	−0·012867	−0·004596
7	2·553231	0·481014	0·096954	0·004189	−0·010346	−0·002935
8	2·741625	0·483494	0·090506	0·003190	−0·008526	−0·001978

f	$24/f$	$\kappa_1/\sqrt{f}$	κ_2	$\kappa_3\sqrt{f}$	$\kappa_4 f^2$	$\kappa_5(\sqrt{f})^3$	$\kappa_6 f^3$
6	4	0·959369	0·477669	0·257082	0·20627	−0·18910	−0·9927
8	3	0·969311	0·483494	0·255989	0·20413	−0·19291	−1·0128
12	2	0·979406	0·489176	0·254427	0·20025	−0·19396	−1·0114
24	1	0·989640	0·494686	0·252418	0·19464	−0·19211	−0·9859
∞	0	1·000000	0·500000	0·250000	0·18750	−0·18750	−0·9375

REFERENCE

PEARSON, KARL (1915). Appendix to Papers by "Student" and R. A. Fisher. Editorial. *Biometrika*, **10**, 522–9.

(ii) Note on Discriminant Functions

BY B. L. WELCH, PH.D.

RECENT years have seen a widespread development of the methods of multivariate analysis. Of the problems which have been discussed one of the simplest is that of apportioning individuals with several measured characters into one or other of two completely specified population groups. Thus if $x_1, x_2, \ldots, x_q$ are the q characters measured upon each individual, and Π_1 and Π_2 denote the two possible populations, we suppose known the probability distributions $p_1 = p(x_1, x_2, \ldots, x_q \mid \Pi_1)$ and $p_2 = p(x_1, x_2, \ldots, x_q \mid \Pi_2)$. In particular it is frequently assumed that Π_1 and Π_2 have the same set of variances and covariances and differ only in the mean values of the characters. In this case R. A. Fisher (1936) has considered the problem of choosing the best linear function X of $x_1, x_2, \ldots, x_q$ to form the basis of classification of the individuals. The solution was obtained by maximizing the absolute value of the ratio of $E(X \mid \Pi_1) - E(X \mid \Pi_2)$ to the standard deviation of X. This is certainly the best discriminant function of any kind, whether linear or not, provided p_1 and p_2 are of the multivariate normal form.

However, without making any assumptions of normality or equality of variances and covariances, the problem of obtaining the best function to discriminate between two completely specified populations may still be solved. The function is simply the ratio of the two probability laws p_1/p_2. This is almost self-evident, but the following demonstrations may be useful.

Suppose in the first instance that it is possible to assess *a priori* probabilities ω_1 and ω_2 that an individual will belong to Π_1 or Π_2 respectively. This was possible, for instance, in the problem of sexing human mandibles, discussed by E. S. Martin (1936). Then, if measurements $x_1, x_2, \ldots, x_q$ are made on the individual, the *a posteriori* probabilities that it will belong to Π_1 or Π_2 will be $\omega_1 p_1/(\omega_1 p_1 + \omega_2 p_2)$ and $\omega_2 p_2/(\omega_1 p_1 + \omega_2 p_2)$, respectively. Now if it is equally important that an individual really belonging to Π_1 should not be classified as belonging to Π_2 and vice versa, then we should assign it to Π_1 provided the *a posteriori* probability $\omega_1 p_1/(\omega_1 p_1 + \omega_2 p_2)$ is greater than $\frac{1}{2}$. Any other assignation would increase the overall chance of misclassifications. We should therefore classify into Π_1 when

$$\frac{p_1}{p_2} > \frac{\omega_2}{\omega_1}. \qquad \ldots\ldots(1)$$

Whatever the prior probabilities the discriminant function to be calculated from the

observed measurements is therefore the ratio of the probability laws; the criterion level to which this function is to be referred does, however, depend on the prior probabilities.

If it is not possible or even appropriate to assess prior probabilities then the determination of a rule of classification must be made to depend on other conditions. For instance, it may be required (*a*) that if an individual really belongs to Π_1 there should be the same chance of it being misclassified as there would be if it belonged to Π_2 and (*b*) that this common chance should be a minimum. If $x_1, x_2, \ldots, x_q$ be represented by a point in q-dimensional space, any rule of classification involves the choice of a region R in this space such that an individual falling into R will be classified into Π_1. The above conditions (*a*) and (*b*) may then be written

$$\int_R p_2 d\tau = 1 - \int_R p_1 d\tau, \qquad \ldots\ldots(2)$$

and

$$\int_R p_2 d\tau \text{ to be a minimum.} \qquad \ldots\ldots(3)$$

The problem is therefore to minimize $\int_R p_2 d\tau$ subject to the condition that $\int_R (p_2 + p_1) d\tau$ is equal to unity. A straightforward application of the calculus of variations shows that R consists of points such that

$$p_1/p_2 > k, \qquad \ldots\ldots(4)$$

where k is chosen to satisfy (2). The discriminant function is the same as before although the criterion level will in general be different.

Whatever the conditions which are imposed, provided they are formulated on a probability basis, it would appear that we shall be led to the same discriminant function. To take a further instance we may mention that in discussing the theory of testing statistical hypothesis, J. Neyman & E. S. Pearson (1932) have considered the problem of testing the hypothesis that a sample comes from a certain population when there is only one possible alternative population. They imposed the conditions (*a*) that the chance of rejection when the hypothesis is true should be a small specified probability ϵ, and (*b*) that the chance of rejection when the alternative hypothesis is true should be a maximum. The solution depended only on the ratio of the probability laws of the sample on the two hypotheses. This ratio has been termed by Neyman & Pearson a "likelihood ratio" and sometimes a "test criterion". It seems to me that "discriminant function" is a better term for this ratio and that "criterion" is best reserved for the particular value of the function which is considered critical. This value will, as we have seen, depend on the manner in which the problem of deciding between the two populations is formulated.

When the populations Π_1 and Π_2 are normal multivariate, differing only in their mean values, it is easy to see that the ratio p_1/p_2 leads to the same linear function of $x_1, x_2, \ldots, x_q$ as is obtained by maximizing $\{E(X \mid \Pi_1) - E(X \mid \Pi_2)\}^2/\sigma_X^2$. For if

$$p(x_1, x_2, \ldots, x_q \mid \Pi_t) = \left(\frac{1}{\sqrt{(2\pi)}}\right)^q \frac{1}{W^{\frac{1}{2}}} \exp\left[-\tfrac{1}{2} \sum_{i,j} W^{ij}(x_i - \theta_{it})(x_j - \theta_{jt})\right] \quad (t = 1, 2), \qquad \ldots\ldots(5)$$

where W is the determinant of the covariances and W^{ij} are the elements of the reciprocal matrix, then

$$\log \frac{p_1}{p_2} = -\tfrac{1}{2} \sum_{i,j} W^{ij}\{(x_i - \theta_{i1})(x_j - \theta_{j1}) - (x_i - \theta_{i2})(x_j - \theta_{j2})\}$$

$$= \sum_{i,j} W^{ij}(\theta_{j1} - \theta_{j2}) x_i - \tfrac{1}{2} \sum_{i,j} W^{ij}(\theta_{i1}\theta_{j1} - \theta_{i2}\theta_{j2}).$$

$\sum_{i,j} W^{ij}(\theta_{j1} - \theta_{j2}) x_i$ is the same linear function as is obtained from the other approach (R. A. Fisher, 1936).

REFERENCES

FISHER, R. A. (1936). "The use of multiple measurements in taxonomic problems." *Ann. Eugen., Lond.*, **7**, 179–88.

MARTIN, E. S. (1936). "A study of an Egyptian series of mandibles, with special reference to mathematical methods of sexing." *Biometrika*, **28**, 149–72.

NEYMAN, J. & PEARSON, E. S. (1932). "On the problem of the most efficient tests of statistical hypotheses." *Philos. Trans.* A, **231**, 289–337.

(iii) Principles of Genetics. By E. W. SINNOTT AND L. C. DUNN. Fourth Edition. London: McGraw-Hill Publishing Company, 1939. Price 21*s*.

This book, which has gone through three editions since 1925, needs no recommendation to geneticists. It is so good on the whole that a short review can be mainly concerned with its defects. Mendel did not use red-flowered peas as stated on p. 41, but purple-flowered (*rot-violett*) and, if he had crossed red and white, would probably have got a purple F_1. The treatment of biometrical methods on pp. 138–51 is unsatisfactory. For example, the account of the significance of a difference of means takes no account of "Student's" work, and is therefore just 31 years out-of-date. There is no need to point out the serious nature of the errors which may arise from this neglect. And since Bateson's original definition of genetics included a study of populations, I cannot but feel that the authors have devoted too little space to this topic. Nevertheless, as an elementary account of modern genetics, the book can be strongly recommended.

J. B. S. H.

(iv) Corrections to formulae in papers on the moments of χ^2. By J. B. S. HALDANE.

THE following errata should be noted:

Biometrika, **29**, 138, equations (3):

For $\mu_3 = \kappa_3 = 8 + 2(11k - 6)\, s^{-1} + (k^2 - 30k + 120)\, s^{-2}$,

read $\mu_3 = \kappa_3 = 8 + 2(11k - 56)\, s^{-1} + (k^2 - 30k + 120)\, s^{-2}$.

Biometrika, **29**, 390, antepenultimate line:

For $\kappa_6 = 3840 + 21{,}300m^{-1} + 249{,}600m^{-2} + 69{,}160m^{-3} + 2004m^{-4} + m^{-5}$,

read $\kappa_6 = 3840 + 124{,}800m^{-1} + 249{,}600m^{-2} + 69{,}160m^{-3} + 2004m^{-4} + m^{-5}$.

Biometrika, **29**, 391, equations (1):

For $\kappa_6 = 3840n + 21{,}300R_1 + 249{,}600R_2 + 69{,}160R_3 + 2004R_4 + R_5$,

read $\kappa_6 = 3840n + 124{,}800R_1 + 249{,}600R_2 + 69{,}160R_3 + 2004R_4 + R_5$.

And line 20:

For $+ 5814{\cdot}04n^2 + 18{,}640{\cdot}492n$,

read $+ 5814{\cdot}04n^2 + 39{,}340{\cdot}492n$.

J. B. S. H.

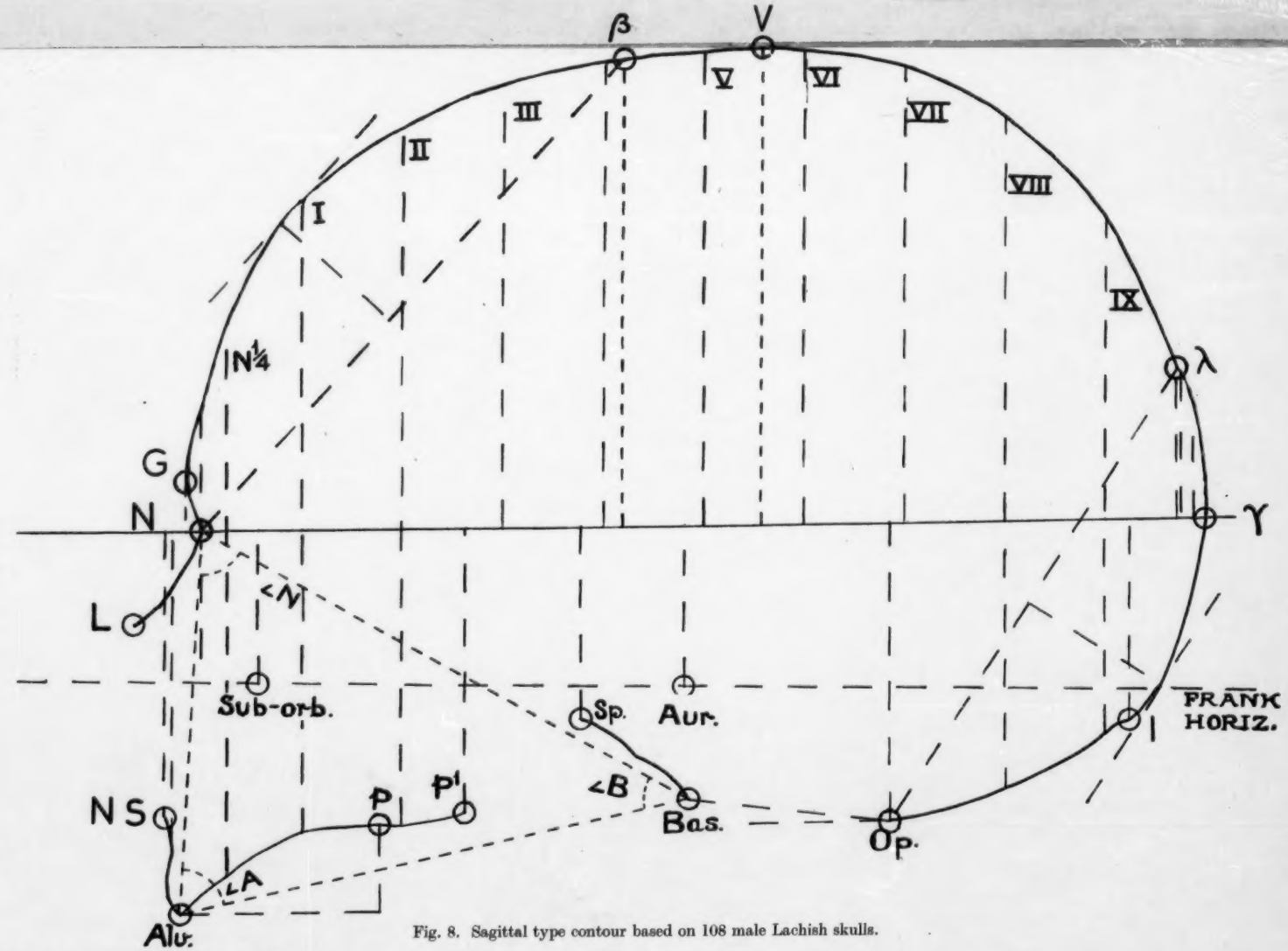


Fig. 8. Sagittal type contour based on 108 male Lachish skulls.

Fig. 8. Stiegler type control based on 108 male Lacerta stirpis.

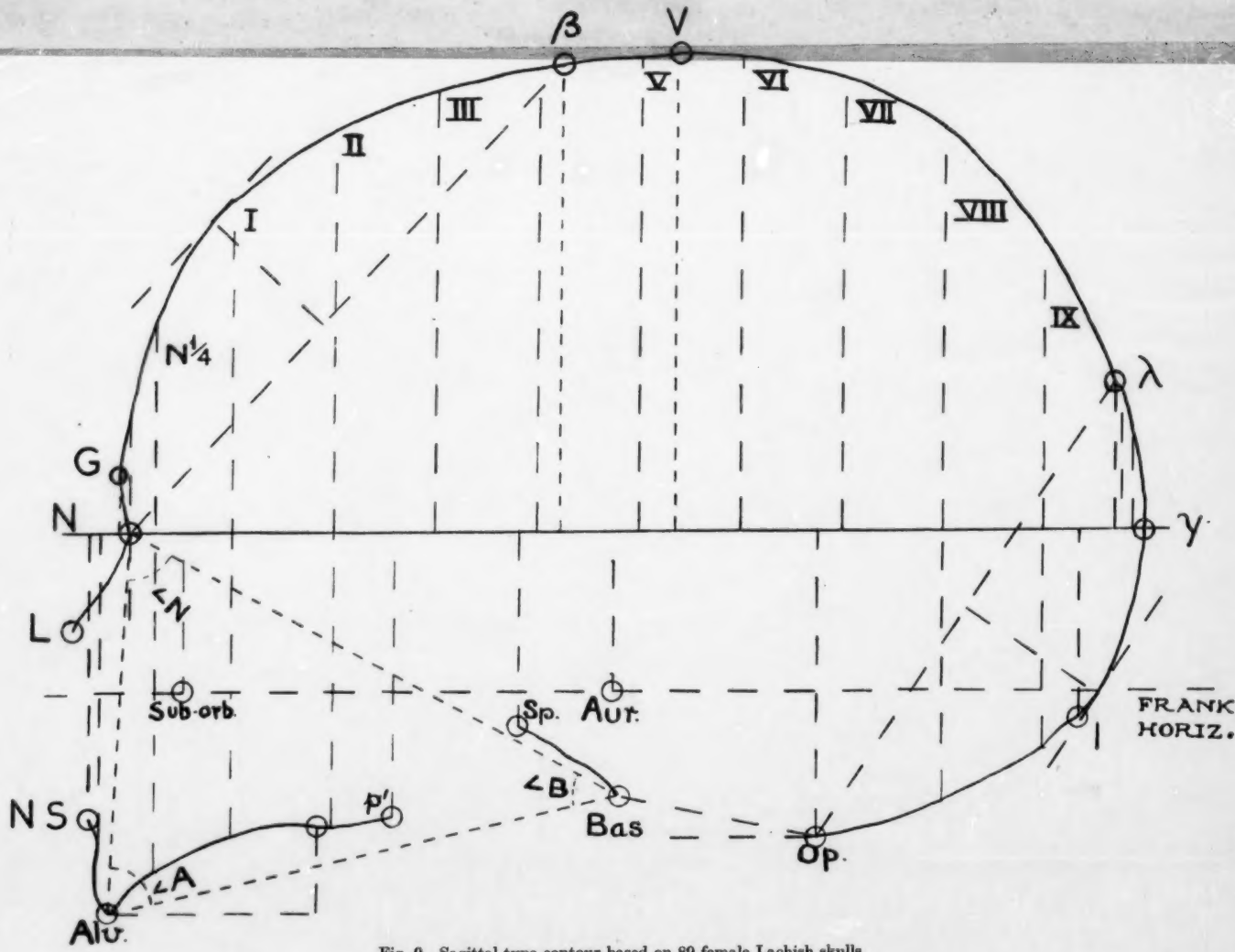


Fig. 9. Sagittal type contour based on 89 female Lachish skulls.

Fig. 9. Spatial type contour based on 89 female Turkish skulls.

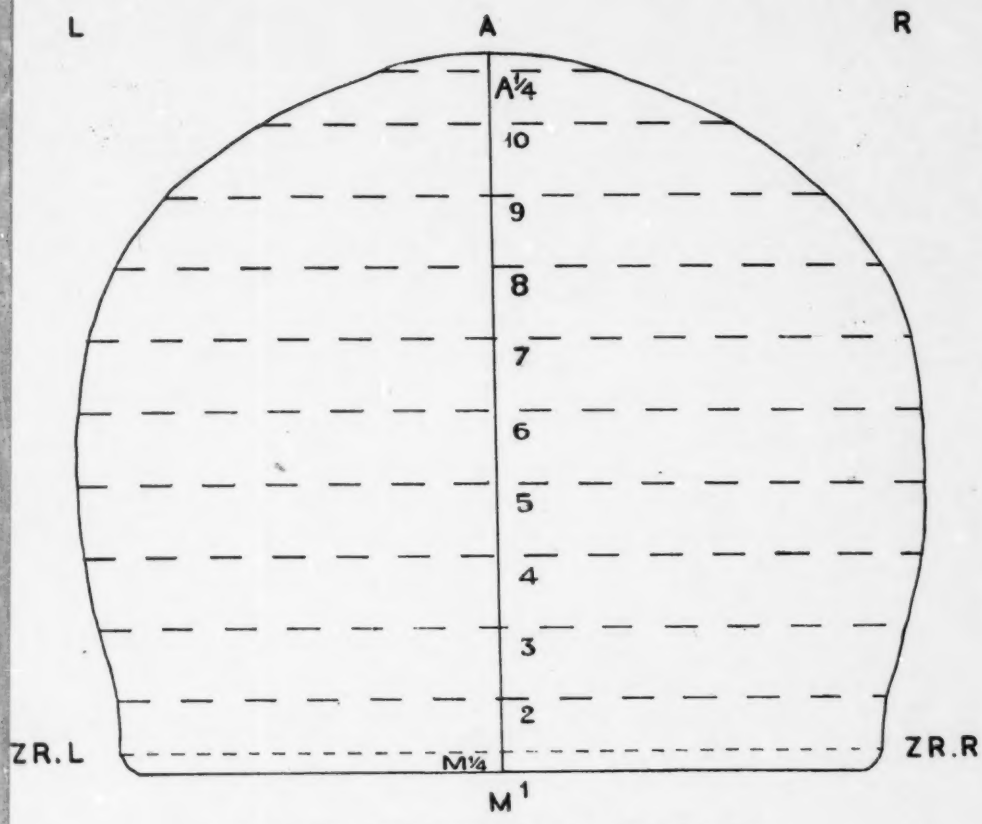


Fig. 4. Transverse type contour based on 108 male Lachish skulls.

Fig. 4. Transverse type contour based on 108 male Lachish skulls.

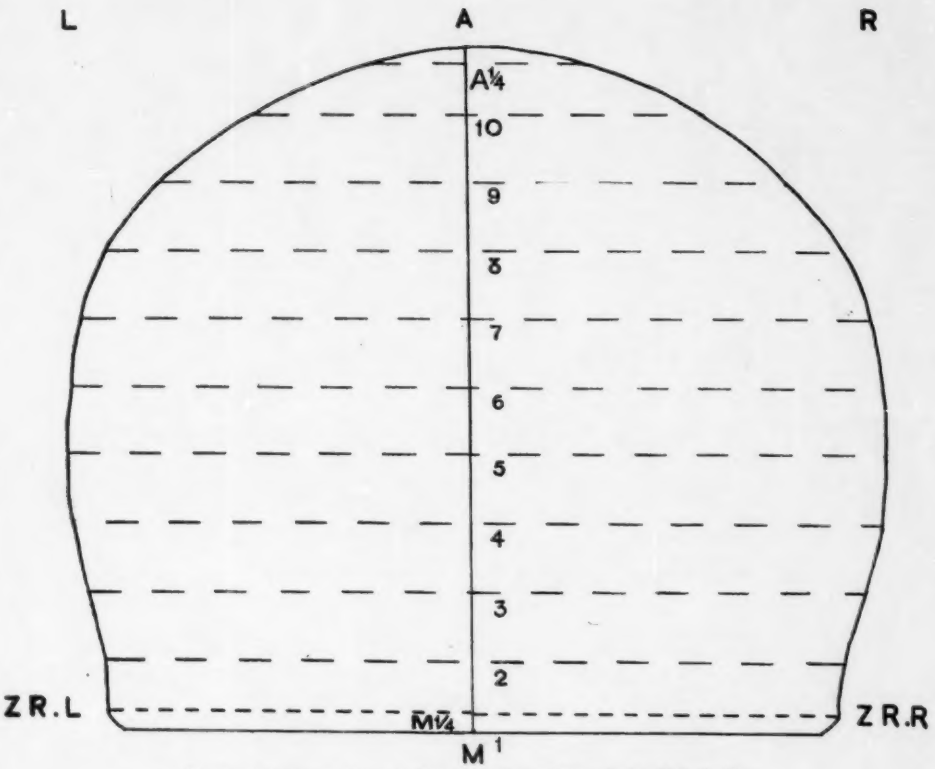


Fig. 5. Transverse type contour based on 89 female Lachish skulls.

Fig. 5. Transverse type contour based on 89 female Lachish skulls.

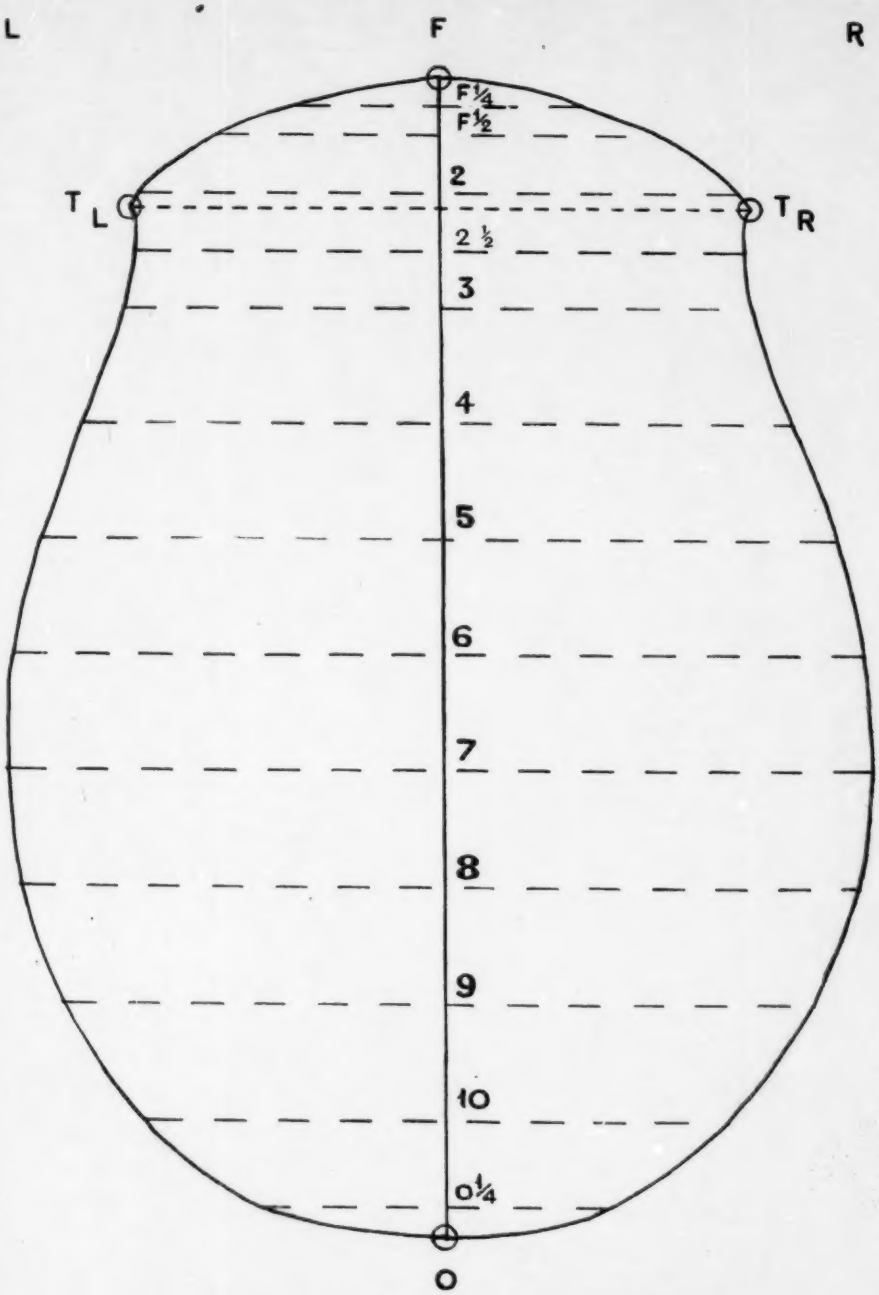


Fig. 6. Horizontal type contour based on 108 male Lachish skulls.

Fig. 6. Horizontal type contour based on 108 male Lachish skulls.

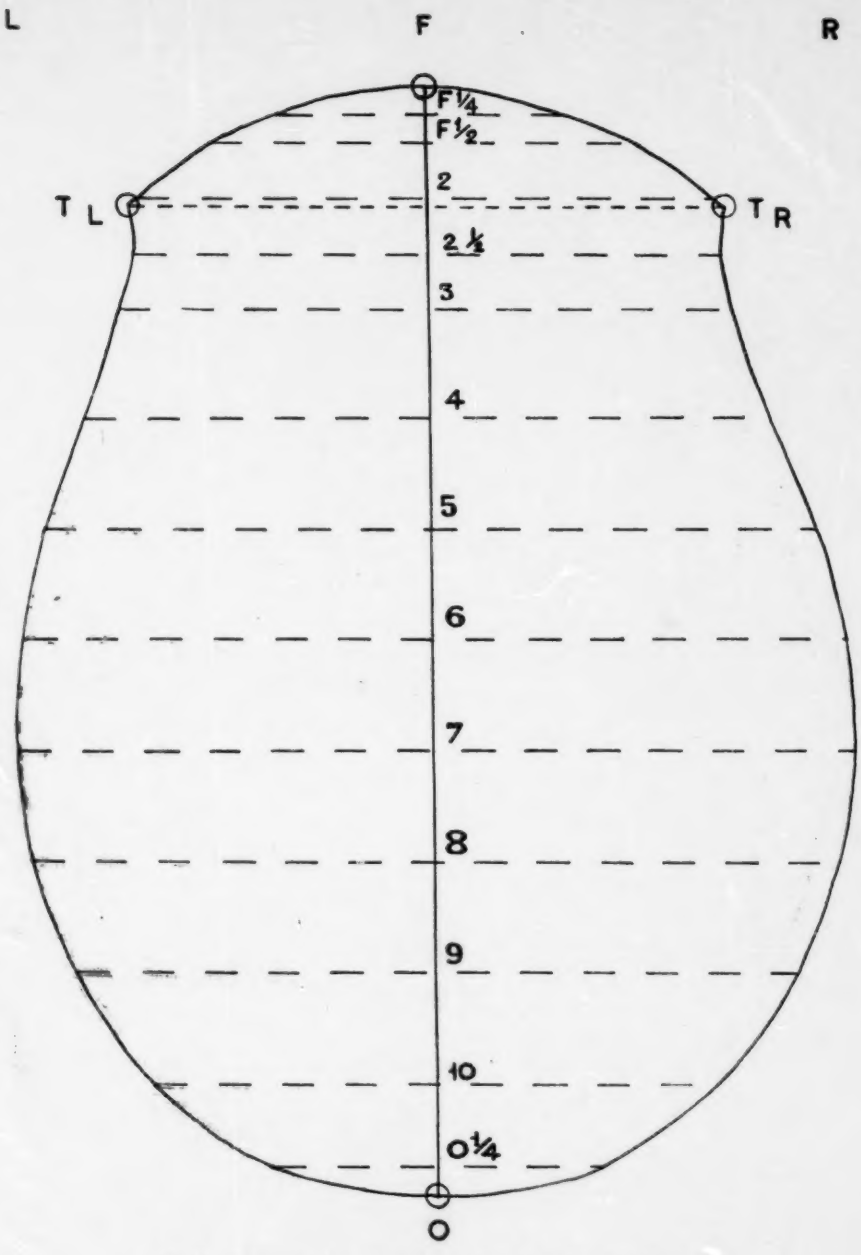


Fig. 7. Horizontal type contour based on 89 female Lachish skulls.

Fig. 7. Horizontal type contour based on 89 female Lachish skulls.